Self-Mix Interferometry

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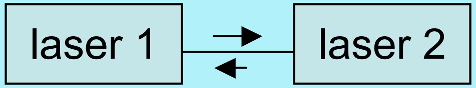
Summary

- Introduction
- Coupling regimes
- Weak coupling and Self-Mixing
- Developing Interferometers
- Experiments
- Conclusions

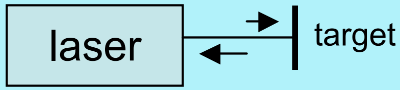
Introduction

◇ Coupling phenomena can take place:

- between two laser sources (and we call them *mutual-coupling* or *injection*)



- in a single source, as self-coupling of field to a remote target (and we call it *self-mixing*)



◇ The level of coupling may be *weak* (fraction of power interacting: down to 10^{-8}) or *strong* (fraction of power up to a few 10^{-2})

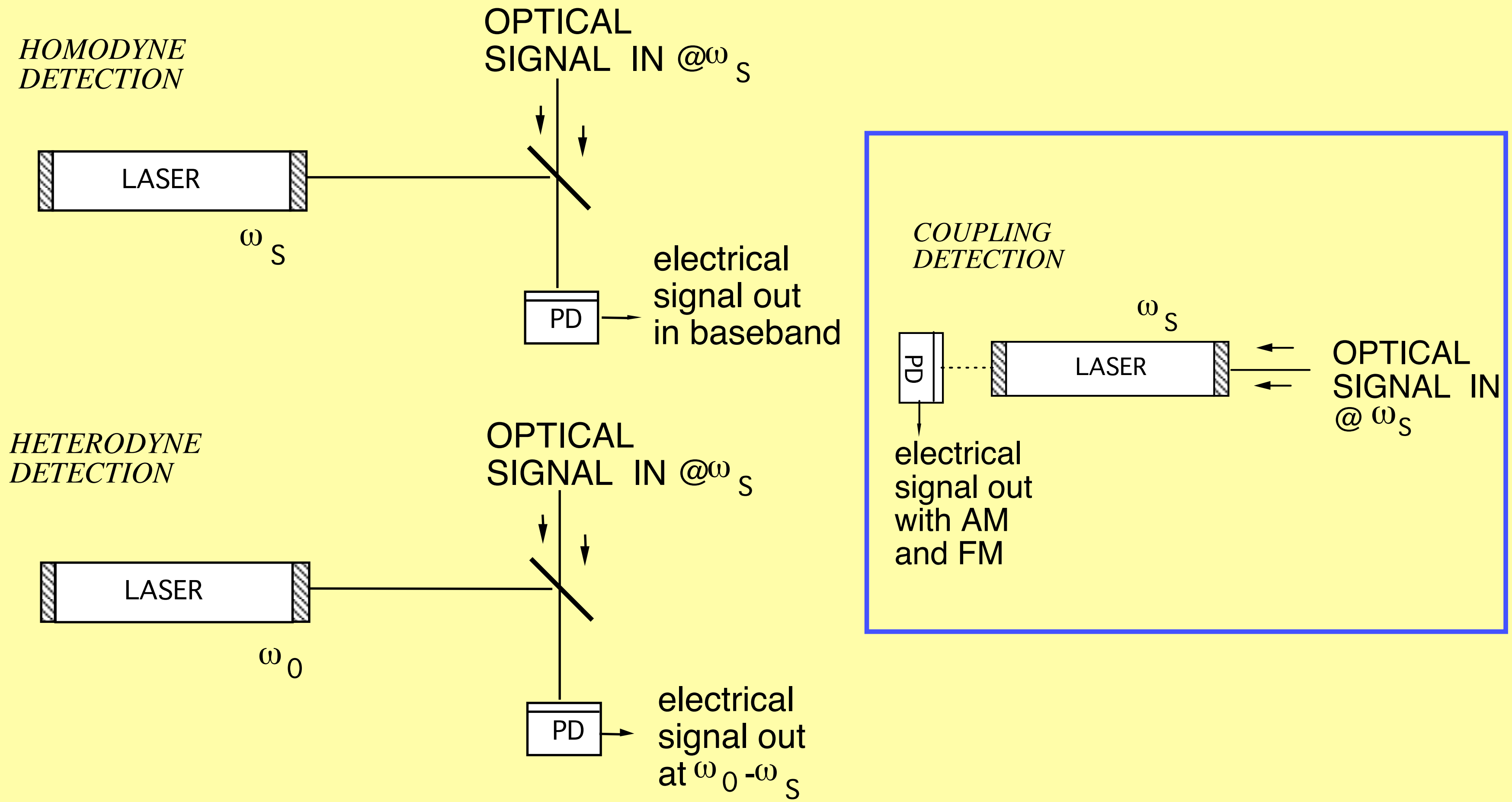
- At *weak level* we observe AM and FM *modulations* of the cavity field, carrying information on the external perturbation (coupled signal in mutual coupling) or amplitude/phase of returning field (in self-mixing)

→ *interferometer* → *coherent (injection) detection*

- At *strong* levels we get chaos, both in mutual coupling and self- mixing schemes)

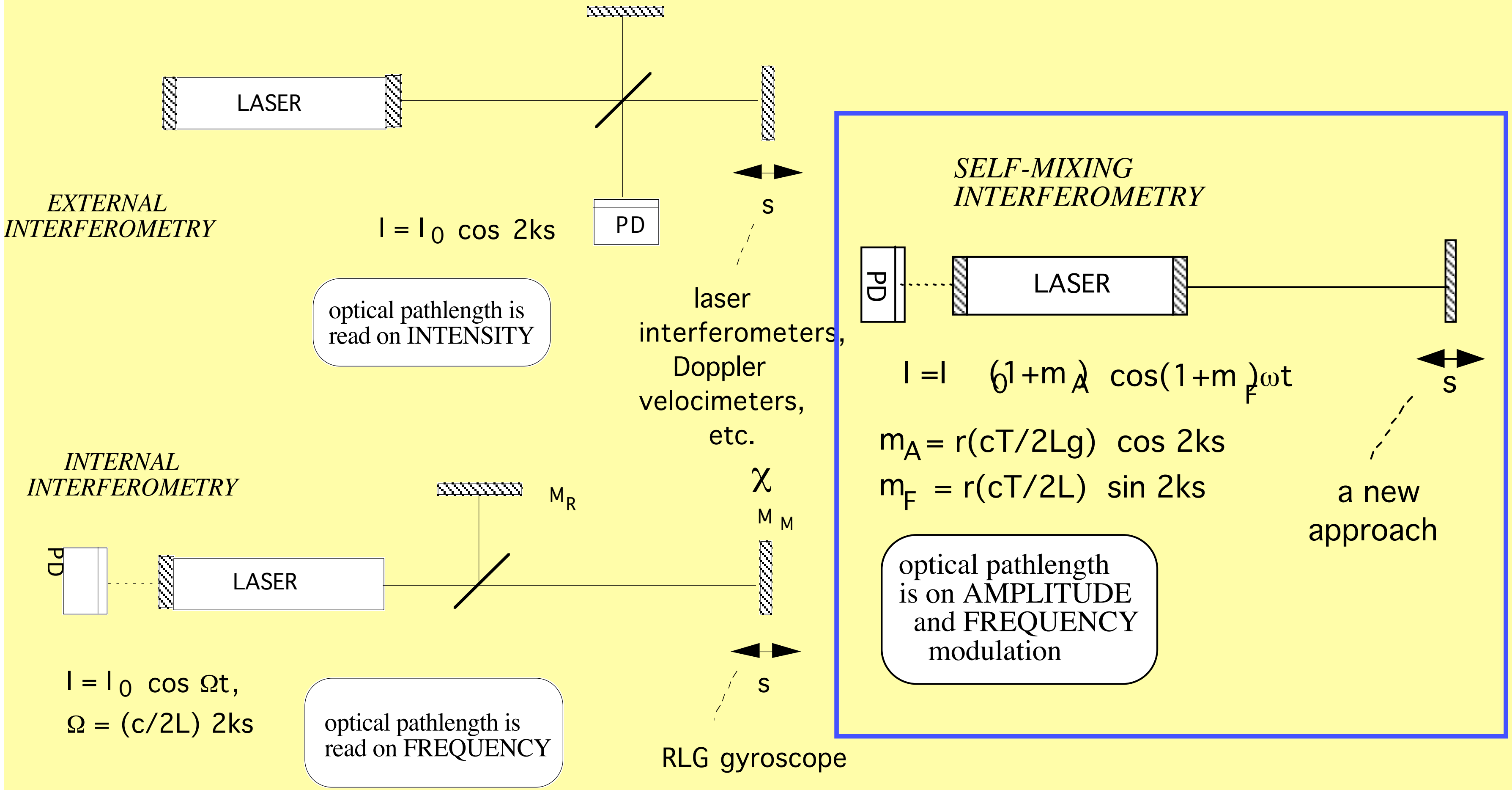
→ *cryptography*

Mutual coupling as a new configuration of coherent detection



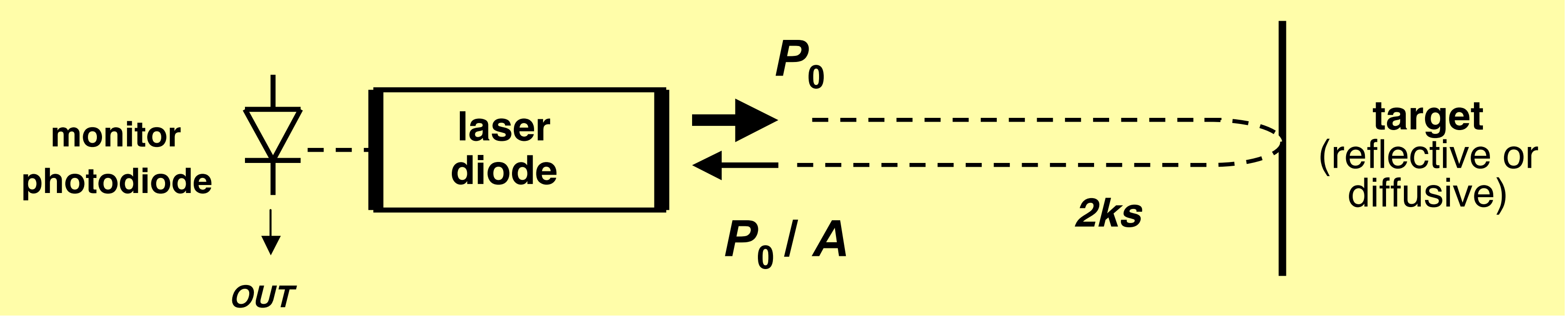
Despite the output signal is different, coupling detection belongs to schemes of coherent detection because dependence is on *field* and it always works in quantum-limited regime

Self-mixing as a new configuration of interferometer



Compared to other schemes of interferometry, self-mixing yields a different output signal yet information contained in it is the same, a sine/cosine function of optical phase length 2ks

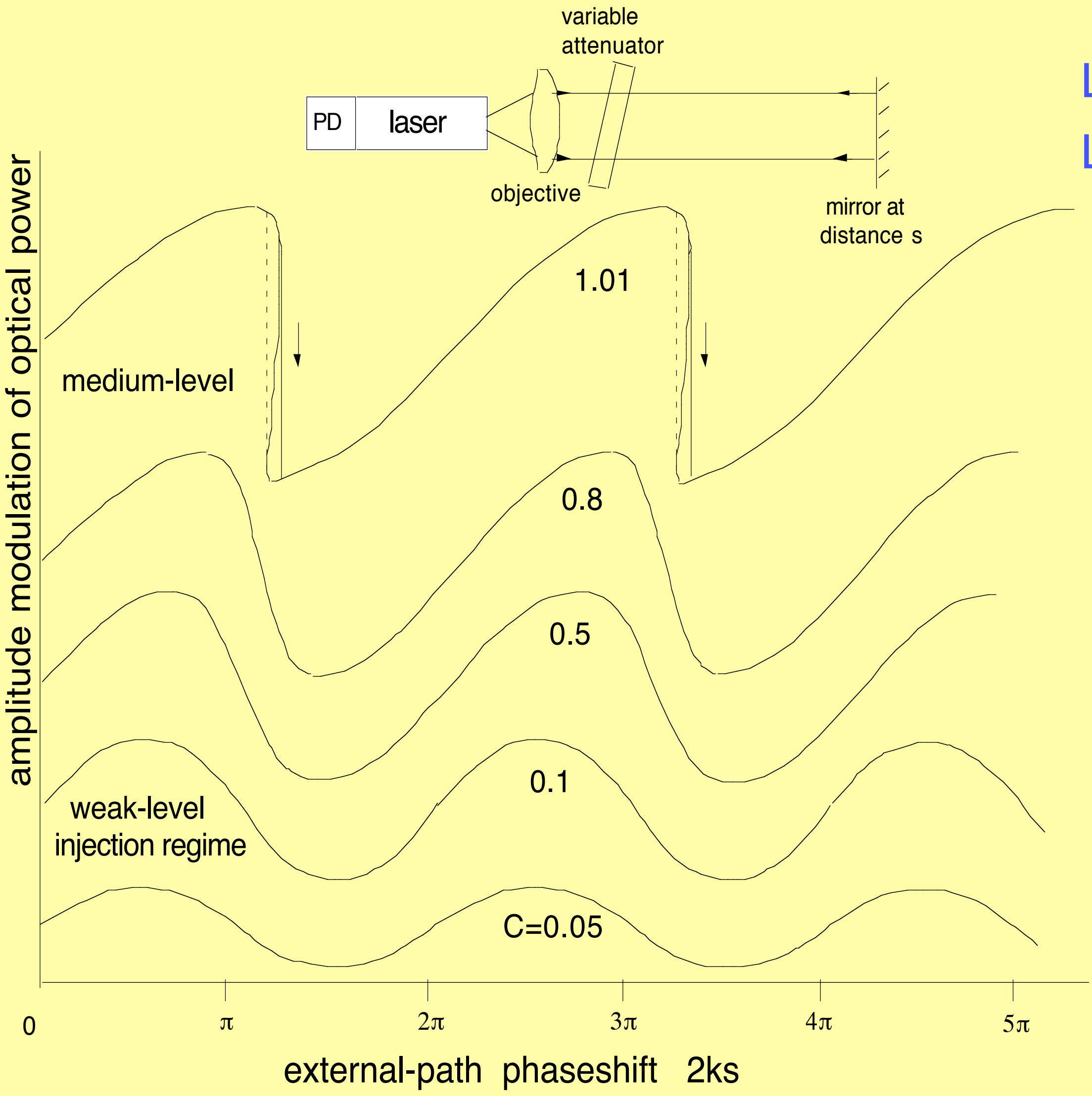
basic self-mix properties



- light propagated to the target and back modulates in amplitude the cavity field and hence the emitted power
- output power from the laser is $P = P_0\,[1+m\cdot F(2ks)]$
- modulation index $m = A^{-1/2}\,[c/2s(\gamma-1/\tau)]$ depends on the *field* attenuation $A^{-1/2}$ (so, self-mix is a *coherent* process)
- waveform F(2ks) is a periodic function of external phase $\phi = 2ks$, and for weak injection is a cosine function. F makes a full cycle every $\Delta s = \phi/2k = 2\pi/2k = \lambda/2$ (as in a plain interferometer)
- In general, the shape of F(…) depends on the injection parameter

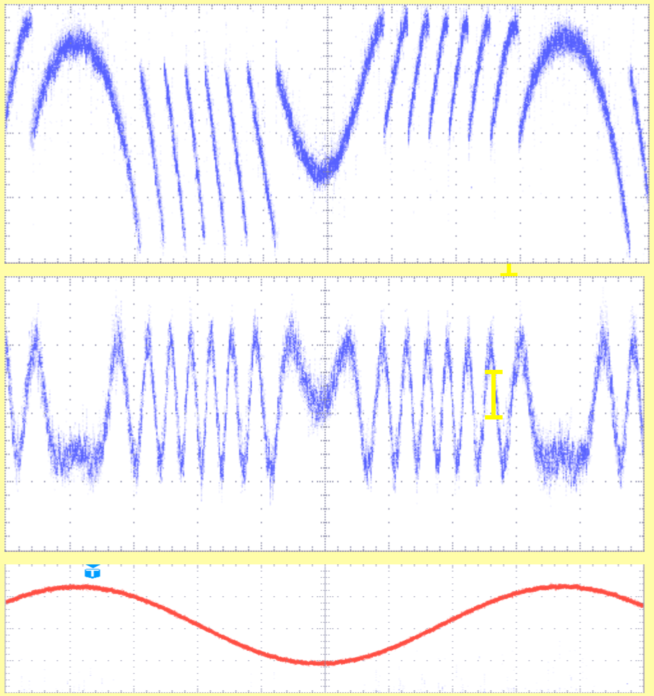
$$C = (1+\alpha^2)^{1/2} A^{-1/2}\,[\varepsilon(1-R_2)/\sqrt{R_2}]\; s/n_{las}L_{las}$$

injection level: weak and moderate



LD 1-channel interferometer

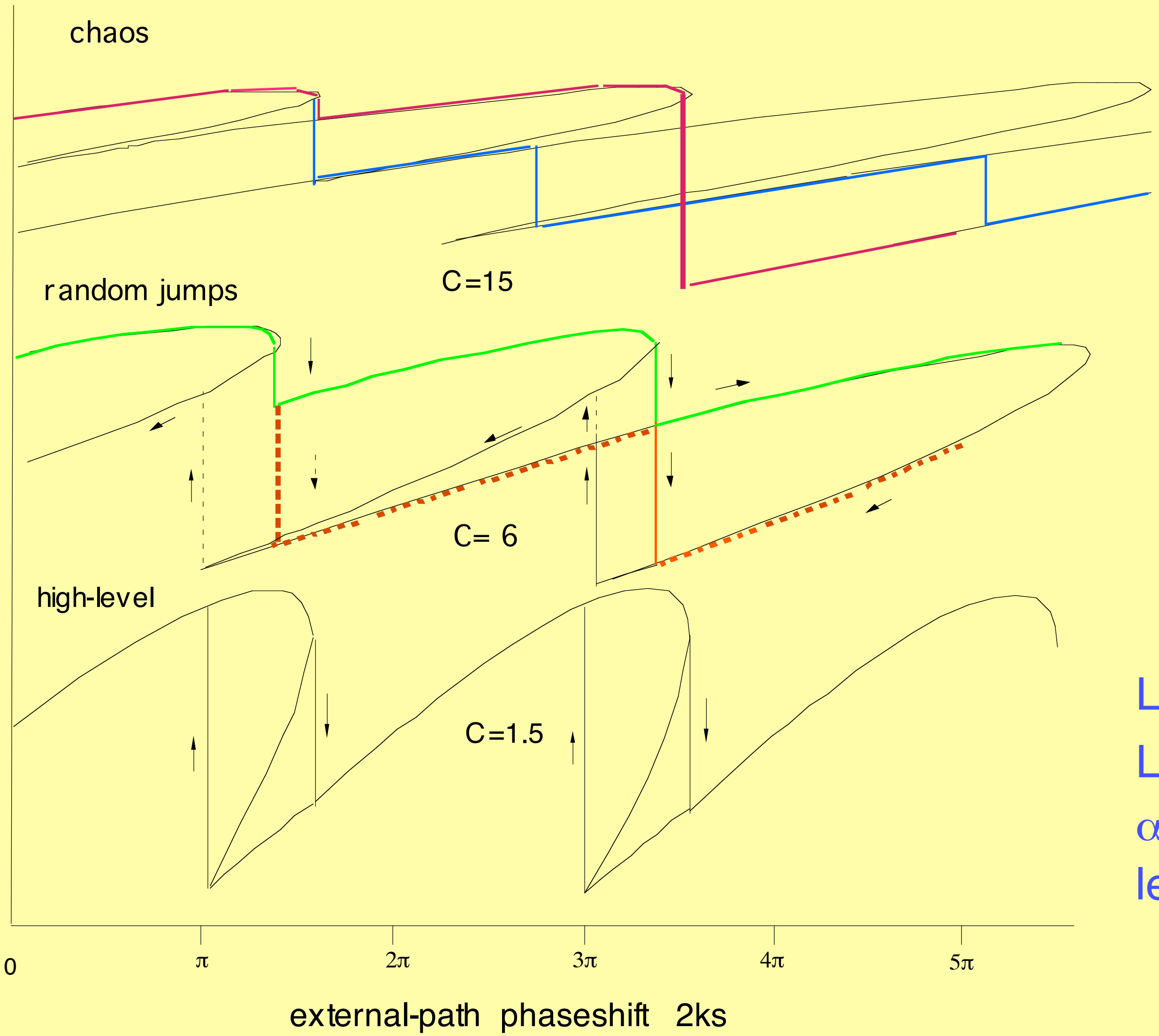
Laser Diode Vibrometer



He-Ne 2-channel interferometer, LDVs

Echo coherent-detectors

injection level: moderate and strong



Chaotic Generators

Angle Sensors

LD 1-channel interferometer
Laser Diode Vibrometer
α-factor and coherence
length measurement

theories for self-mixing

◊ rotating-vector addition

qualitative and easy, but few results deduced

◊ 3-mirror model

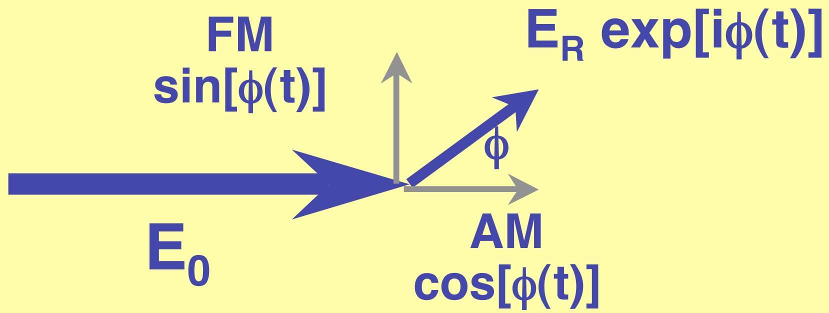
basic results deduced with a simple analysis

◊ Lang-Kobayashi (laser diode) equations

a complete description, yields a powerful treatment

rotating-vector addition

- In the laser cavity, frequency and amplitude modulation of the lasing field occur

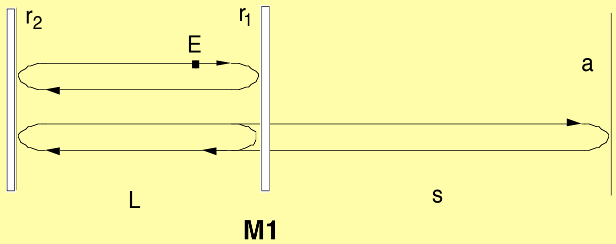


ϕ = 2ks
k = $2\pi/\lambda$
s(t) = target distance

- **AM is easily detected in a DL as a modulation superposed on the average power emitted by the source**

- **FM requires a frequency down-conversion, and we can only get it in a dual-mode, frequency-stabilized He-Ne laser**

3-mirror model



The II Barkhausen condition is applied to balance at M1: $E\ r_1\ r_2 \exp 2\alpha^* L \exp i2kL = E\ a \exp i2ks$

perturbed loop gain then follows as:

$G_{loop} = r_1\ r_2 \exp 2\alpha^* L \exp i2kL + a \exp i2ks$

and the zero-phase condition is

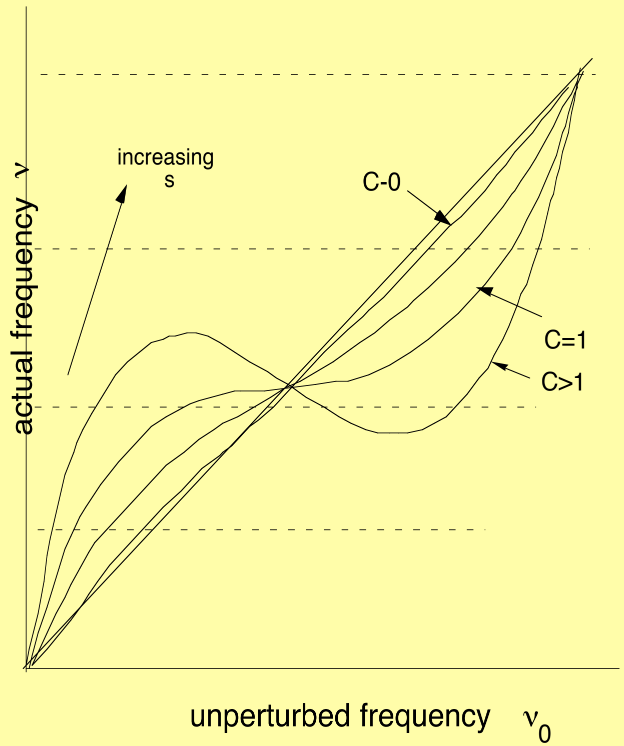
$r_1\ r_2 \exp 2\alpha^* L \sin 4\pi L n_l(\nu - \nu_0)/c + a \sin 2ks = 0$

The diagram at right

$$\nu = \nu_0 + (c/4\pi L n_l)\ a \sin 4\pi s/\lambda$$

is obtained for injection-perturbed frequency ν vs unperturbed frequency ν_0

Diagram shows that for C<1 there is one solution for ν, whereas for1<C<4.6 there are 3 solutions and **ECM** (ext cavity modes) start to be excited



Lang-Kobayashi equations

These Equations are the well-known Lamb's equation for an adiabatic active medium, adapted to a semiconductor medium where density of carriers is coupled to photon density (or field amplitude), see R. Lang, K. Kobayashi, *IEEE J. Quantum Electron.*, 1988

$$\frac{dE_0(t)}{dt} = \frac{1}{2}[G_N(N(t)-N_0)-\frac{1}{\tau_p}]E_0(t) + \frac{\chi}{\tau_L}E_0(t-\tau)\cos[\omega_0\tau+\phi(t)-\phi(t-\tau)]$$

$$\frac{d\phi(t)}{dt} = \frac{1}{2}\alpha G_N(N(t)-N_T) - \frac{\chi}{\tau_L}\frac{E_0(t-\tau)}{E_0(t)}\mathrm{sen}[\omega_0\tau+\phi(t)-\phi(t-\tau)]$$

$$\frac{dN(t)}{dt} = R_P - \frac{N(t)}{\tau_S} - G_N[N(t)-N_0]E_0^2(t)$$

Solutions reveal: F(ϕ) waveforms, AM/FM modulation, C factor, bi- and multi-stability, line broadening, route to chaos, etc. Of course, equations are easily re-written for mutual coupling of E_1 and E_2.

features of self-mixing interferometer

Injection (of Self-mixing) interferometer vs conventional types

advantages:

- optical part-count is minimal
- self-aligned setup (measures where spot hits)
- no spatial, λ or stray-light filters required
- operates on a normal diffusing target surface
- signal is everywhere on the beam, also at the target side
- resolution is λ/2 with fringe counting and sub-λ with analog processing
- bandwidth up to hundreds kHz or MHz

disadvantages:

- reference is missing (in the basic setup)
- wavelength accuracy and long-term stability is poor (with LD)
- little flexibility of reconfiguration

Dolly on self-mixing applications

Metrology

- Displacement
- Vibration
- Velocity
- Distance
- Angle

Physical Quantities

- Coherence Length
- α - linewidth enhancement factor
- Remote echoes
- Return loss and Isolation factor

Sensing

- CD readout
- Scroll sensor

but...

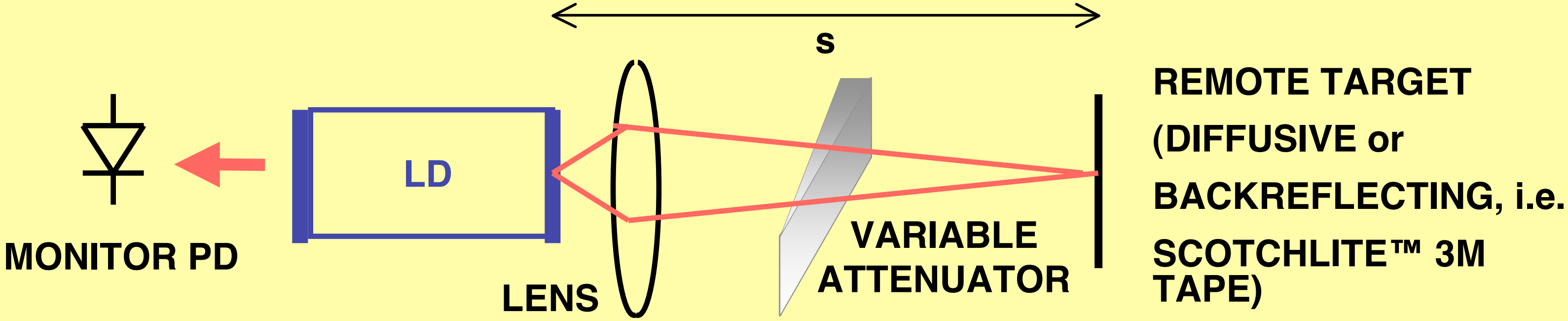
there are problems to be solved on the way
of selfmix technology ..!

a) The first is:

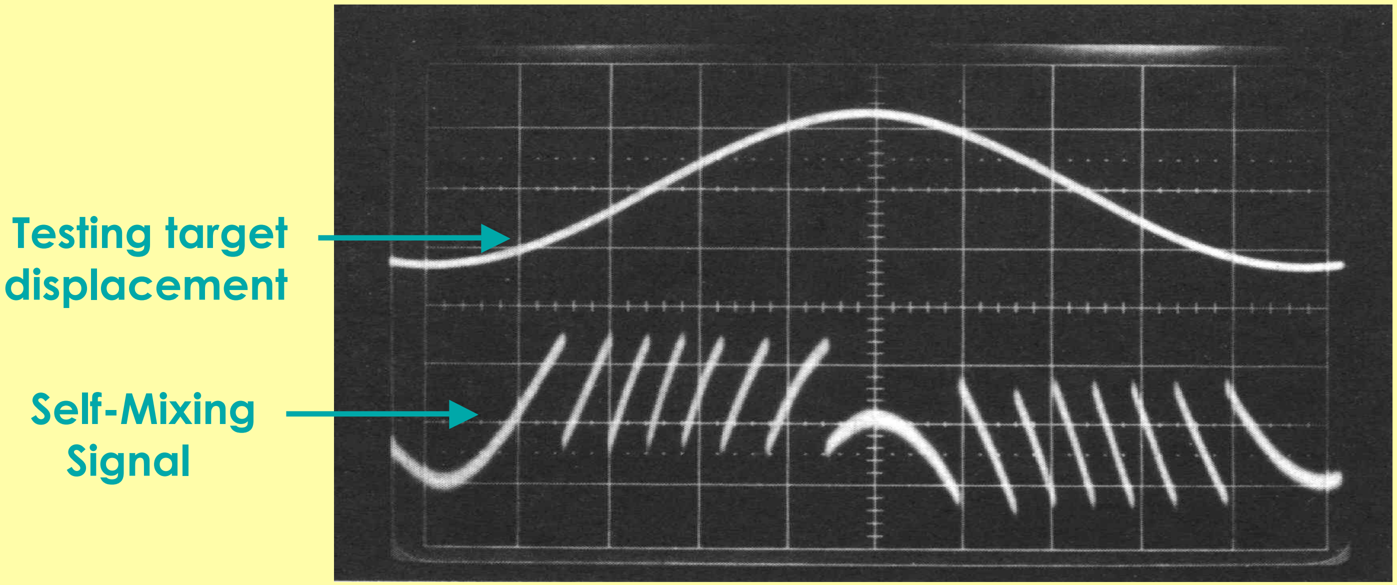
we need a second signal, *sin 2ks* or something equivalent to that, for a *digital processing,* because the plain *cos 2ks* signal is not enough to measure $\lambda/2$ displacements without sign ambiguity

- luckily enough, it happened that ....

Measuring displacements



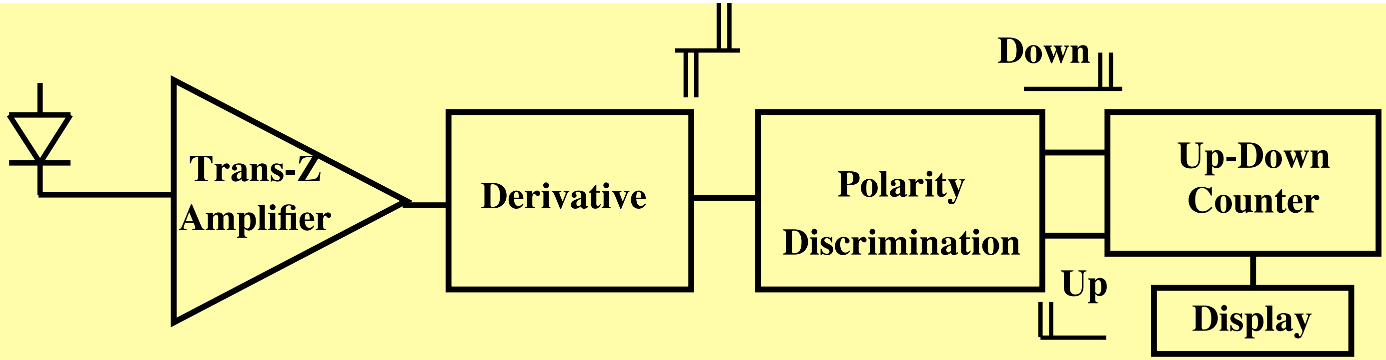
- best regime: moderate feedback C > 1, but also C< 4.6
- principle: counting of fast signal transitions with polarity

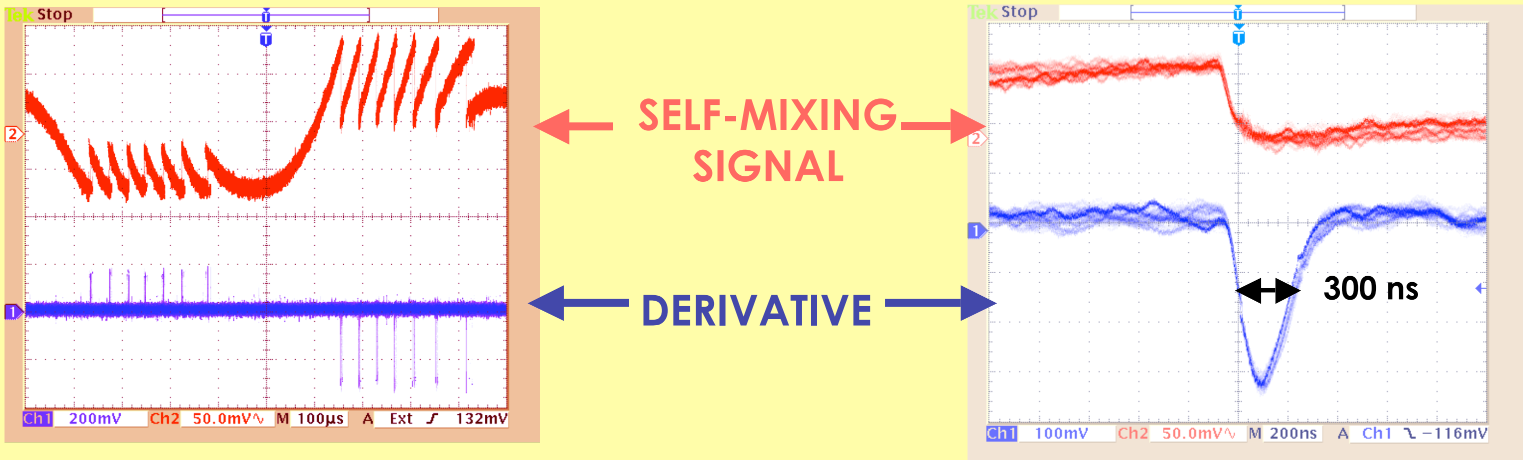


S.Donati, G.Giuliani, S.Merlo, J.Quant.El. <u>31</u> (1995) pp.113-19

cited by 115
(Google Scholar)

Displacement: circuit functions





100 μs/div

0.2 μs/div

- Resolution: 420 nm
- Max. Target speed: 0.4 m/s
- Distance range 0.4 ÷ 1.6 m

S.Donati, G.Giuliani, S.Merlo, J.Quant.El. <u>31</u> (1995) pp.113-19 **cited by 115**

Displacement: pushing the performance limit

On a corner-cube, the self-mix measures displacement up to ≥2m, in $\lambda/2$=0.42 µm steps, with a few ppm accuracy (see figure, from Donati et al., Trans. IM-45, 1996, pp.942-947).
Using a DFB laser, λ-drifts of ≤ 10^{-7} per year should be achieved.
Instead, on a diffuser target, signal is lost because of the speckle pattern **fading**

S.Donati, L.Falzoni, S.Merlo, Trans.Instr.Meas. <u>45</u> (1996) pp.942-47 **cited by 23**

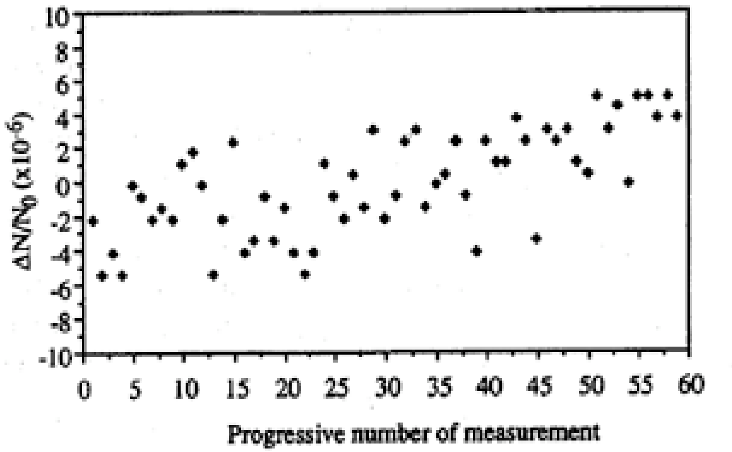


Fig. 5. Experimental residual error obtained with standing target after compensation of laser temperature variations. Every data point corresponds to a 2.28 °C temperature sweep of the laser.

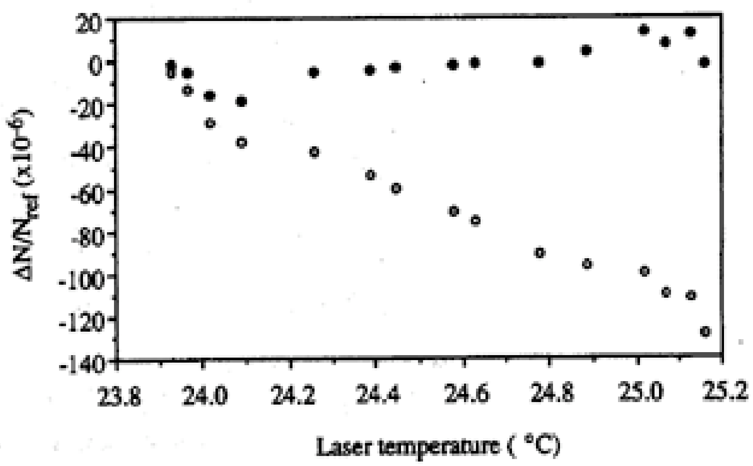
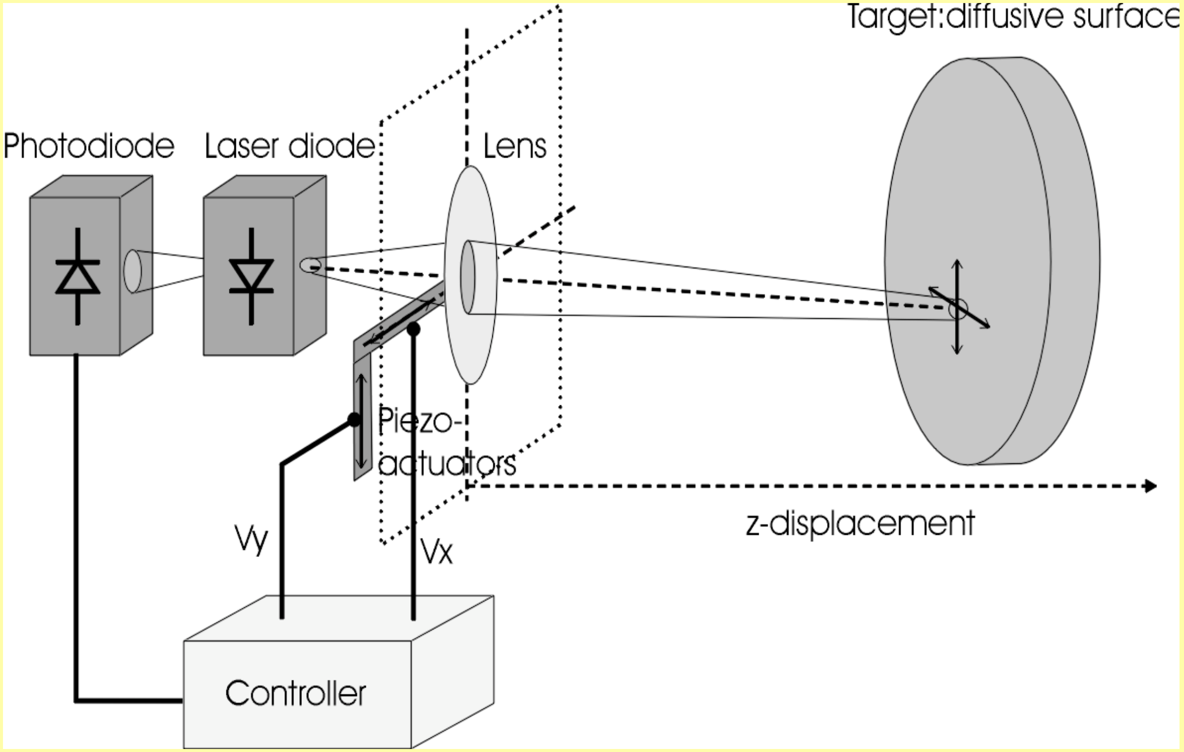


Fig. 6. $\Delta N/N_{ref}$ versus laser temperature, obtained for 22-cm target displacements: ○ experimental data before compensation, ● compensated results.

b) second problem: we need eliminate the speckle pattern statistics that gives *fading* of the selfmix signal because we want to be able to operate on diffuser (not a *specular*) target surface

- We may try tracking the bright speckle …

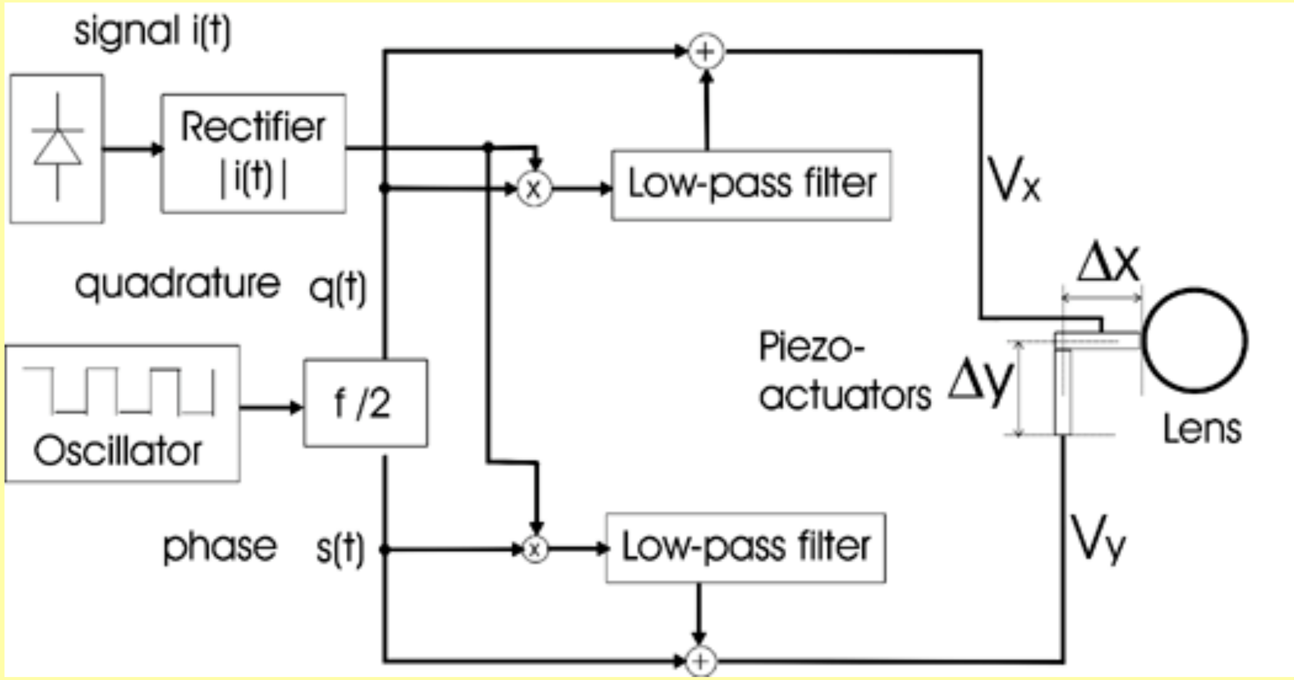
Displacement: the bright-speckle tracking (BST)



Tracking a bright-speckle permits to stay on a maximum of intensity and avoid fading. Operation on a diffuser target is then allowed, with little added error

S.Donati, M.Norgia, J.Quant.El. 37 (2001), pp.800-06 cited by 21

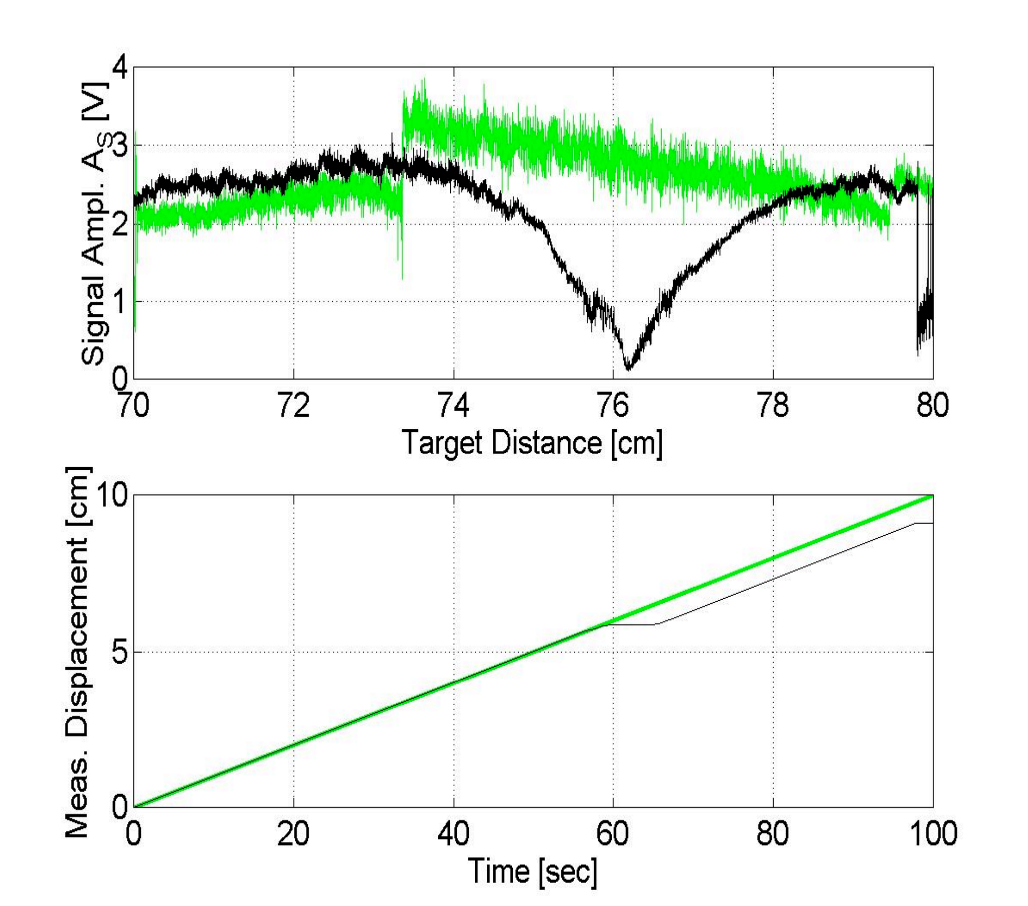
Speckle tracking technique



Block scheme of the speckle-tracking circuit. Signal from the photo-diode is rectified peak-to-peak and demodulated respect to the dither frequency, in phase and quadrature. Results are the X and Y error signals that, after low-pass filter, are sent to the piezo-actuators X and Y to track the maximum amplitude or stay locked on the bright speckle

BST improvement

Top: signal amplitude with (green line) and without (black line) speckle-tracking system, reveals that a fading (at 76 cm) has been removed
Bottom: corresponding displacement as measured by the SMI



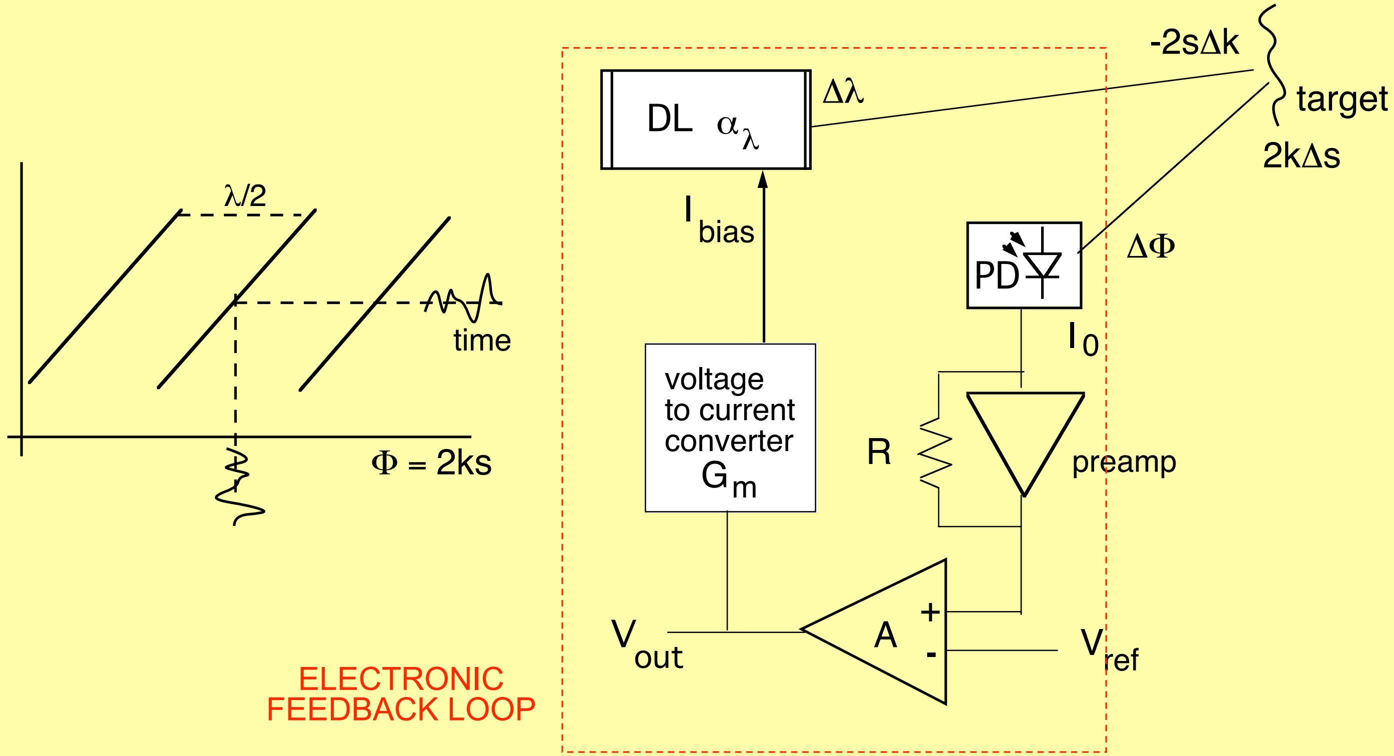
S.Donati, M.Norgia, Trans. Instr. Measur. IM-52 (2003), pp.1765-70

… and now that the digital measurement is OK

c) we want to make an *analogue processing* to measure nanometer (or <<λ) vibration amplitudes

we may do so if we are able to lock at half fringe

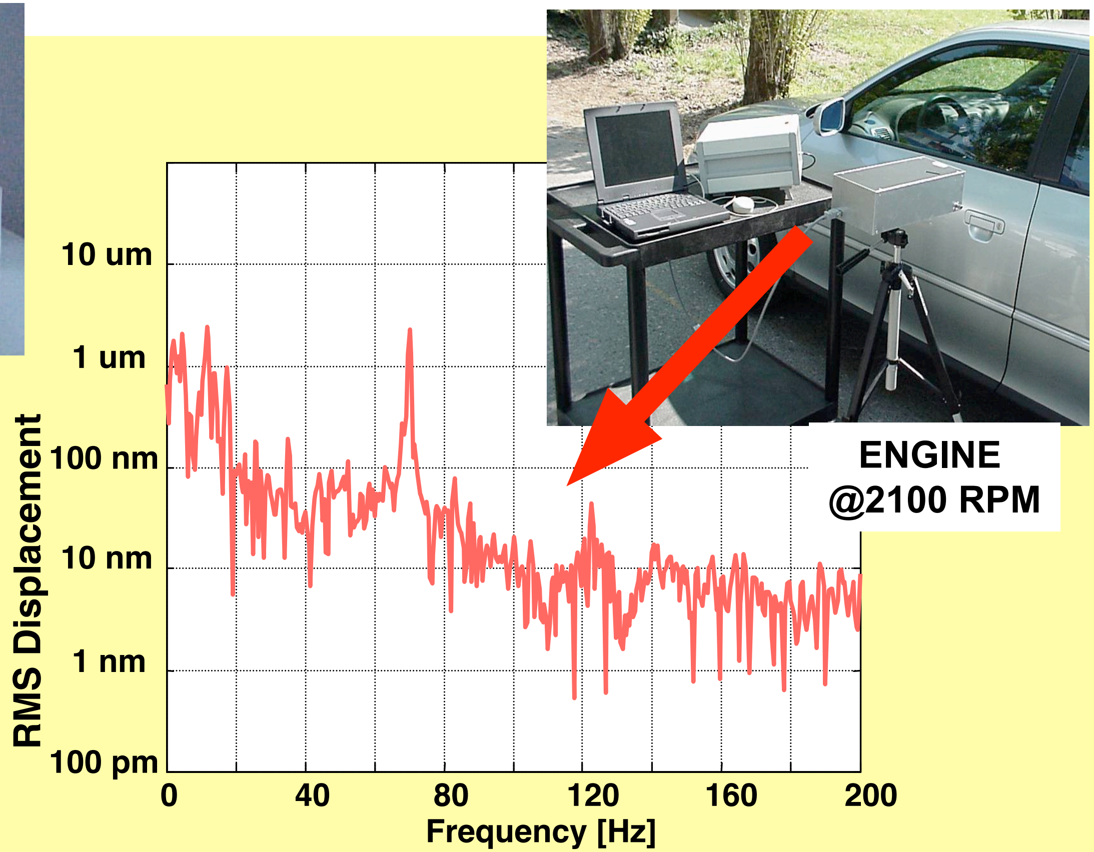
Vibration, mechanical



at C>1, fringe response is linear. With this circuit, we can lock the working point to half-fringe, through an active phase nulling. Output signal is the error signal **$\Delta V = [\alpha G_m]^{-1} \Delta s\ (\lambda/s)$**, independent from signal amplitude and speckle (if loop gain $G_{loop} = RG_m \alpha (s/\lambda) \sigma P_0$ is large)

S.Donati, G.Giuliani: Meas. Science Techn.,14, 2003, pp.24-32

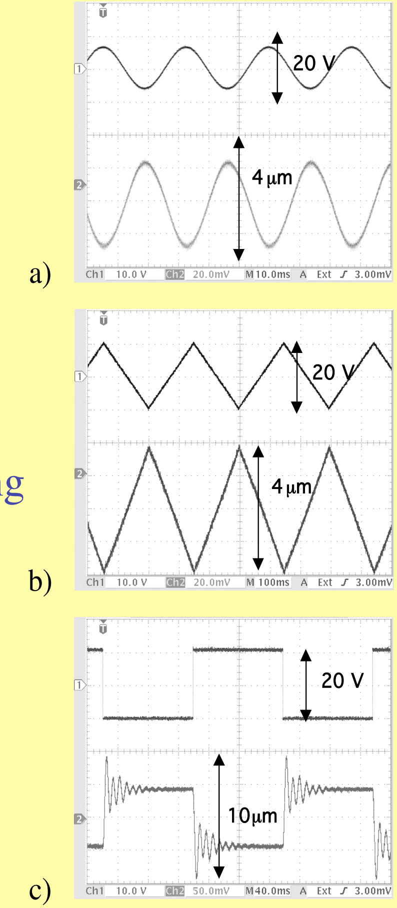
Vibration: application to the automotive

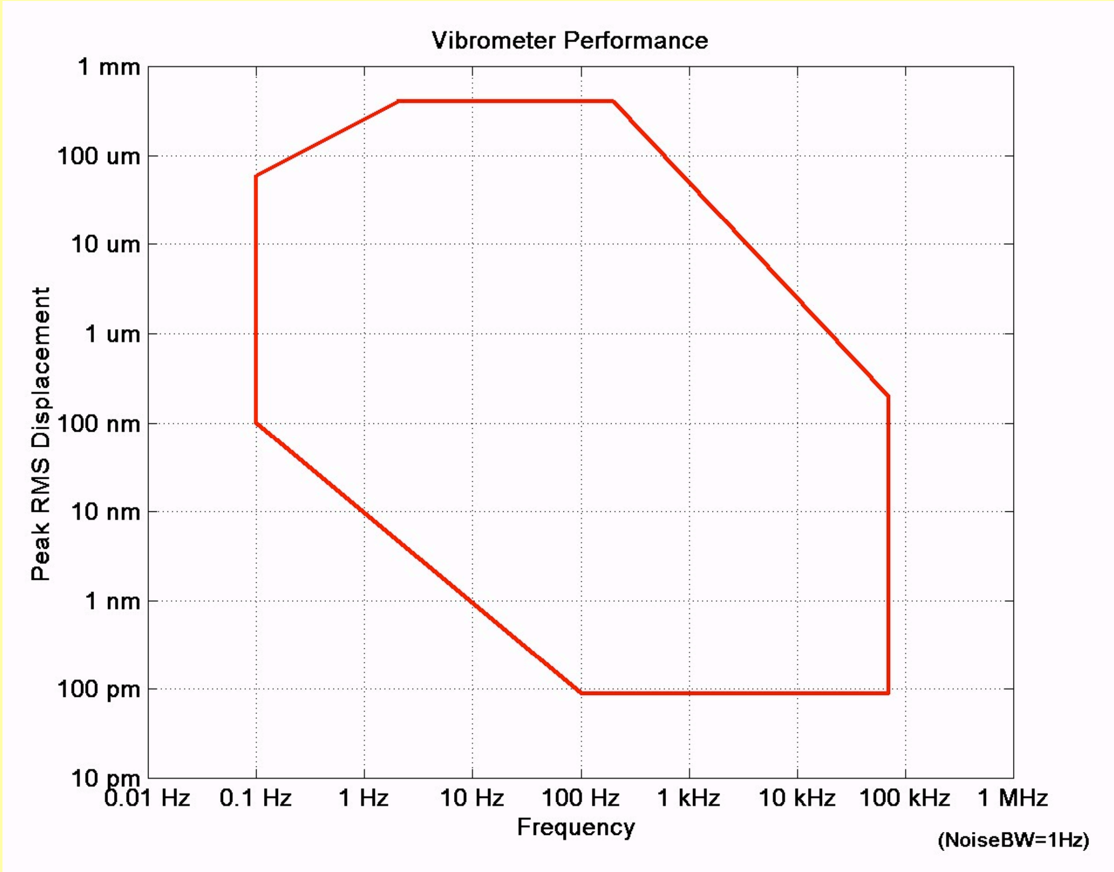


A developmental unit to test automotive vibrations has the following performances: detectable amplitude ≈100 pm/√Hz; max. amplitude: 600 μmp-p; bandwidth:70 kHz; dyn. Range is > 100 dB

performance of self-mix vibration pick-up

measuring PZT response





Because of the servoing arrangement, the vibration signal finds a dynamic range much larger than λ/2 (in practice, up to ≈200 µm) (Donati et al., J.Optics A, vol.4 (2002), pp.S283-94).

d) last, we want to procure a *reference* to our measurement, so be able to measure nanometer (or <<λ) amplitudes of vibrations superposed to large (micrometer, or even hundreds of micrometer) common-mode signals

a reference channel is finally added to self-mix

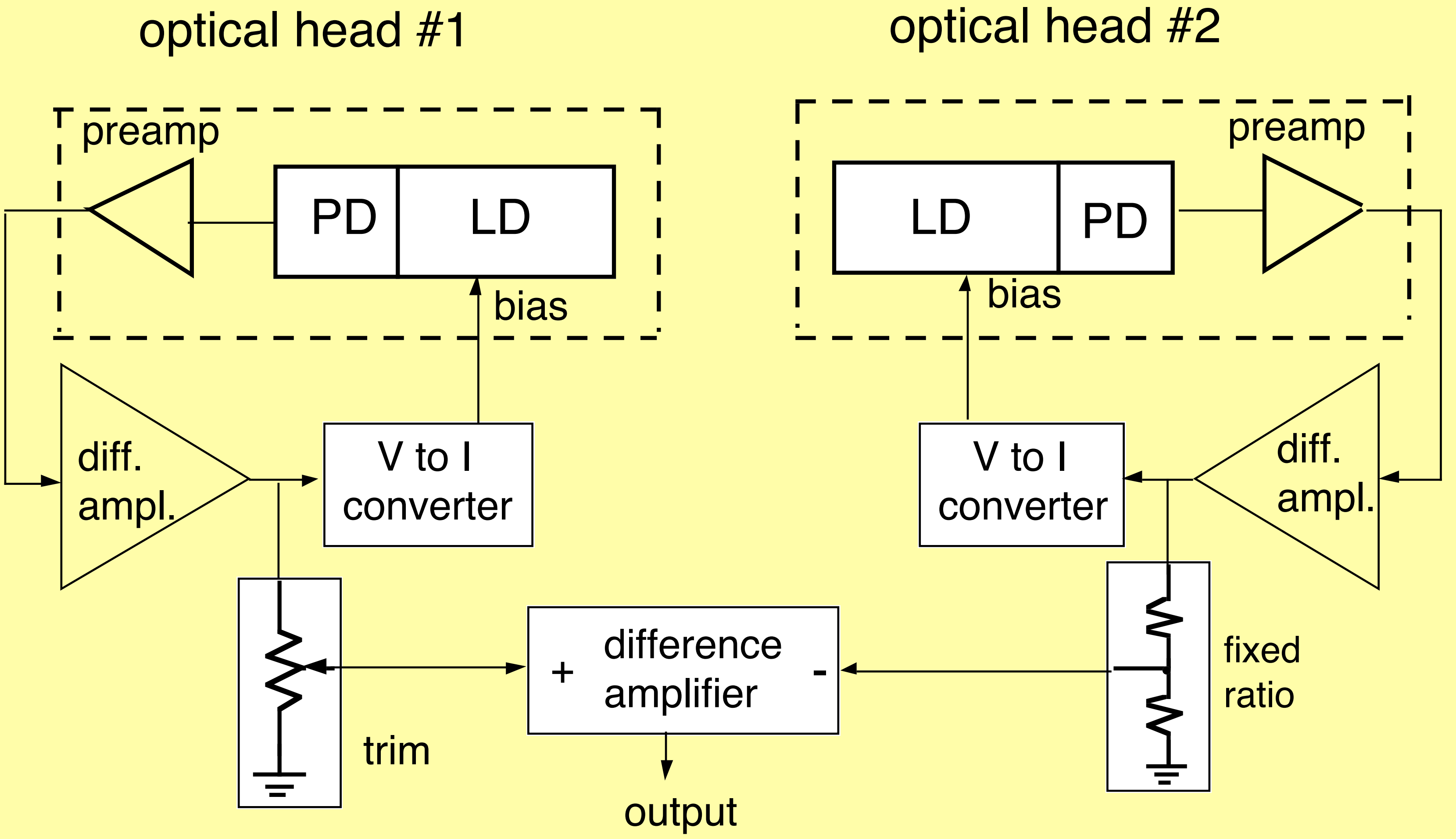
Using a large feedback-loop gain,

$$G_{loop}=RG_m\alpha(s/\lambda)\sigma P_0 >>1 \quad (\approx 200 \text{ in our case})$$

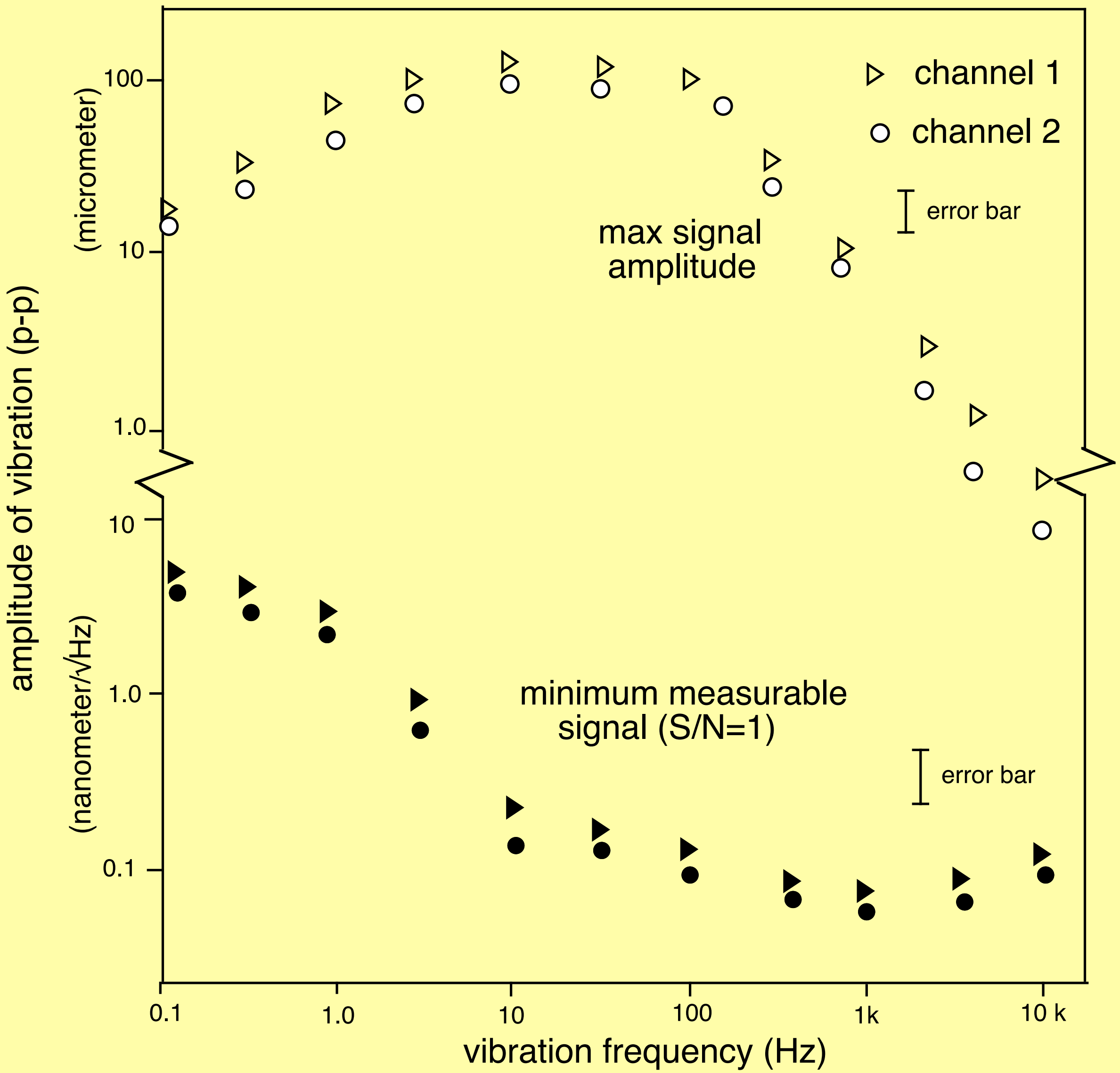
- the dynamic range is increased by G_{loop} respect to the initial $\lambda/2$ value and effects of perturbations/disturbances are reduced by G_{loop}
- output is the amplified error ΔV, equal to $[\alpha G_m]^{-1}\,\Delta s\,(\lambda/s)$, independent from amplitude P of received signal
- speckle amplitude-fading is compensated out, it only affects the loop gain available for the servo action
- thus, we can use <u>two channels</u> and make a reliable subtraction of the common mode displacement/vibration

S.Donati, M.Norgia, G.Giuliani: Applied Optics 45 (2006), pp.7264-68

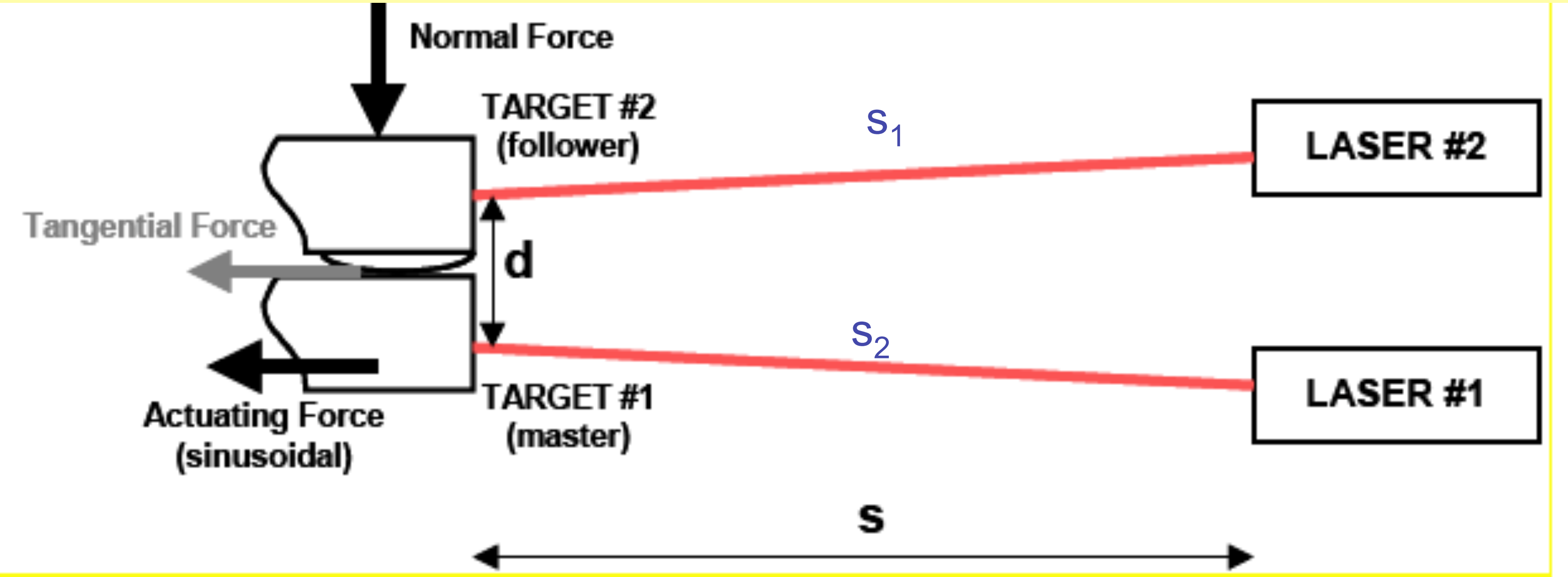
the differential (2-channel) self-mix Vibrometer



performance of the differential self-mix

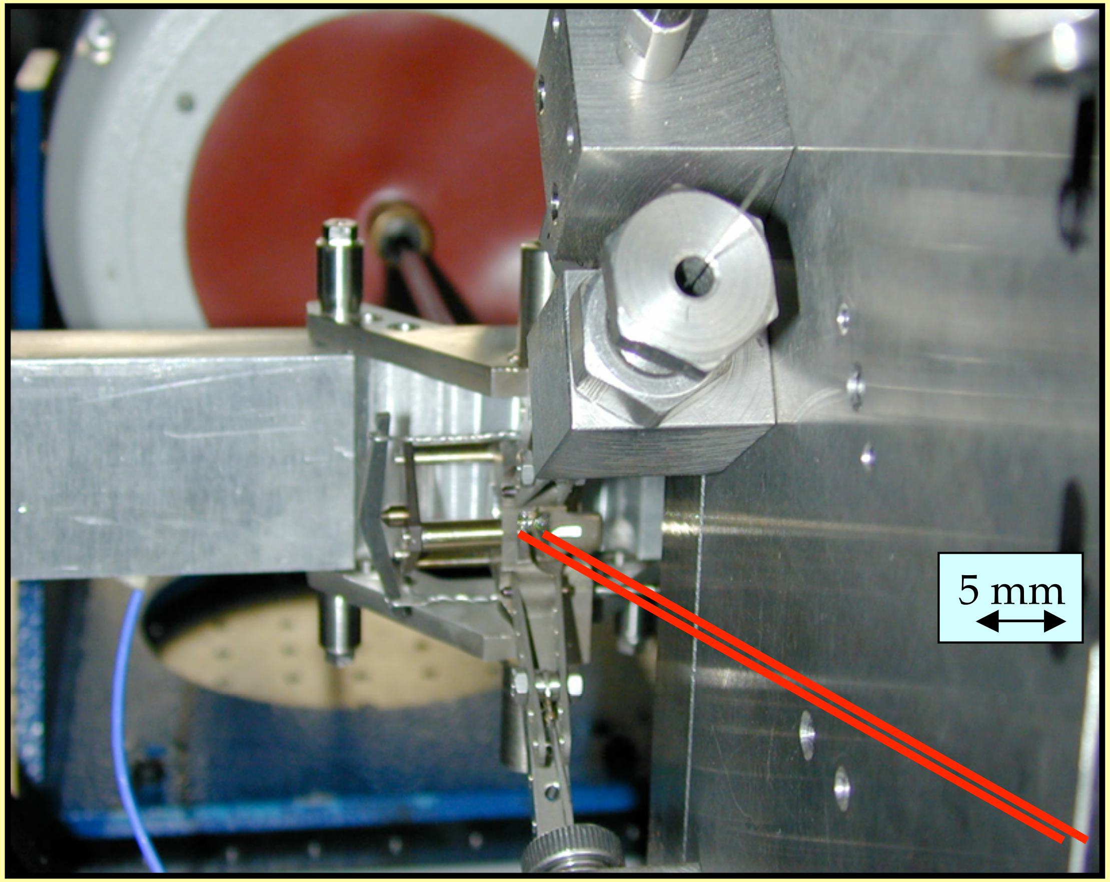


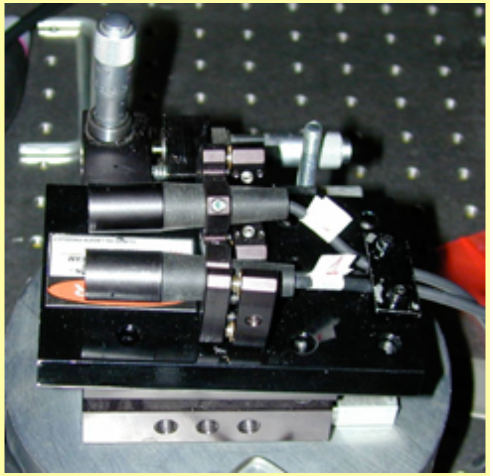
the differential (2-channel) measurement



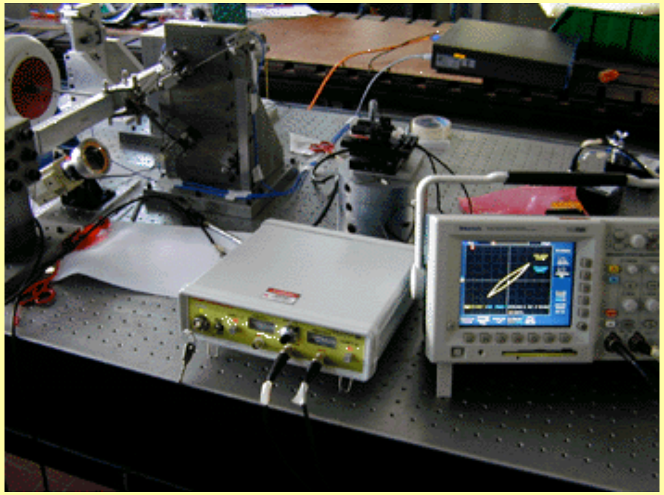
we can now measure the differential displacement s_1-s_2 (the strain of bead #2) as a function of actuating force F (the stress)

Differential vibrometer: application to fatigue study





Laser head

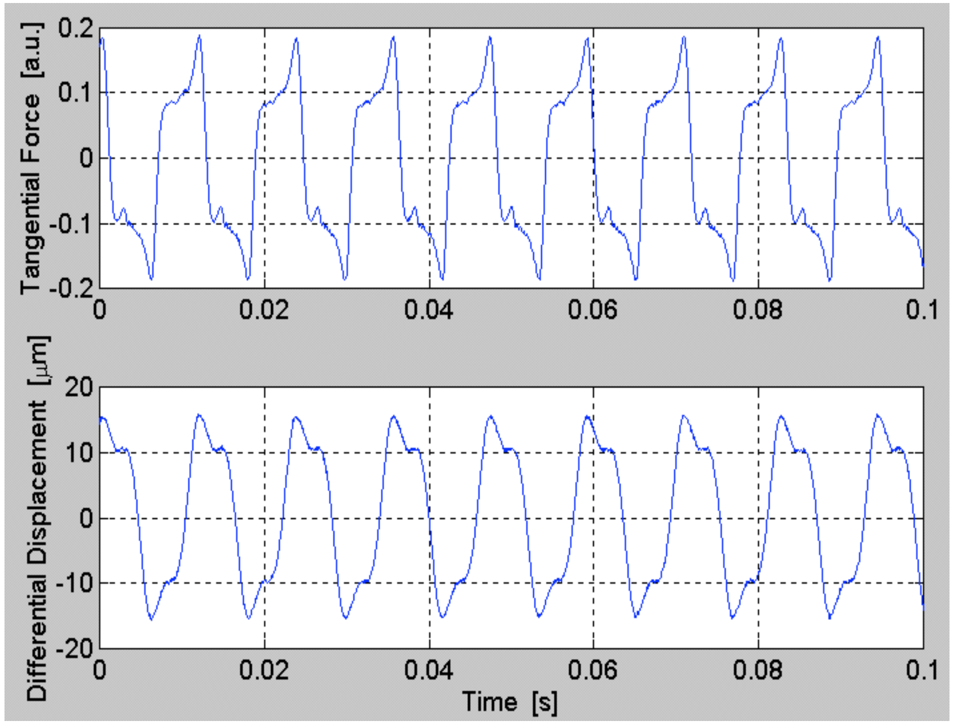


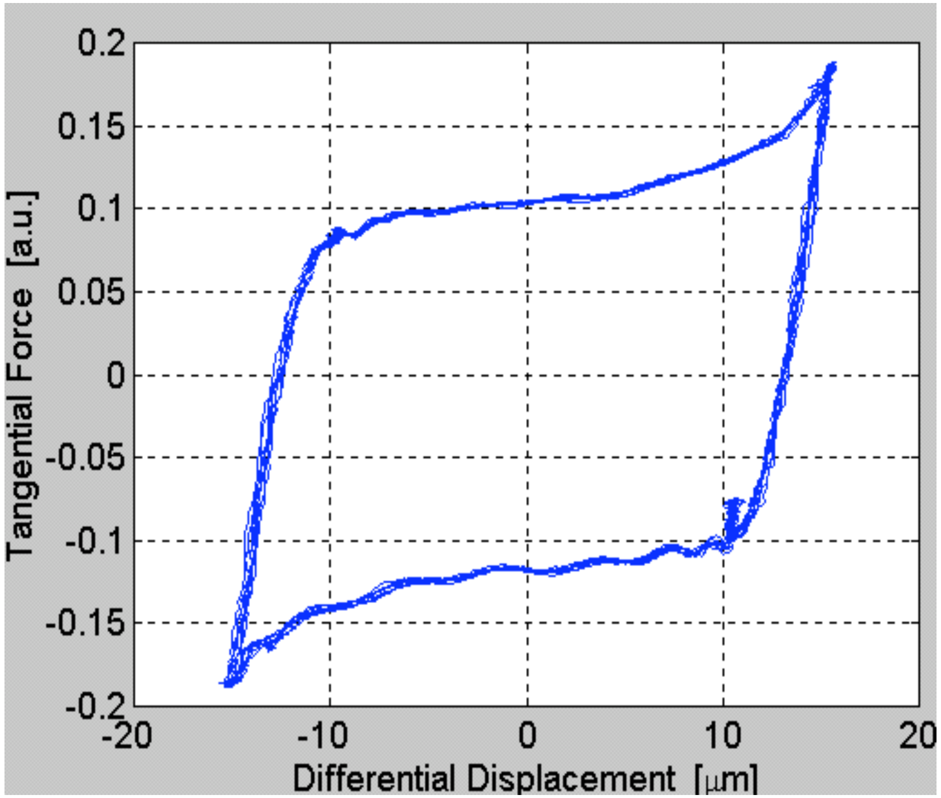
instrumentation

because of the servo loop, the vibrometer can work in differential mode despite the speckle statistics

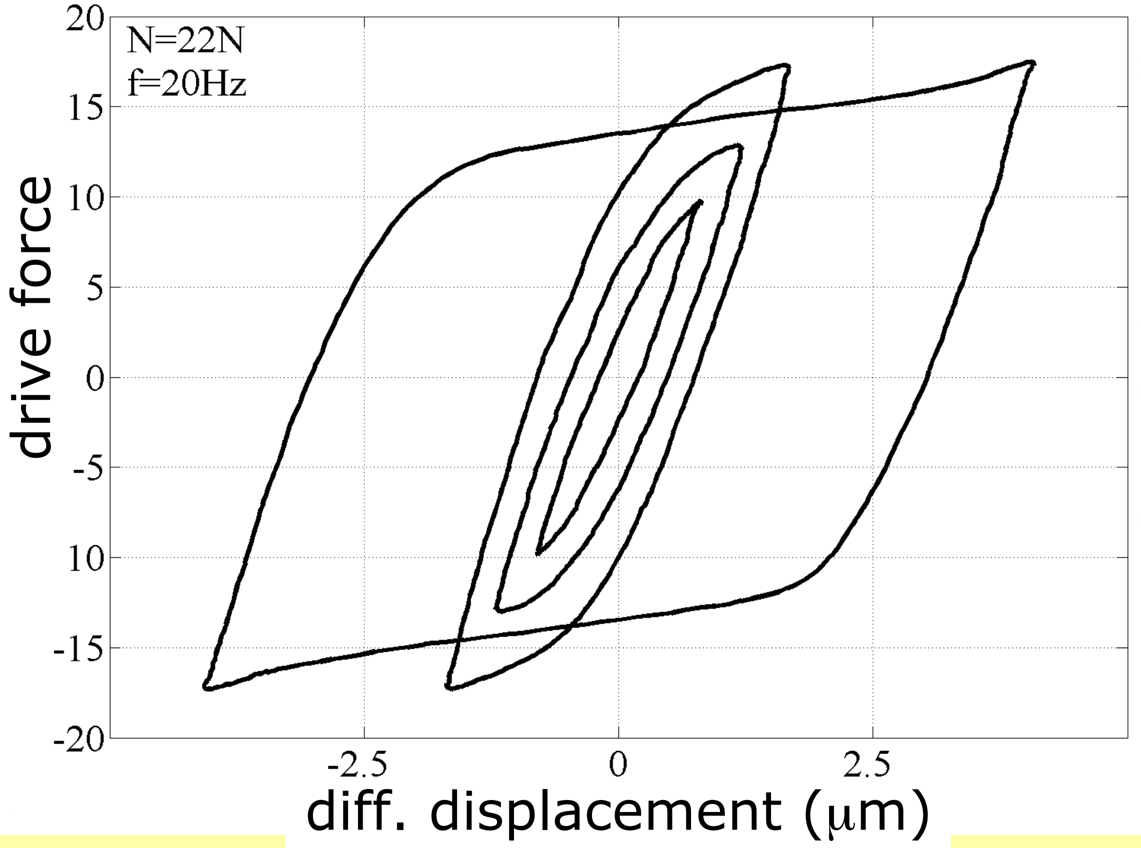
Differential Vibrometer: measuring the F-D diagram

Force-Displacement or Strain-Stress diagram is measured optically for the first time





hysteresis stress-strain cycle



at increasing levels of excitation, the transition from *elastic* to *slip* regimes is evident

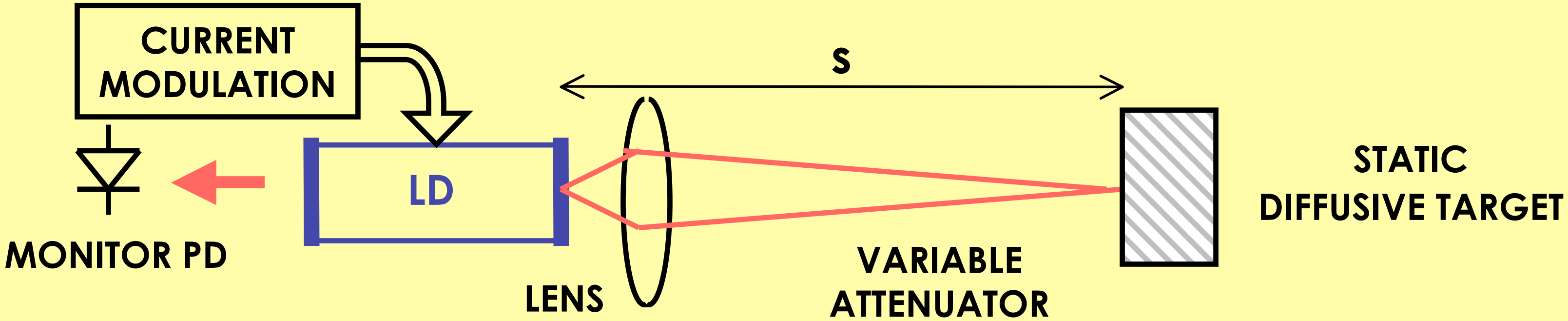
S.Donati, M.Norgia, G.Giuliani: Applied Optics 45 (2006), pp.7264-68 optical phase Φ servoing by modulation of drive current hints another seemingly unattainable application of selmix:

- ***distance*** measurement

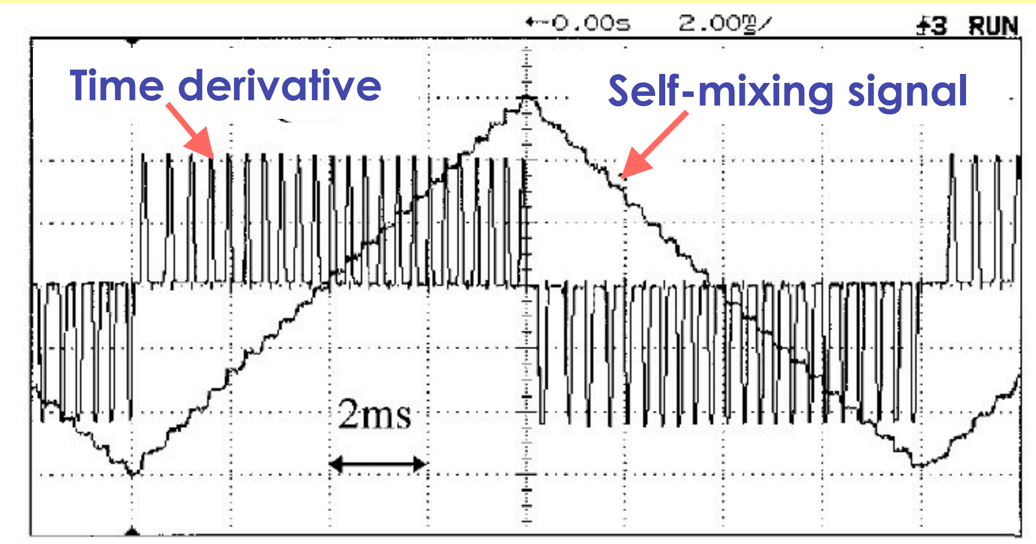
indeed, by sweeping $\lambda=2\pi/k$, we sweep $\Phi=2ks$ and get a rate of 2π–crossings (or, of $\lambda/2$'s) proportional to s

thus, we may be able to make a low-cost selmix rangefinder

Absolute Distance Measurement



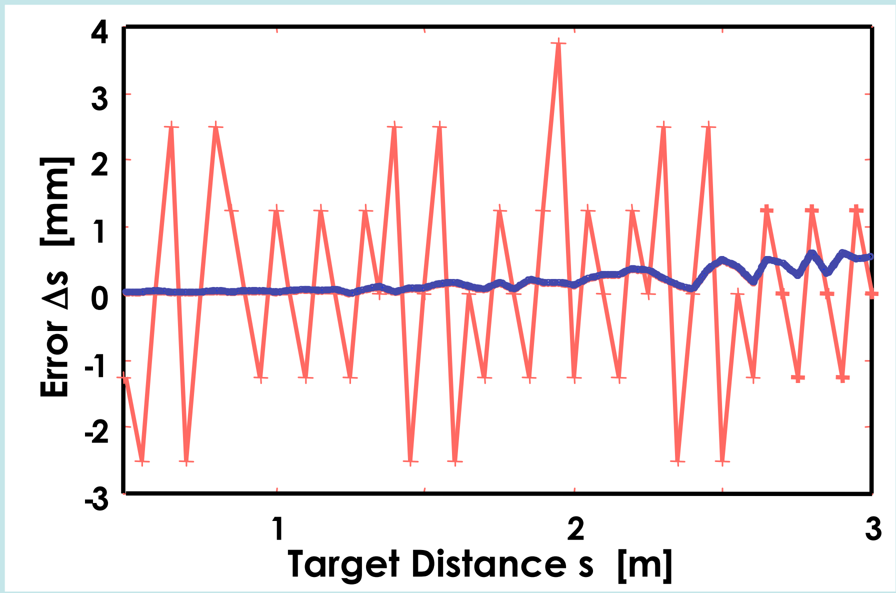
LD wavelength (Dl) is modulated via triangle current waveform, and fringe number N is counted, then distance is found as:



$$s = N\,(\lambda_0^2/2\Delta\lambda)$$

from: S Donati, T Bosch, G Giuliani, M Norgia, J. Opt. A, 2002 Cited by 50

Absolute Distance: accuracy



Pulse counting

"Beat frequency"

Accuracy limits: • LD I- tuneability , • temp. effects (I- modulation distortion),
• discretization errors

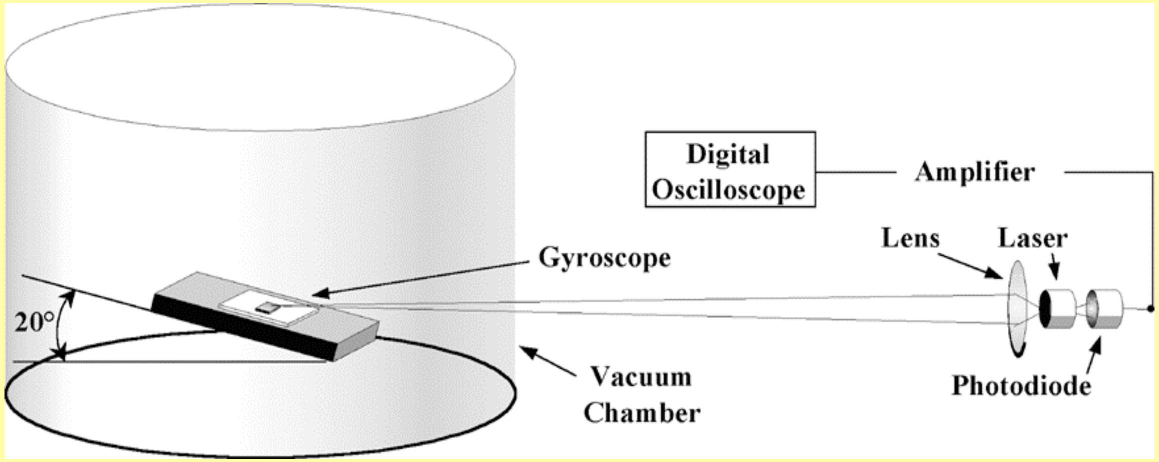
Measured accuracy: 4-mm @ s= 60-cm (with $\Delta\nu$=36 GHz, F-P laser)
and 0.5-mm @ s= 60-cm (with $\Delta\nu$=375 GHz, DBR)

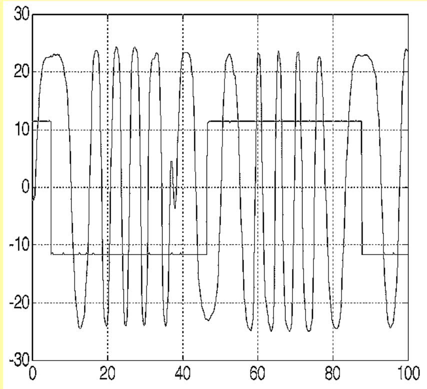
Averaging ("beat frequency" measurement) + thermal effects compensation:
<1- mm @ s=1¸3 m (with $\Delta\nu$ = 36 GHz F-P laser)

e) but, you don't need a *double*-channel vibrometer at all times. To measure large (eg, μm-amplitude) vibrations, a single channel, plain selmix interferometer will do, as illustrated by

- MEMS testing
- 4-mass gyro (MEMS again) trimming
- micromirrors checking
- biological signal pickup (optical stethoscope)

Vibration, MEMS

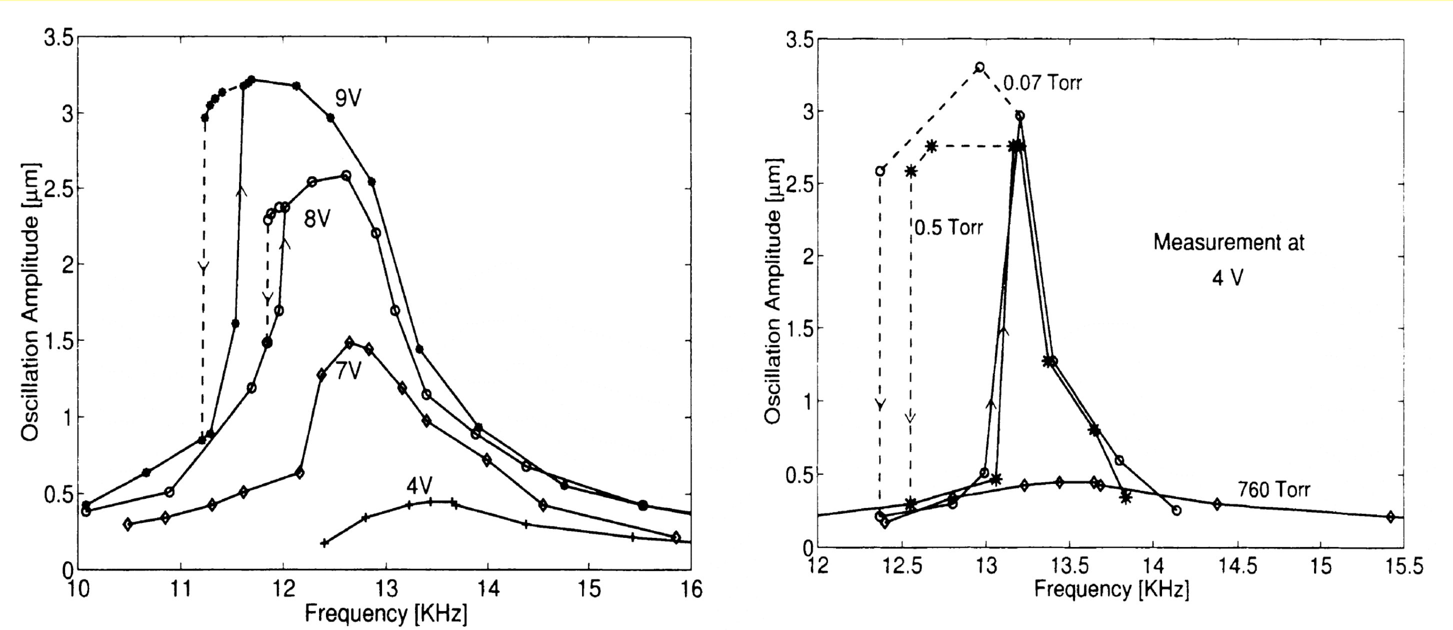




The self-mixing vibrometer has been used to measure the mechanical properties of Si-machined MEMS. Light from the laser is focused on the vibrating mass of the MEMS chip through the plain glass wall of the vacuum chamber. Light on still parts or outside target doesn't disturb operation. The out-of-plane vibration of the MEMS mass is viewed at an angle ($\Phi \approx 20°$), and the appropriate cos Φ correction on s(t) is applied to the fringe signal (left) giving the displacement waveform.

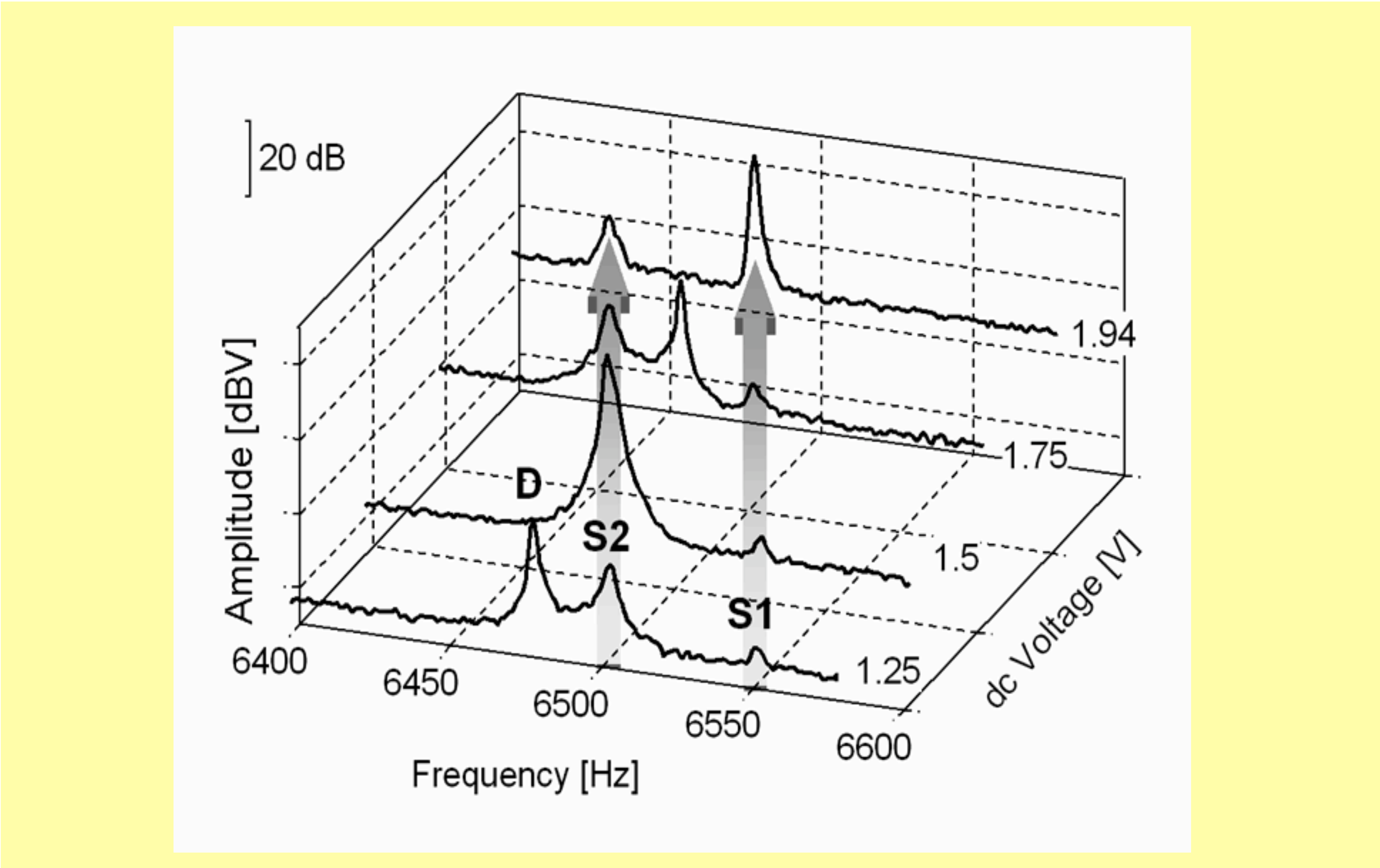
(Annovazzi, Donati, Merlo: Trans. Mechatr. vol.1, 2001,pp.1-6).

testing MEMS response

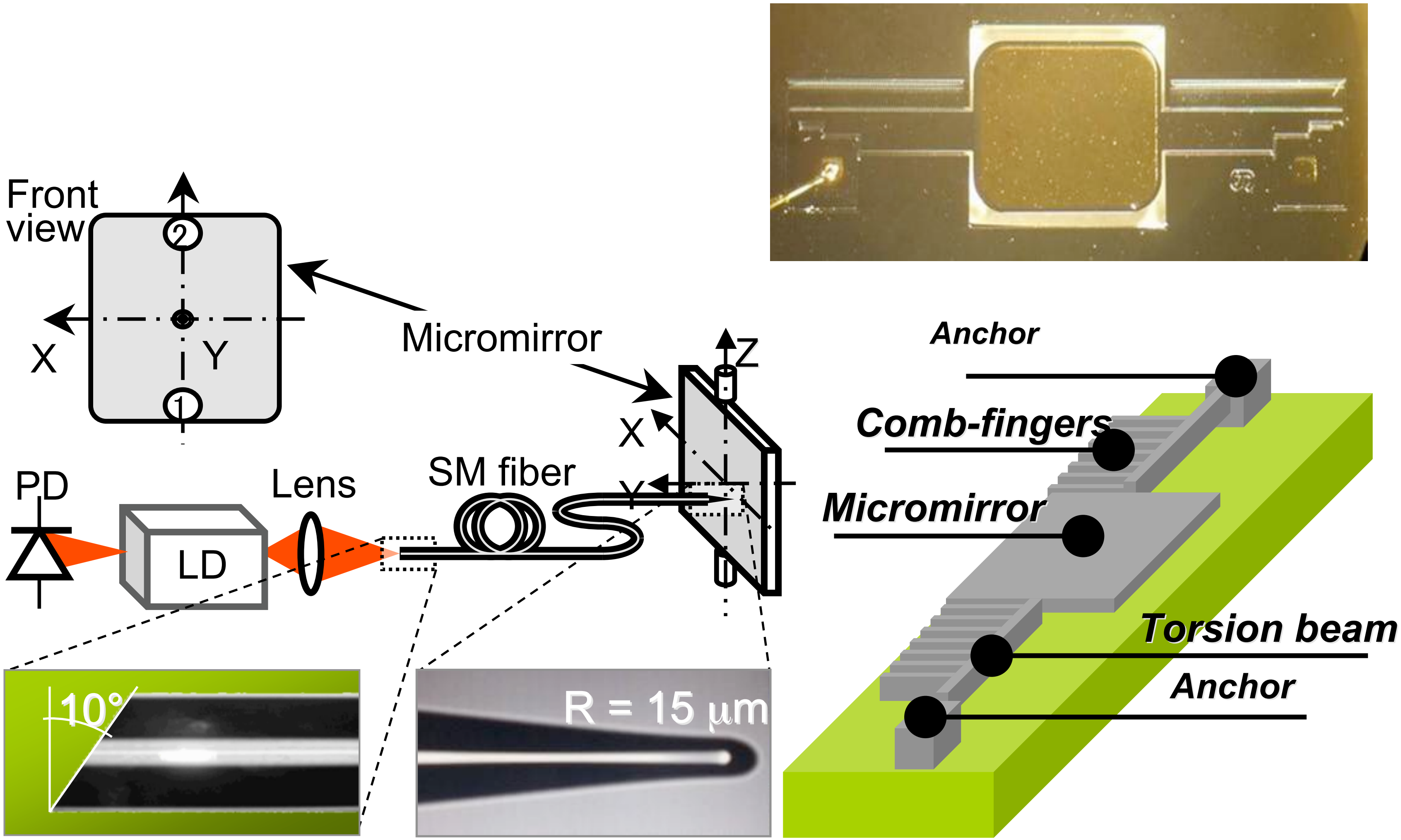


MEMS frequency response measured by the SMI vibrometer.
Left: drive voltage is increased up to incipient hysteresis at 8-9 V, indicating the risk of creep in the structure. Right: at increasing ambient pressure, the Q-factor is damped because of air friction.
(S.Donati et al.: Proc. LEOS Workshop on MEMS, 2000, pp.89-90).

4-masses gyroscope: SMI helps trimming operation

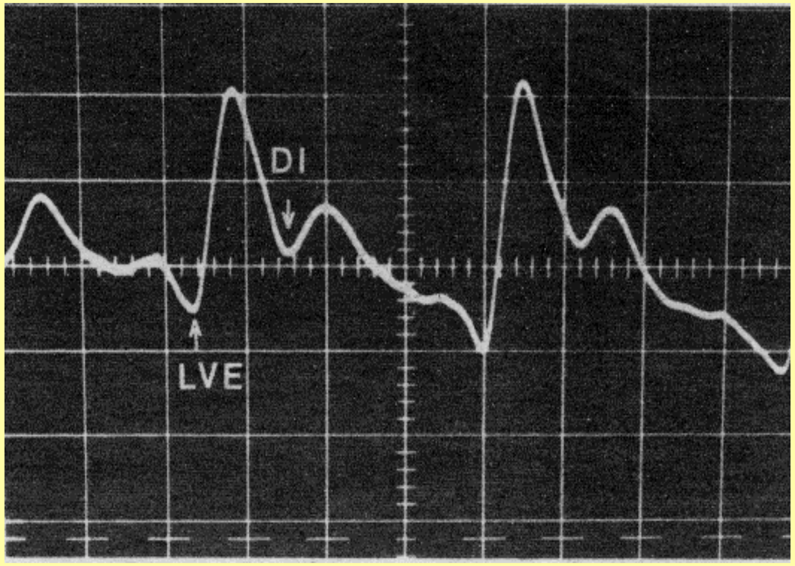


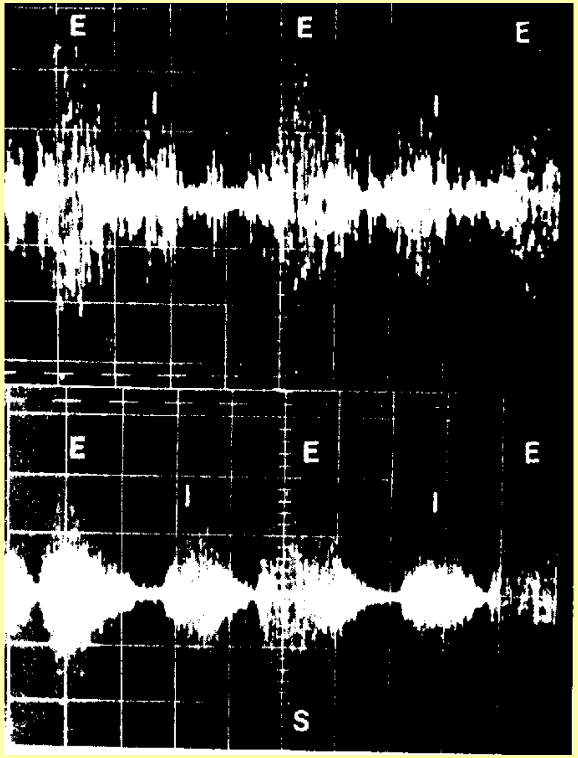
Measurement of MOEMS: micro-mirrors



(V.Annovazzi, M.Benedetti, S.Merlo: J.Select.Topics Quant.Electr.10, 204, pp.536-44)

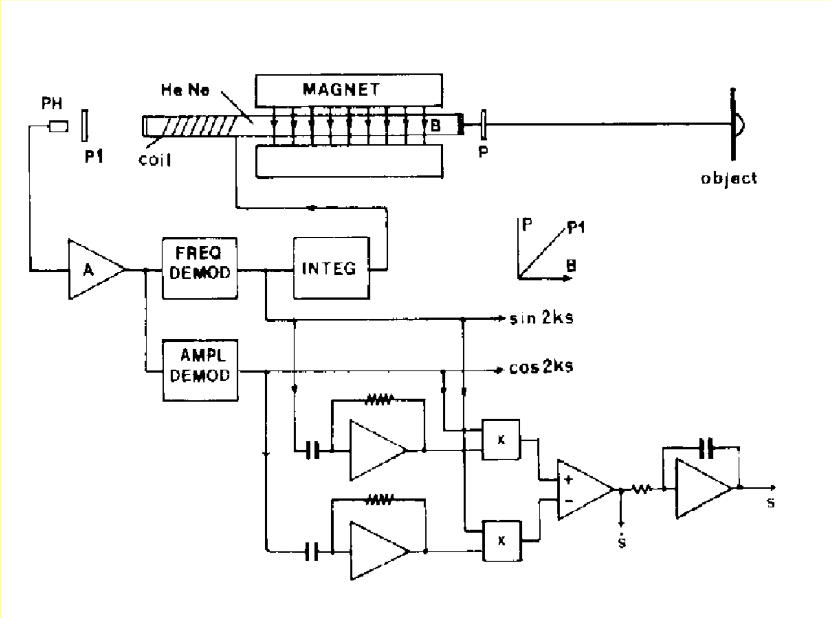
bio signals pick-up

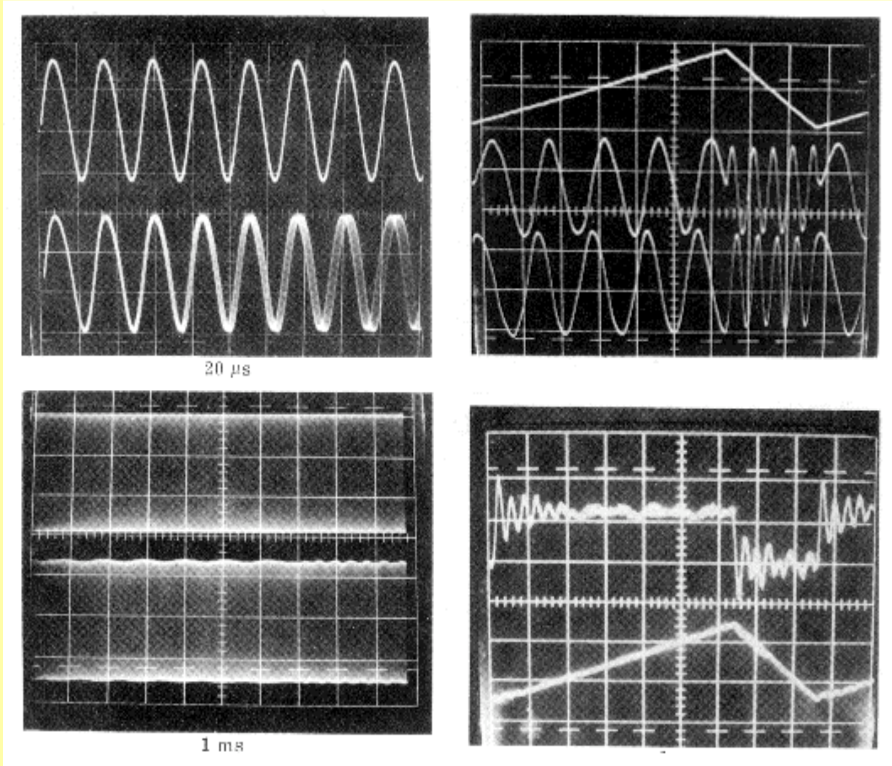




Two samples of biomedical signals measured by the He-Ne SMI
left: pulsation of blood on a finger tip (0.5 μm/div, 0.3 s/div),
right: respiratory sounds detected on the back compared to acoustical.
(S.Donati, V.Speziali, Laser+ElektroOptik. vol.12 (1980), pp.34-5).

bio signals pick-up 2





The first injection interferometer, based on a He-Ne Zeeman laser, frequency stabilized, was reported as early as year 1978 (S. Donati, J. Appl. Phys., vol.49, p.495-498) and employed a dual mode scheme to heterodyne <u>both</u> AM and FM signals down to electrical frequency Later, was developed further by Smith et al. (Optical Engineer.34,1995,p.2802).

Last, looking at the amplitude of the selfmix signal we can exploit

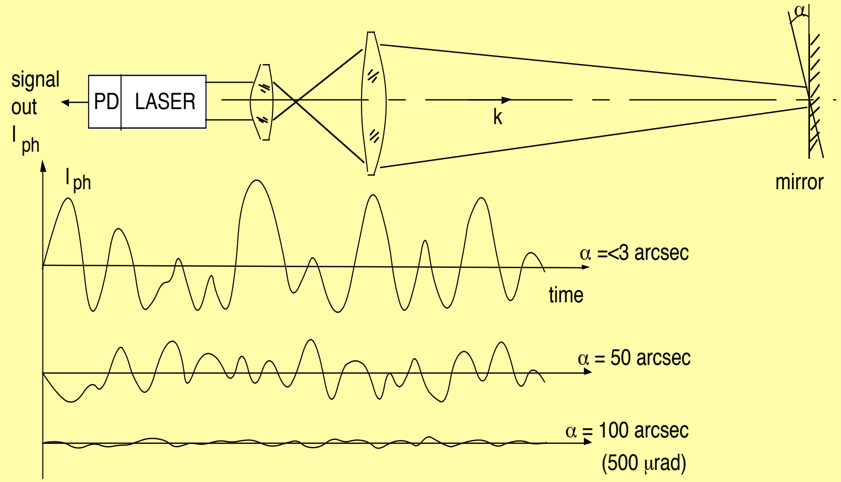
- angle measuremt (autocollimator)
- a very sensitive *optical echo detector* for:
    - return-loss measurements
    - CD pit readout
    - scroll sensor gauge

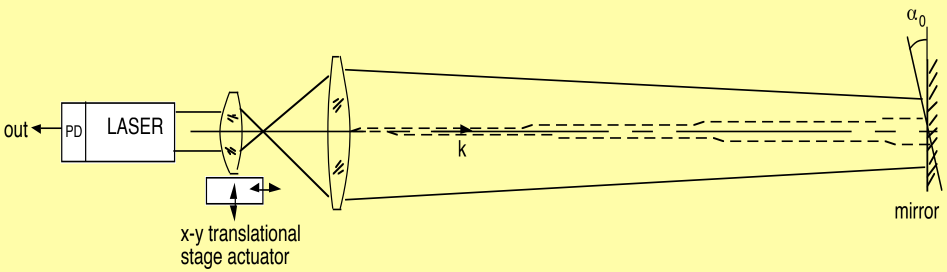
whereas looking at waveform details of the selmix signal we can go back to physical parameters like:

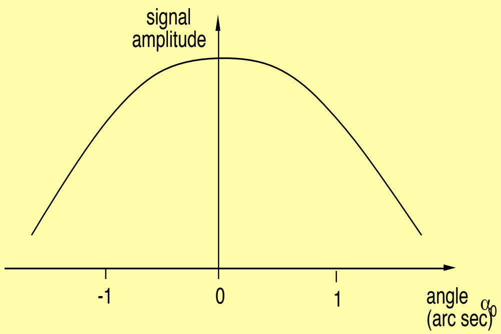
- laser linewidth $\Delta\nu$
- linewidth enhancement factor α

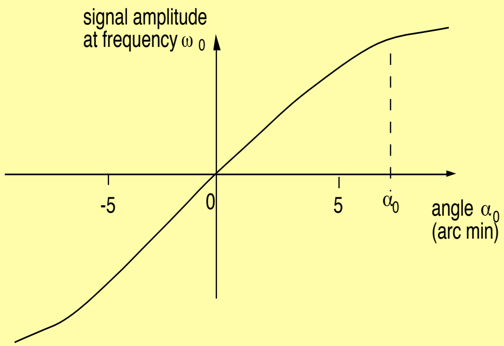
Angle measurement

Matsumoto (Appl Opt. 19, 1980) used microphonic (stray) vibrations as a signal to align a remote mirror to He-Ne beam, getting sensitivity down to 3 arcsec





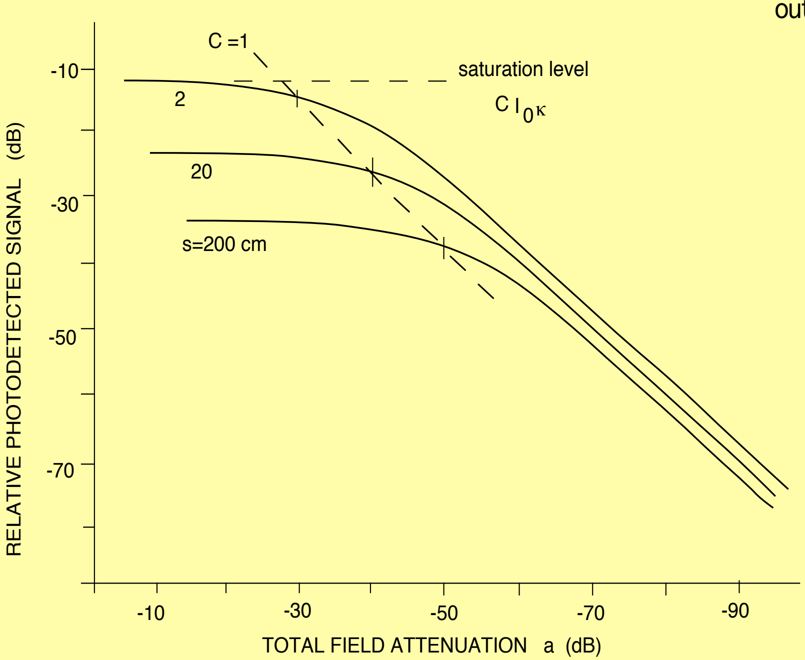


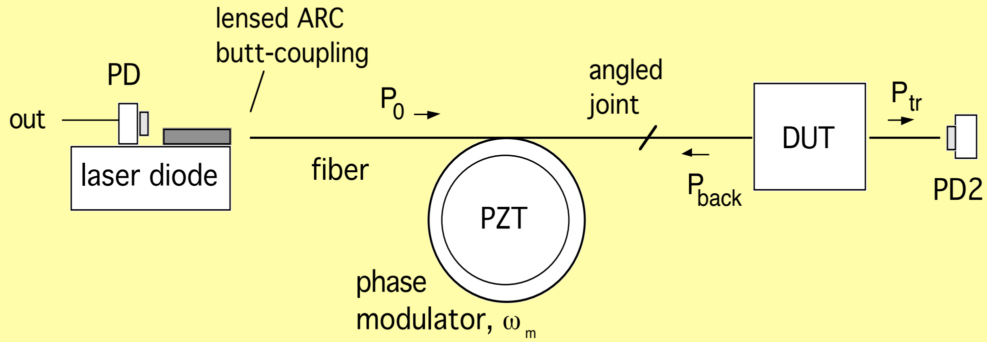


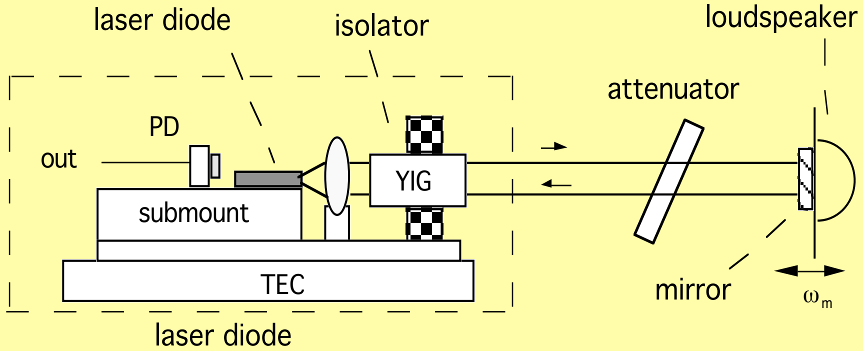
An improved version of the selmix autocollimator uses modulation of the aiming angle (by means of PZT), so that response is trans-formed from quadratic to linear and dynamic range is expanded.

Noise-limited resolution is ≈0.2 arcsec and dynamic range is ≈5 arcmin [Donati, Giuliani: Opt. Engin. vol.40, 2001, pp.95-99] α

Return loss and Isolation loss



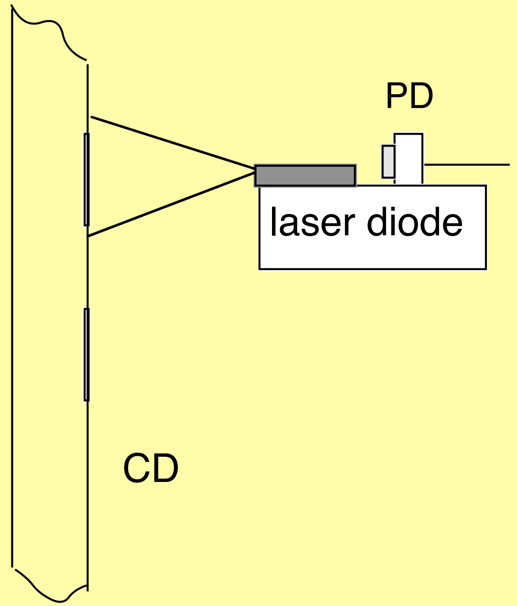


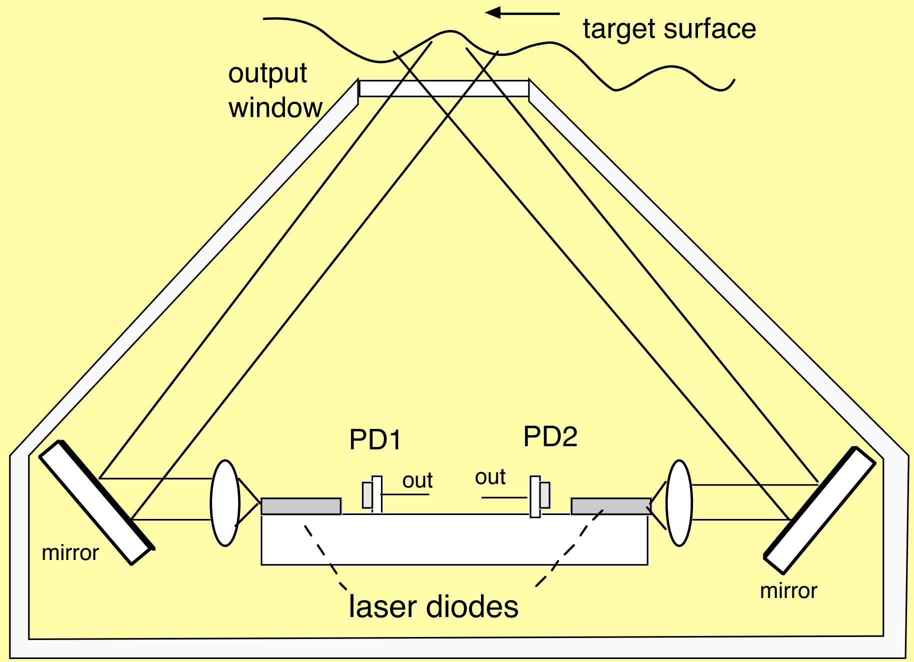


To measure the return loss or the isolation loss, we add modulation of a path length at ω_0 through an in-line PZT Φ-modulator or a remote loudspeaker.The SMI signal output is then on a carrier ω_0 and its amplitude provides the RL or the IL.

S.Donati, M.Sorel: Proc.OFC'97, paper WJ8; Phot Techn Lett.28 (1996),pp.43-49

Consumer applications: CD readout and Scroll sensor





Unwritten portions of the CD surface reflect light and give a large signal, whereas pits diffuse light and give a small return. Signal is detected by the rear PD photodiode.
Ukita,Uenishi,Katagiri:Appl.Opt. 33,1994 pp.5557-63

Two laser beams shine the target at ±45°. Signals 2k×s returned to each laser are opposite in sign, and after subtraction of PD currents, speed and the direction of external target are obtained.
Hewett: Meas.Sci. Technol., 13,2002, pp.2001-06; also: Philips US Patent

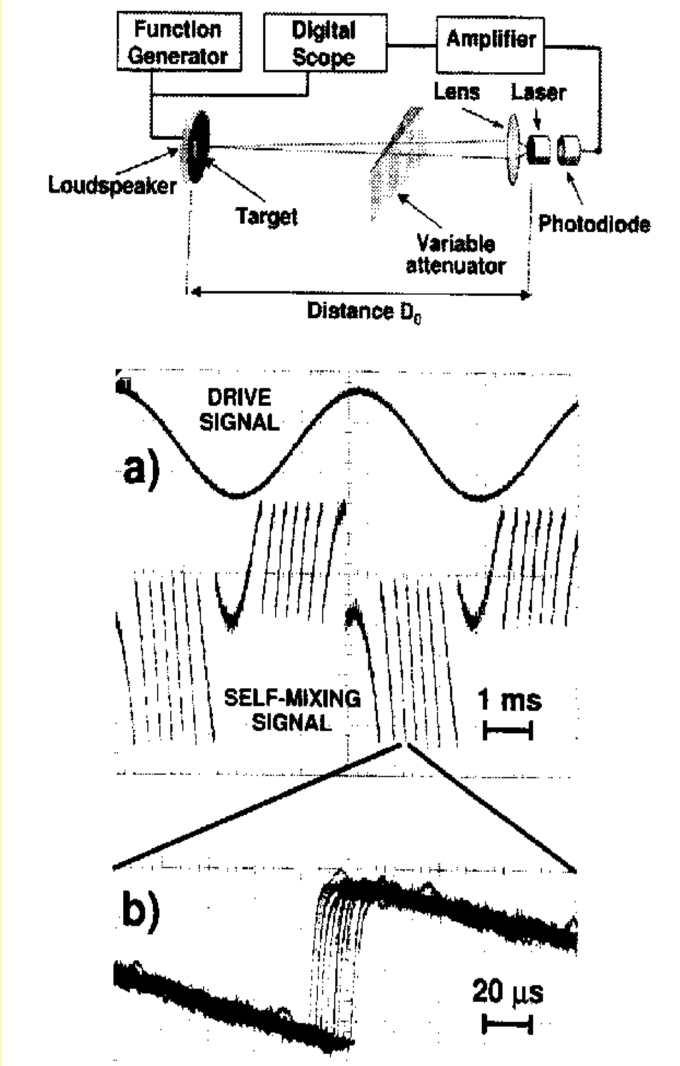
Laser linewidth measured by self-mix

phase variance caused by a target displacement ΔL around L_0 is:

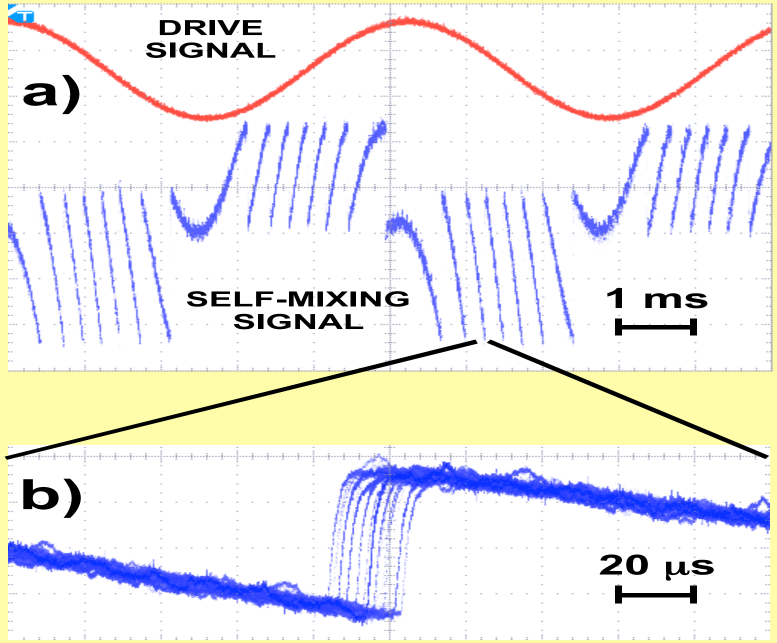
$$\langle \Delta\phi^2 \rangle = \langle \Delta(2ks)^2 \rangle$$
$$= 4\,[k_0^2 \langle \Delta L^2 \rangle + L_0^2 \langle \Delta k^2 \rangle]$$
$$= (4\pi/c)^2\,[\nu_0^2 \langle \Delta L^2 \rangle + L_0^2 \langle \Delta \nu^2 \rangle]$$

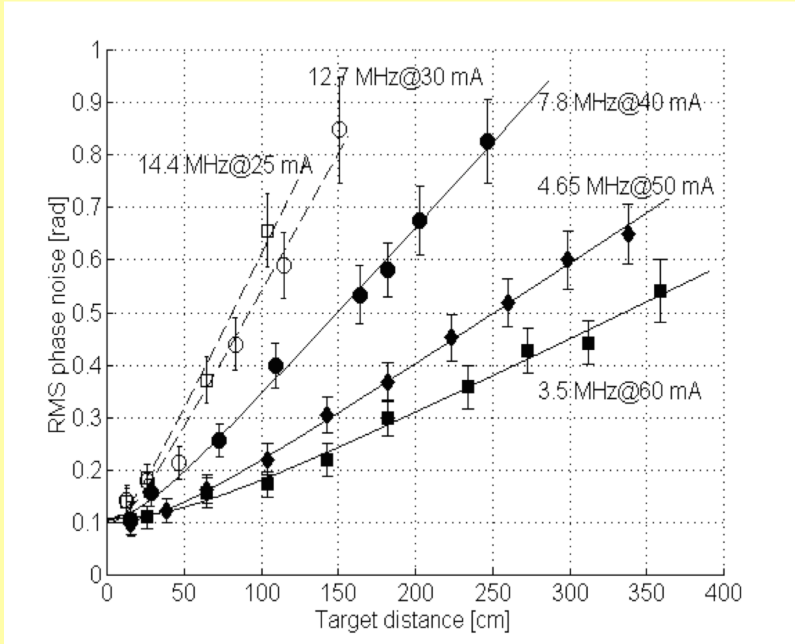
applying a sawtooth drive, we can measure the phase jitter $\langle \Delta\phi^2 \rangle$ and fit it to a line $L_0^2 \langle \Delta\nu^2 \rangle$ + const.

The method gives the coherence length as $L_c = c/\sqrt{\langle \Delta\nu^2 \rangle}$ and it requires much less lab space that the usual one based on arm mismatch (or delayed heterodyne)



Laser linewidth: results

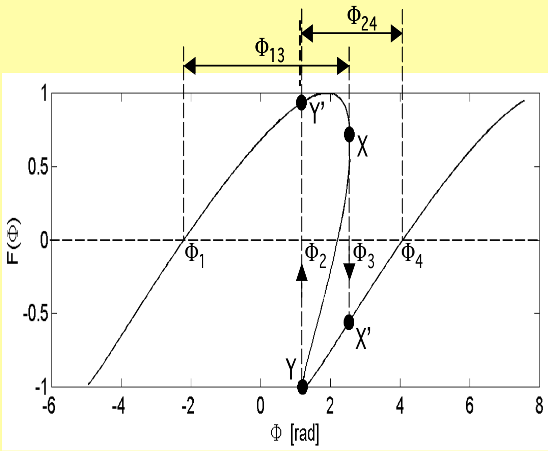


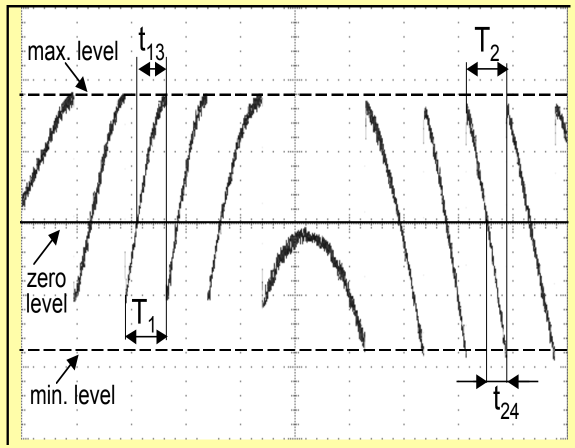


Jitter of switching time (measured at left) allows us to compute the phase variance $\langle\Delta\phi^2\rangle$ dependence versus L_0^2 and hence the linewidth. Typical range of measured linewidth $\Delta\nu$ is 0.5 to 50 MHz at L_0= 2 m.

G.Giuliani, M.Norgia: Phot.Techn.Lett,. vol.PTL-12 2000, pp.1028-30

Measuring the α (linewidth enhancement factor) by self-mixing - of course





The F(ϕ) waveform depends on the alfa factor, as

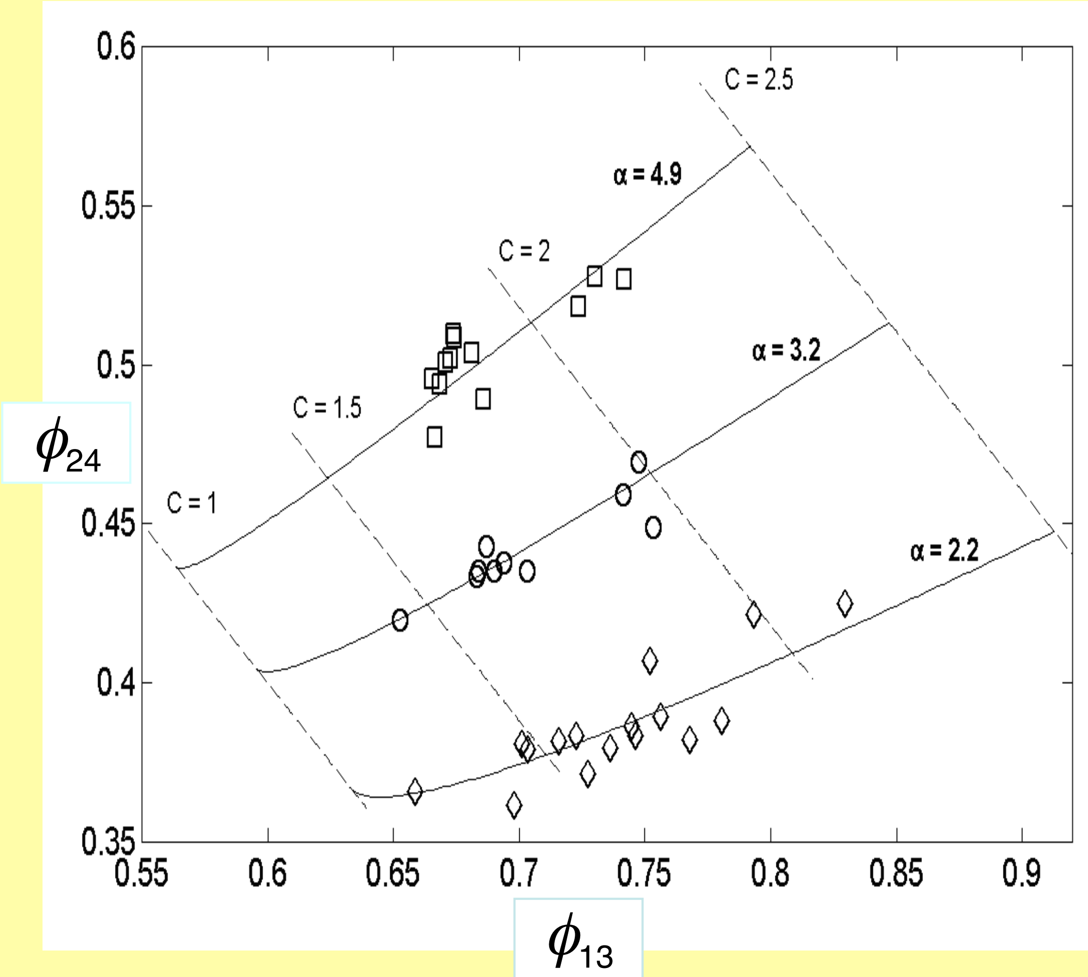
$$\phi_{13} = \sqrt{C^2 - 1} + \frac{C}{\sqrt{1+\alpha^2}} + \arccos(-\frac{1}{C}) - \arctan(\alpha) + \frac{\pi}{2}$$

$$\phi_{24} = \phi_{24} + 2\arctan(\alpha) + \pi$$

we first draw a nomogram of (ϕ_{13} ϕ_{24}) as a function of (α,C), then measure waveform details and plot the resulting ϕ's to extract parameters related to α

α - linewidth enhancement factor: results

The method also provides the C - factor of injection

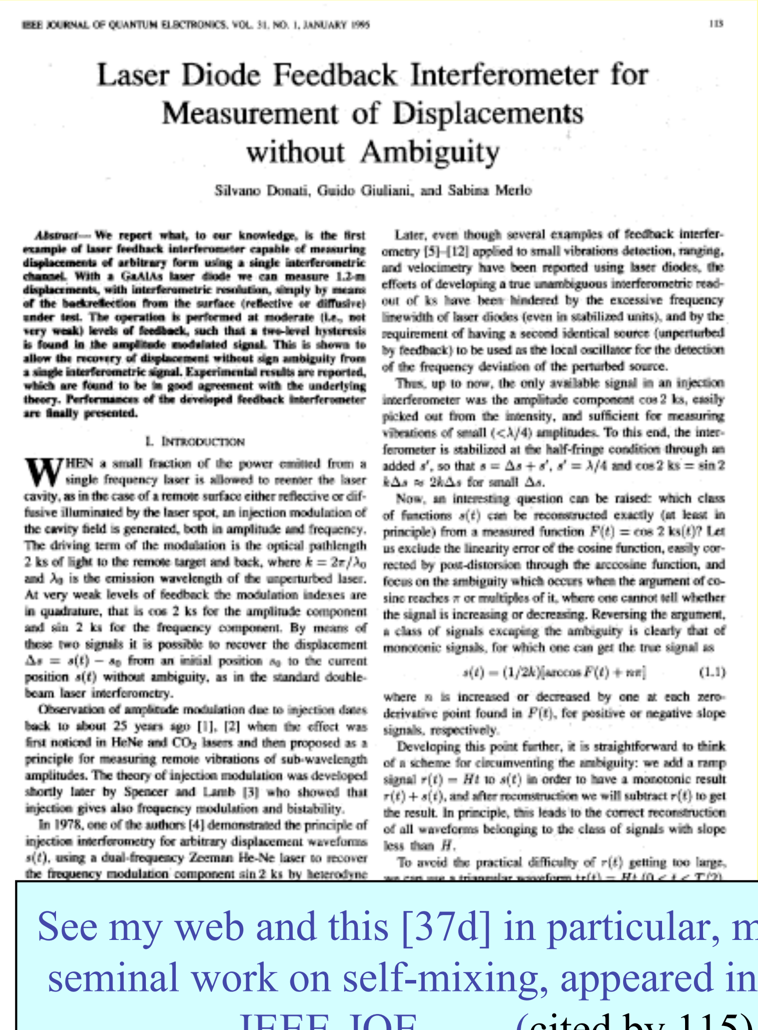


Y.Yu, G.Giuliani, S.Donati: Phot.Techn.Lett,. 16, 2004, pp.990-93 cited by 32

in conclusion …

COUPLING PHENOMENA PROVIDE A RICH PHENOMENOLOGY, AND ARE USEFUL FOR NEW *INSTRUMENTAL* TECHNIQUES FOR THE MEASUREMENTS OF DISTANCE AND OF PHYSICAL PARAMETERS

To probe further, part I



IEEE JOURNAL OF QUANTUM ELECTRONICS, VOL. 31, NO. 1, JANUARY 1995 113

Laser Diode Feedback Interferometer for Measurement of Displacements without Ambiguity

Silvano Donati, Guido Giuliani, and Sabina Merlo

***Abstract*— We report what, to our knowledge, is the first example of laser feedback interferometer capable of measuring displacements of arbitrary form using a single interferometric channel. With a GaAlAs laser diode we can measure 1.2-m displacements, with interferometric resolution, simply by means of the backreflection from the surface (reflective or diffusive) under test. The operation is performed at moderate (i.e., not very weak) levels of feedback, such that a two-level hysteresis is found in the amplitude modulated signal. This is shown to allow the recovery of displacement without sign ambiguity from a single interferometric signal. Experimental results are reported, which are found to be in good agreement with the underlying theory. Performances of the developed feedback interferometer are finally presented.**

I. INTRODUCTION

WHEN a small fraction of the power emitted from a single frequency laser is allowed to reenter the laser cavity, as in the case of a remote surface either reflective or diffusive illuminated by the laser spot, an injection modulation of the cavity field is generated, both in amplitude and frequency. The driving term of the modulation is the optical pathlength 2 ks of light to the remote target and back, where $k = 2\pi/\lambda_0$ and λ_0 is the emission wavelength of the unperturbed laser. At very weak levels of feedback the modulation indexes are in quadrature, that is cos 2 ks for the amplitude component and sin 2 ks for the frequency component. By means of these two signals it is possible to recover the displacement $\Delta s = s(t) - s_0$ from an initial position s_0 to the current position $s(t)$ without ambiguity, as in the standard double-beam laser interferometry.

Observation of amplitude modulation due to injection dates back to about 25 years ago [1], [2] when the effect was first noticed in HeNe and CO_2 lasers and then proposed as a principle for measuring remote vibrations of sub-wavelength amplitudes. The theory of injection modulation was developed shortly later by Spencer and Lamb [3] who showed that injection gives also frequency modulation and bistability.

In 1978, one of the authors [4] demonstrated the principle of injection interferometry for arbitrary displacement waveforms $s(t)$, using a dual-frequency Zeeman He-Ne laser to recover the frequency modulation component sin 2 ks by heterodyne

Later, even though several examples of feedback interferometry [5]–[12] applied to small vibrations detection, ranging, and velocimetry have been reported using laser diodes, the efforts of developing a true unambiguous interferometric readout of ks have been hindered by the excessive frequency linewidth of laser diodes (even in stabilized units), and by the requirement of having a second identical source (unperturbed by feedback) to be used as the local oscillator for the detection of the frequency deviation of the perturbed source.

Thus, up to now, the only available signal in an injection interferometer was the amplitude component cos 2 ks, easily picked out from the intensity, and sufficient for measuring vibrations of small ($<\lambda/4$) amplitudes. To this end, the interferometer is stabilized at the half-fringe condition through an added s', so that $s = \Delta s + s'$, $s' = \lambda/4$ and cos 2 ks = sin 2 $k\Delta s \approx 2k\Delta s$ for small Δs.

Now, an interesting question can be raised: which class of functions $s(t)$ can be reconstructed exactly (at least in principle) from a measured function $F(t) = \cos 2\, ks(t)$? Let us exclude the linearity error of the cosine function, easily corrected by post-distorsion through the arccosine function, and focus on the ambiguity which occurs when the argument of cosine reaches π or multiples of it, where one cannot tell whether the signal is increasing or decreasing. Reversing the argument, a class of signals excaping the ambiguity is clearly that of monotonic signals, for which one can get the true signal as

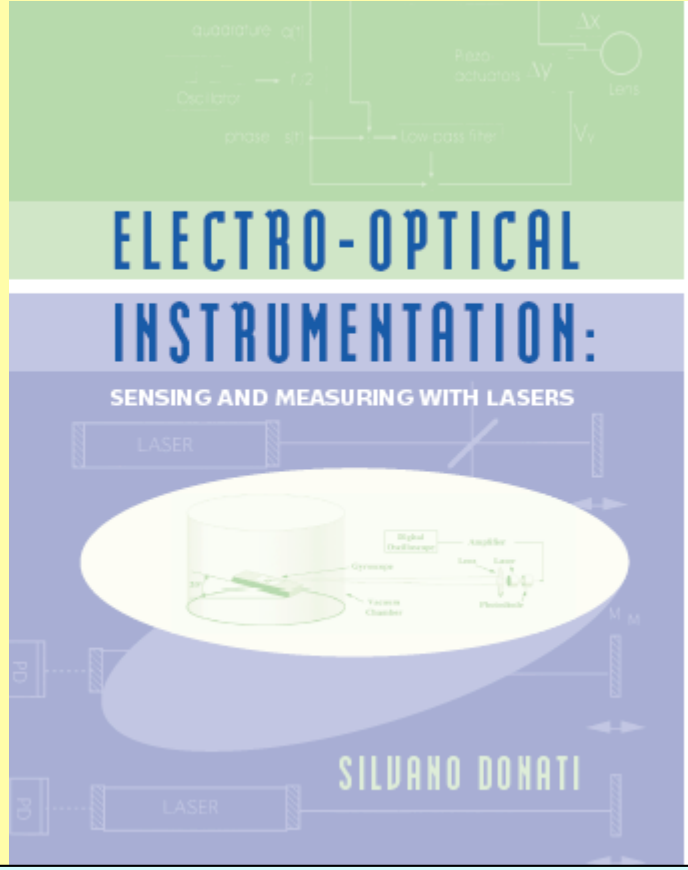
$$s(t) = (1/2k)[\arccos F(t) + n\pi] \quad (1.1)$$

where n is increased or decreased by one at each zero-derivative point found in $F(t)$, for positive or negative slope signals, respectively.

Developing this point further, it is straightforward to think of a scheme for circumventing the ambiguity: we add a ramp signal $r(t) = Ht$ to $s(t)$ in order to have a monotonic result $r(t) + s(t)$, and after reconstruction we will subtract $r(t)$ to get the result. In principle, this leads to the correct reconstruction of all waveforms belonging to the class of signals with slope less than H.

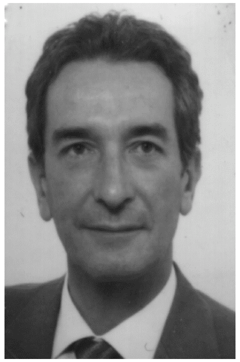
To avoid the practical difficulty of $r(t)$ getting too large,

See my web and this [37d] in particular, my seminal work on self-mixing, appeared in IEEE-JQE (cited by 115)



A book on electrooptical methods for measurements, Prentice Hall 2004, treats self -mix interferometry in detail

I acknowledge my Group of "Optoelectronics", University of Pavia



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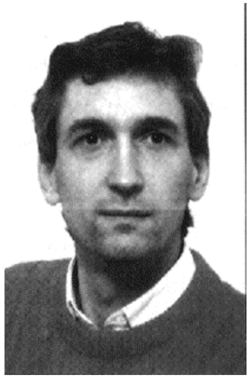

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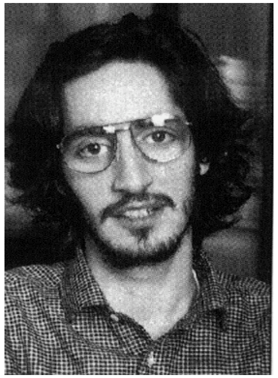

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and the Photonics Society of IEEE for awarding me the DL

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