“Optical Measurements”

Master Degree in Engineering
Automation-, Electronics-, Physics-,
Telecommunication- Engineering

LASER Sources:
Properties and Applications

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Main Properties of LASER Sources

- Properties of LASERS
  - **Monochromaticity** ($\Delta\nu_{\text{laser}} \sim 10^{-6} \div 10^{-9} \times \Delta\nu_{\text{spectral lamp}}$)
  - **Brightness** $B=P/A\Omega$ ($B_{\text{laser}}=10^5 \div 10^8$ W/m^{2}sr)
  - **Stability** in amplitude ($\Delta P/P \sim 10^{-5}$) and frequency ($\Delta\nu/\nu \sim 10^{-15}$)
  - **Ultrashort pulses** ($\sim 10^{-15}$ s), **high peak power** ($\sim 10^{15}$ W)
  - **Size** (from ~1 μm up to a few km)
  - Propagation: in **free space** or in **optical fibers**
  - Commercialization: **reduced costs** and **unbulky**

Typical properties and Applications of LASER Sources

- Typical properties of LASERs
  - **Spatial quality of the beam** SPATIAL COHERENCE
  - **Spectral quality ("monochromaticity")** TEMPORAL COHERENCE
  - **Wavelength**
  - **Optical Power or pulse energy**
  - **“SOP” State Of Polarization**

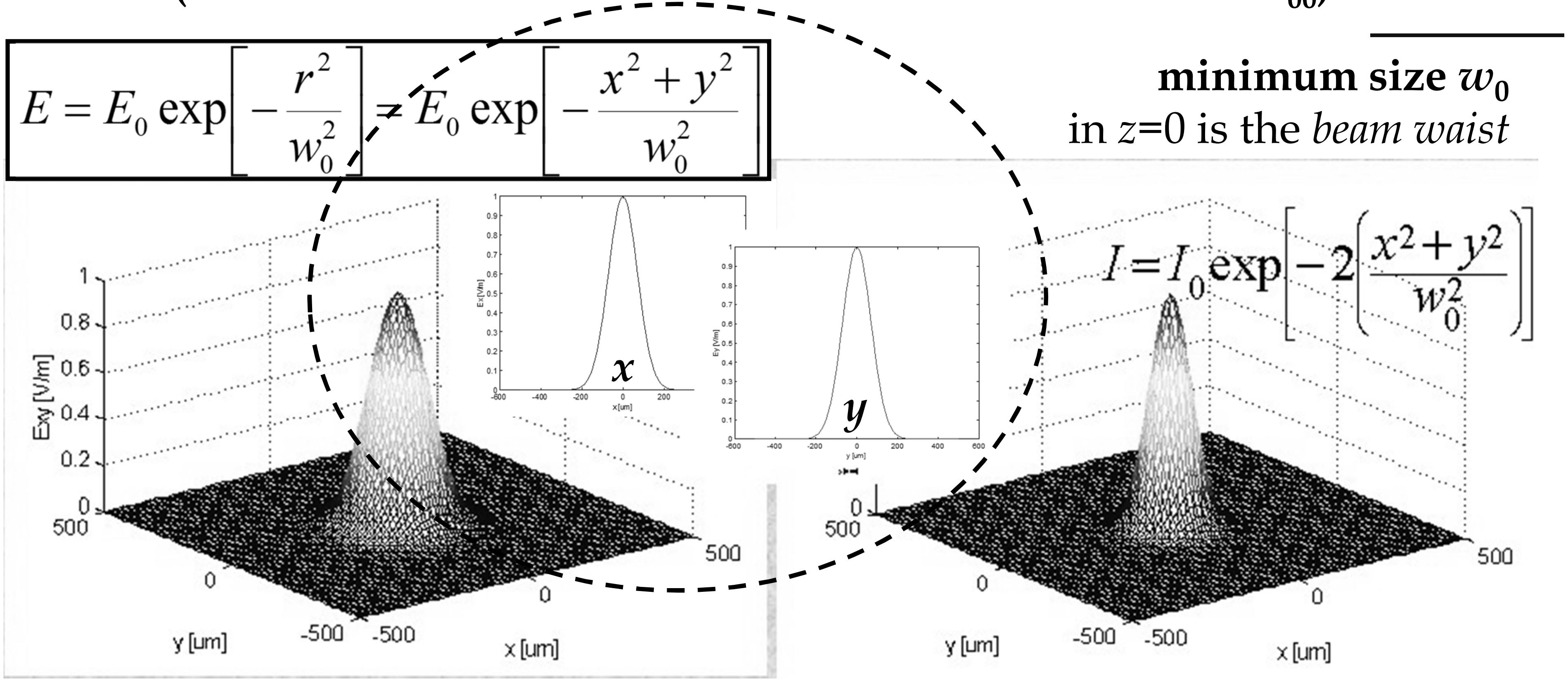
- Applications of LASERs
  - Experiments of Physics, Metrology, Telemetry, Interferometry, Optical Communications, industrial processing, printers, scanners, bar-code readers, pointers, Optical References, **optical measurement and optoelectronic sensors**

Properties of LASER beams

(TRANSVERSE PROFILE and fundamental mode TEM$_{00}$)

$$\boxed{E = E_0 \exp\left[-\frac{r^2}{w_0^2}\right] = E_0 \exp\left[-\frac{x^2+y^2}{w_0^2}\right]}$$

minimum size w_0
in z=0 is the *beam waist*



$$I = I_0 \exp\left[-2\left(\frac{x^2+y^2}{w_0^2}\right)\right]$$

- Optical intensity profile [W/m^2] in a plane (***x-y***) transverse to the propagation direction (z): depends on the spatial mode and its “width”
- Fundamental LASER mode TEM$_{00}$ is symmetric in both directions x and y with a GAUSSIAN profile, both for electric field and intensity, with a spot size or “width” w_0 , *beam waist*, (37%E_0 o 13.5%I_0) [$w_0 \rightarrow w$ for $z \neq 0$]

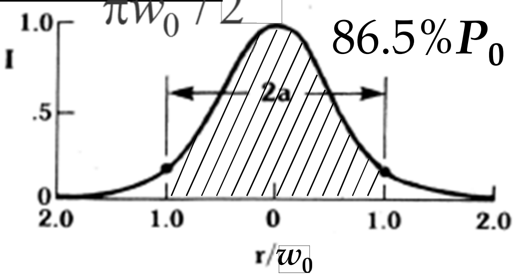
Properties of LASER beams

(TRANSVERSE PROFILE and "width" of the spot – *spot size*)

$$I(r)=I_0\exp\left(-2\frac{r^2}{w_0^2}\right)\quad \text{with}\quad I_0=\frac{P_0}{\pi w_0^2/2}$$

The optical power P is the integral of the intensity I over a collecting surface S (*e.g.* circle of radius r)

$$\mathbf{I}_{(\mathbf{r})}=\frac{\mathbf{2\,P_0}}{\pi w_0^2}\mathbf{e}^{-2r^2/w_0^2}$$



$$P(S)=P(r)=\int_S I\,\mathrm{d}S=I_0\int_0^r \exp\left[-2\left(\rho^2/w_0^2\right)\right]2\pi\rho\,\mathrm{d}\rho\quad \text{with}\quad S=\pi r^2$$

by substituting $\xi=2\dfrac{\rho^2}{w_0^2}$ and $P_0=\dfrac{1}{2}\pi w_0^2 I_0$ and integrating we obtain :

$$P(r)=P_0\int_0^{2(r^2/w_0^2)} \mathrm{e}^{-\xi}\,\mathrm{d}\xi=P_0\left[1-\exp\left(-2\frac{r^2}{w_0^2}\right)\right]$$

if e^{-2}=13.5%

$1-e^{-2}$=86.5%

power collected within a circle of radius r **P_0 is the whole power of the beam** (with r "big" $r\rightarrow\infty$)

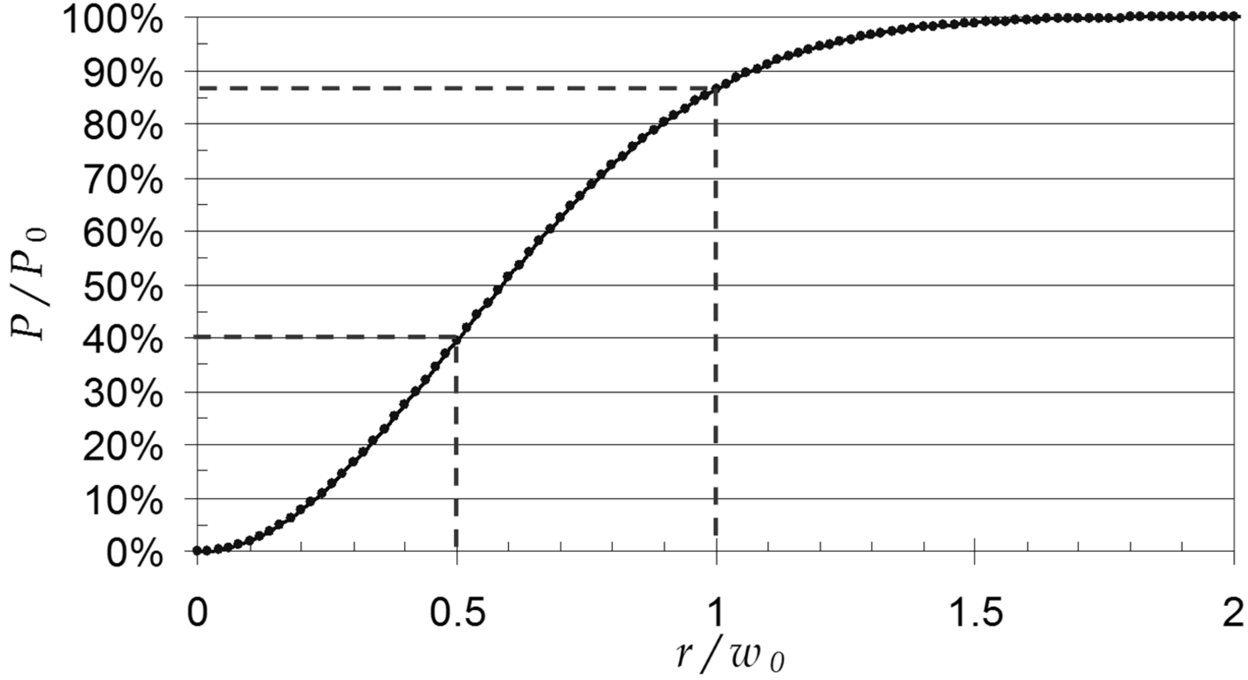
- **Within a circle of radius $r=w_0$ we find <u>86.5% of the whole power P_0</u>** of the laser beam. We shall call w_0 **the beam (standard) <u>spot size</u> or <u>*beam waist*</u> of the laser beam (at 1/e² of the peak intensity)**

Properties of LASER beams

(TRANSVERSE PROFILE and integration over a circle)

Power P (normalized to P_0) as the integral of the intensity I over a circle of radius r (normalized to w_0)

$$P(r)/P_0 = \int_0^{2(r^2/w_0^2)} \mathrm{e}^{-\xi}\,\mathrm{d}\xi = \left[1-\exp\left(-2\frac{r^2}{w_0^2}\right)\right]$$



- **For r =(0.5, 1, 2) $\times w_0$ we get P=(40%, 86.5%, 99.97%) $\times P_0$**

Properties of LASER beams

(TRANSVERSE PROFILE and higher order modes TEM$_{lm}$)

- More in general the transverse profile of the electric field (transverse mode) can be described by the product of two (Hermite) polynomials, in directions x and y, and a 2D Gaussian curve with radial symmetry. Index, hence degree, of the two polynomials sets the **order of the mode: TEM$_{lm}$**

$$E = E_0 H_l^{(x)}\left(\frac{x}{w_0/\sqrt{2}}\right) H_m^{(y)}\left(\frac{y}{w_0/\sqrt{2}}\right)\exp\left[-\frac{x^2+y^2}{w_0^2}\right]$$

$H_0(x) = 1$
$H_1(x) = 2x$
$H_2(x) = 4x^2 - 2$
$H_3(x) = 8x^3 - 12x$
$H_4(x) = 16x^4 - 48x^2 + 12$
$H_5(x) = 32x^5 - 160x^3 + 120x$

Hermite polynomials
of order 0, 1, 2, ..., 5

Zero order = unitary constant

Odd/even order corresponds to odd/even symmetry

i-th order $\Rightarrow$ i “zeros” on the axis

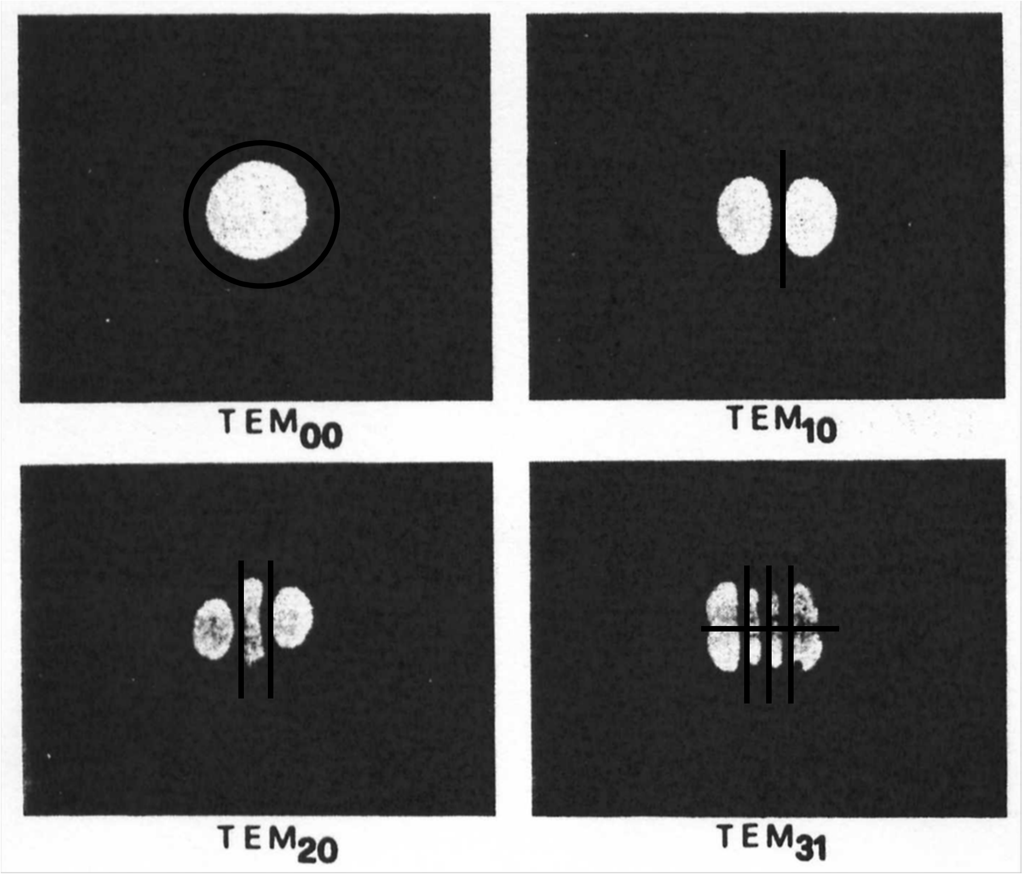
Properties of LASER beams

(TRANSVERSE PROFILE and first higher order modes TEM$_{lm}$)

$$E = E_0 H_l\left(\frac{\sqrt{2}x}{w_0}\right) H_m\left(\frac{\sqrt{2}y}{w_0}\right)\exp\left[-\frac{x^2+y^2}{w_0^2}\right]$$

The order of the polynomial tells us **how many zeros cut the Gaussian curve along the corresponding axis**

- For l=0 and m=0, from $H_0 \equiv 1$ we get TEM$_{00}$ with Gaussian (2D) profile



Also for higher order modes we define a **spot size** **equal to the radius $w_{0,lm}$ of the circle containing 86.5% of the whole optical power of the mode**

At increasing mode order, the spot size increases and **divergence** as well!!! (spatial beam quality gets worse compared to TEM$_{00}$)

then $w_0 \rightarrow w(z)$…

Free-space propagation

- "Spot broadening" (divergence)… $w=w(z)$

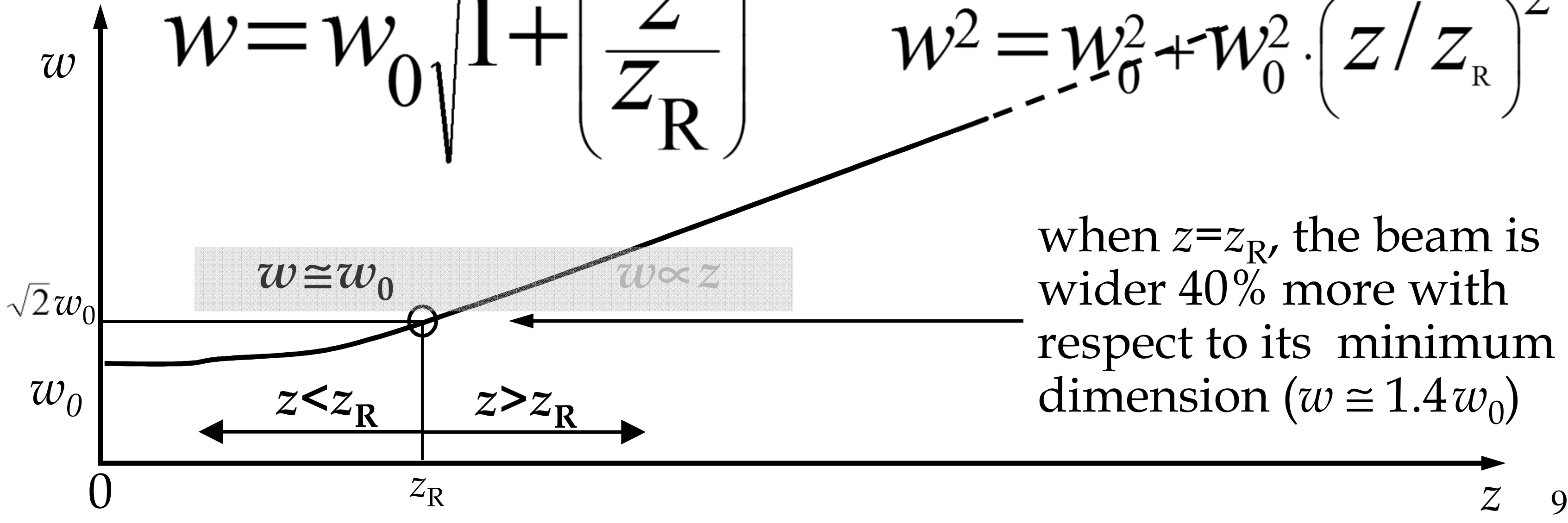
$$w^2 = w_0^2 + \left(\lambda z / \pi w_0\right)^2$$

"*w* grows" during propagation of the beam/mode along *z* axis

We call **Raileigh distance/range** $\boxed{z_R = \frac{\pi w_0^2}{\lambda}}$

$$w = w_0 \sqrt{1 + \left(\frac{z}{z_R}\right)^2}$$

$$w^2 = w_0^2 + w_0^2 \cdot \left(z / z_R\right)^2$$



when $z=z_R$, the beam is wider 40% more with respect to its minimum dimension ($w \cong 1.4 w_0$)

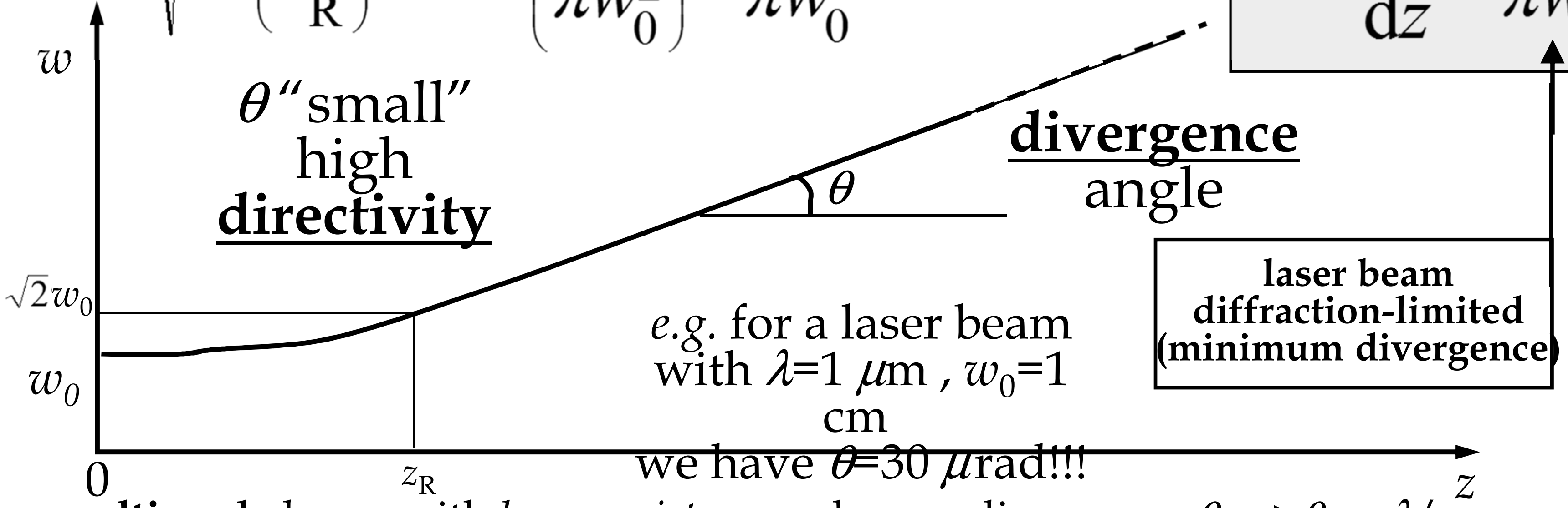
Near/far field and beam divergence

- We have two asymptotic work regions:
  near field when $z \ll z_R$ and the “beam is collimated”

$$w = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2} \cong w_0$$

far field when $z \gg z_R$ and the “beam is divergent" (linearly)

$$w = w_0\sqrt{1+\left(\frac{z}{z_R}\right)^2} \cong w_0\left(\frac{z\lambda}{\pi w_0^2}\right) = \frac{\lambda}{\pi w_0} z = \theta z \quad \text{with} \quad \boxed{\theta = \frac{\mathrm{d}w}{\mathrm{d}z} = \frac{\lambda}{\pi w_0}}$$



θ “small”
high
directivity

divergence
angle

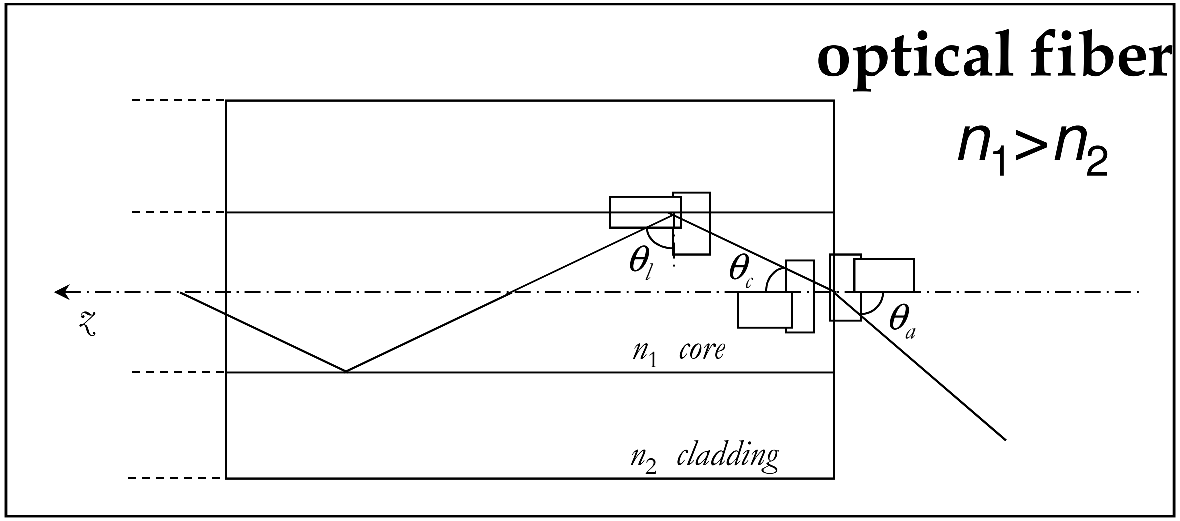
e.g. for a laser beam with λ=1 μm , w_0=1 cm
we have θ=30 μrad!!!

laser beam
diffraction-limited
(minimum divergence)

In a **multimode** beam with *beam waist* w_0 we have a divergence $\theta_{MM} > \theta_{DL} = \lambda/\pi w_0$ and can define a factor $\boldsymbol{M^2 = (\theta_{MM}/\theta_{DL}) > 1}$ stating the spatial quality of the beam

Guided propagation (optical fiber)

- **Guided mode** HE_{11}
- **Low attenuation** $\alpha < 0.2$ dB/km at 1.55 μm (= -4.5%/km)



SM fiber

$n \cong 1.45$

$\Delta n \cong 5 \times 10^{-3}$

ϕ_{core}**=9 µm**

ϕ_{clad}**=125 µm**

$$NA = \sin\theta_a = \sqrt{n_1^2 - n_2^2} = \sqrt{(n_1 + n_2)(n_1 - n_2)} = \sqrt{2n\Delta n}$$

Numeric **A**perture

Problems: Chromatic dispersion [ps/(nm×km)]
Polarization dispersion [ps/km$^{-1/2}$]

For PMD “dispersion” is differential delay, st.dev. in [ps], from a variance [ps^2/km] after a propagation length L [km]

Properties of LASER beams

(AMPLITUDE NOISE)

- Electric field varying in time with **<u>amplitude fluctuations</u>**

$$E(t) = E_0\left[1 + \underset{\longleftrightarrow}{a(t)}\right]\exp\left[-j2\pi\nu_0 t\right] \qquad \text{with } a(t) << 1$$

- A small-variations analysis of the laser system (in response to small variations of the pump rate or of the cavity losses) indicates the phenomenon of **relaxation oscillations**
  - oscillation frequency $$f_{RIN} = \left[\frac{x-1}{\tau_c \tau_{sp}}\right]^{1/2}$$ ($x = P/P_{\mathrm{th}}$ is the extra-threshold)
  - (exponential) decay time $$\tau_{RIN} = \frac{2\tau_{sp}}{x}$$

Need for **<u>stabilization</u> systems** (passive/active)

Properties of LASER beams
(FREQUENCY NOISE)

- Electric field varying in time with **frequency fluctuations**

$$E(t)=E_0\exp\left\{-j\left[2\pi\nu_0 t+\underbrace{\phi(t)}\right]\right\} \text{ with } 1/(2\pi)\,\mathrm{d}\phi/\mathrm{d}t = \Delta\nu \ll \nu_0$$

“instantaneous” frequency or **average frequency**?

$$\boldsymbol{\nu(t)} = [1/(2\pi)]\,\mathrm{d}\phi_{\mathrm{tot}}/\mathrm{d}t = \nu_0+(1/2\pi)\,\mathrm{d}\phi/\mathrm{d}t = \boldsymbol{\nu_0+\Delta\nu}$$

- From the equation of the resonator mode frequencies:

$$\nu = m\cdot\frac{c}{2L} \quad\Rightarrow\quad \Delta\nu = m\cdot\frac{c}{2L^2}\left(-\Delta L\right) \quad\Rightarrow\quad \frac{\Delta\nu}{\nu} = -\frac{\Delta L}{L}$$

- Strong **dependence of the laser frequency on *L***
  - *e.g.* for an Nd:YAG LASER (λ=1.064 µm, $\nu\cong$300 THz) with L=30 cm if ΔL=-1 μm, one has $\Delta\nu$=1 GHz!!! (1nm→1MHz)

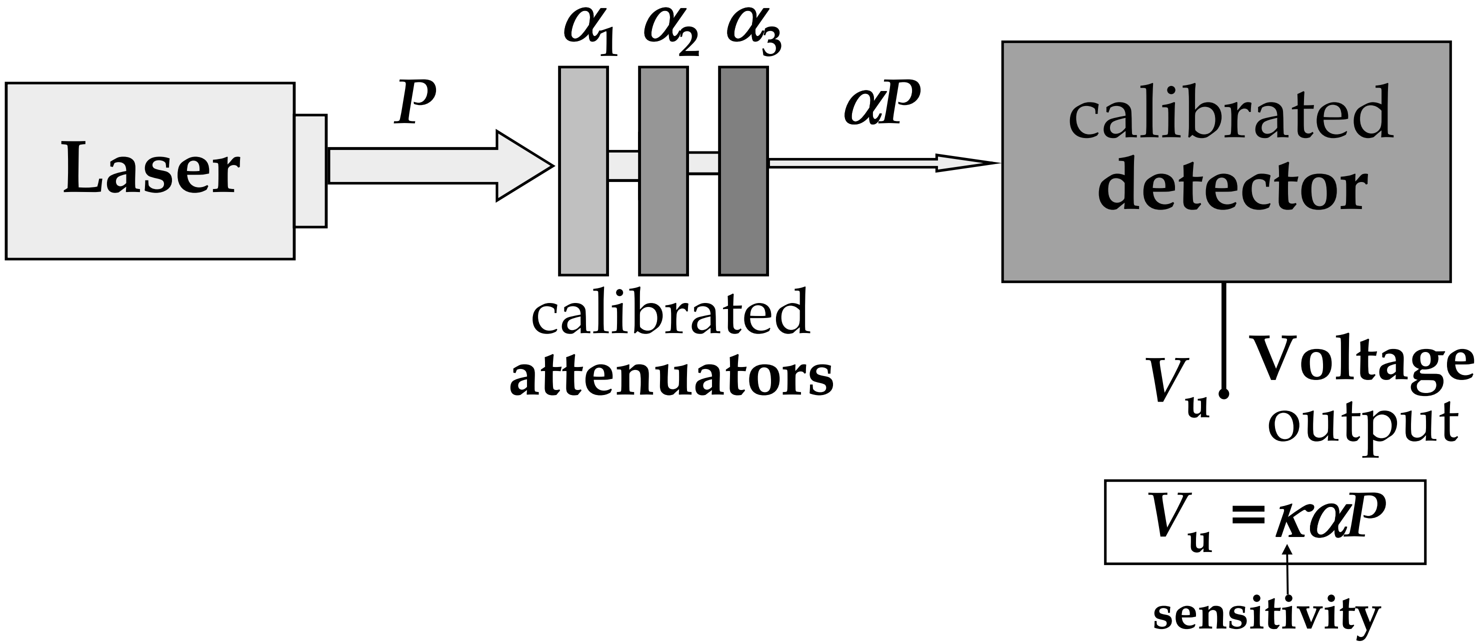
Need for **stabilization systems** (passive/active)

Optical Power

(DEFINITIONS and MEASUREMENT METHODS)

- $E = E_0 \exp(-j\omega_0 t)$ Electric field [V/m]
- $I_0 = \frac{EE^*}{\eta_0} = \frac{E_0^2}{\eta_0}$ Intensity [W/m^2]

  $\eta_0 = (\mu_0/\varepsilon_0)^{1/2} = 377\ \Omega$
  characteristic impedance
  of the vacuum
- $P = \int I \mathrm{d}S$ Power [W] with $E_0 = \sqrt{P_0\ \eta_0 / \left(\pi w_0^2/2\right)} \propto \sqrt{P_0}$



α_1 α_2 α_3

P

αP

Laser

calibrated
detector

calibrated
attenuators

V_u

Voltage
output

$\boxed{V_u = \kappa\alpha P}$

sensitivity

Photo-voltaic /-conductive Detectors

(Si, HgCdTe, Ge, GaAs, GaAsP, InGaAs) $0.1\mu m \le \lambda \le 10\mu m$

$$\boxed{h\nu > E_g}$$

Quantum efficiency $\boxed{\eta = \frac{\Delta N_e}{\Phi \Delta t}} = \frac{\text{(Nr. photoelectrons)}}{\text{(Nr. incident photons)}} \; [\%]$

MICRO

photon flux (#phot./s)

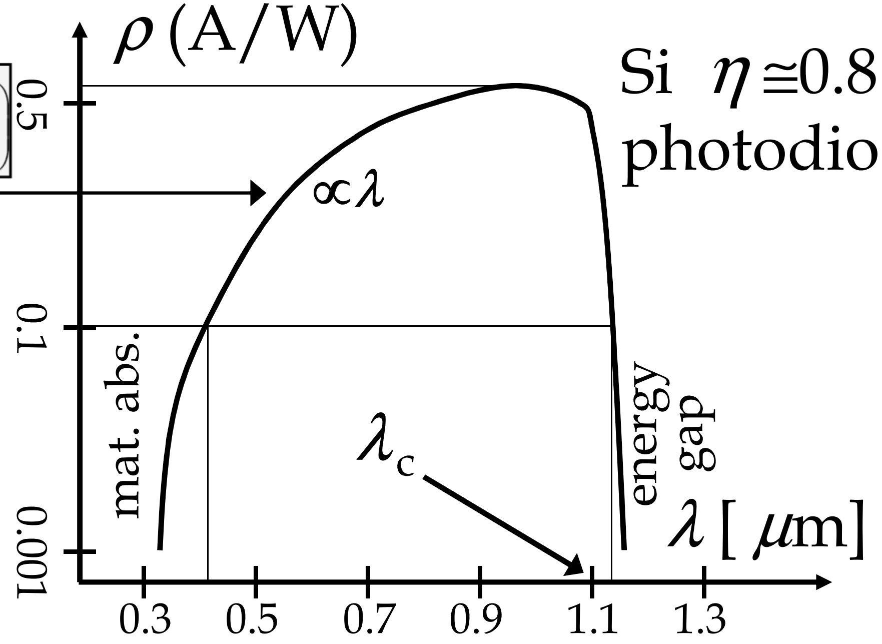
Responsivity $\boxed{\rho = \frac{i}{P} \quad (\text{A/W})}$

MACRO

$$i = \frac{e\Delta N_e}{\Delta t}$$

$$P = \Phi \cdot h\nu$$

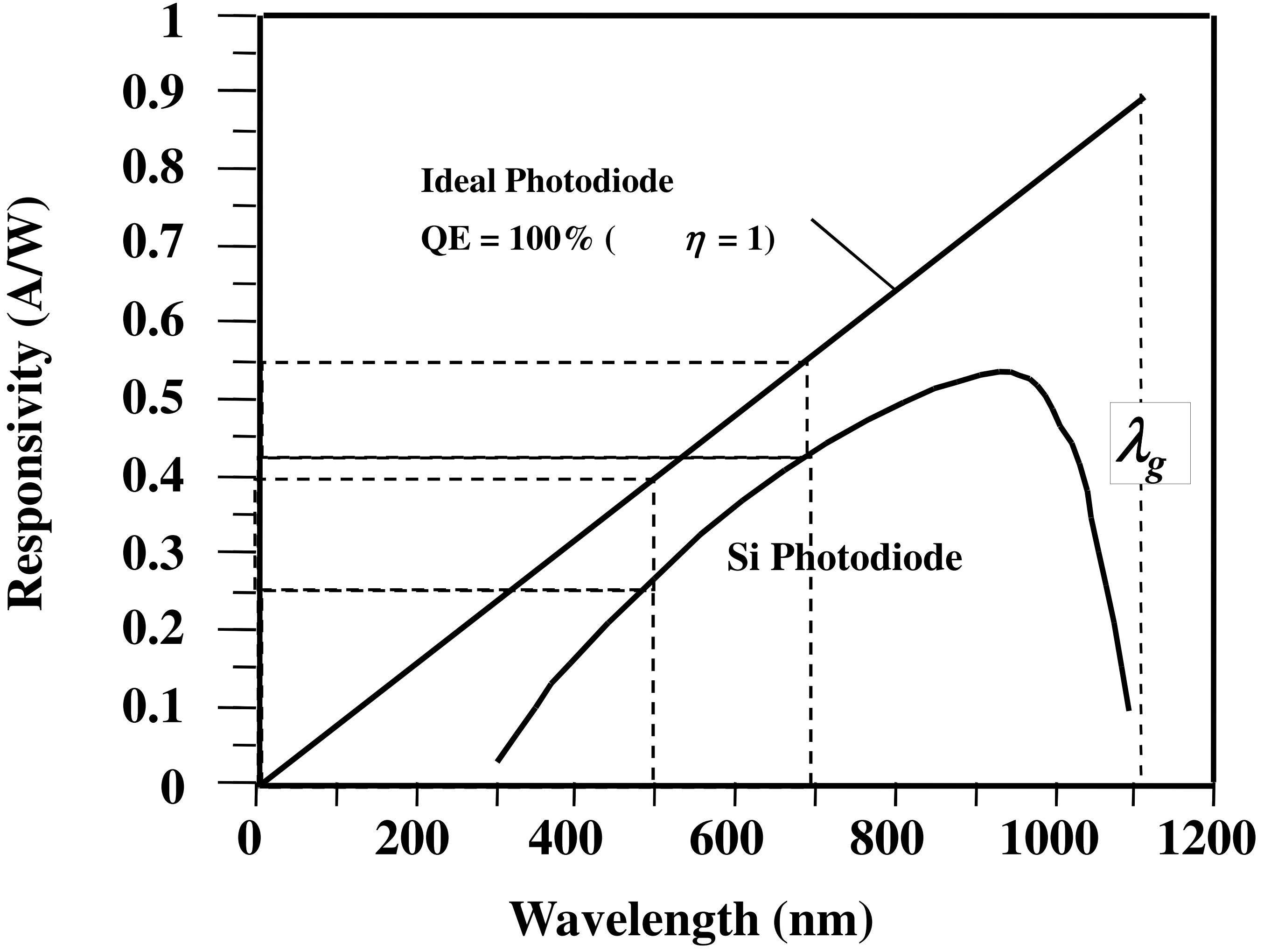
$$\boxed{\rho = \frac{\eta e}{h\nu}} = \frac{\eta e \lambda}{hc}$$



Si $\eta \cong 0.8$
photodiode

Typical Si photodiode responsivity

$\eta_{(500\ \text{nm})}=0.25/0.40=0.623$ $\eta_{(700\ \text{nm})}=0.425/0.55=0.77$



Photodiodes (output current)

When an optical beam (uniform) with intensity I (W/m^2) impinges on a photo detector with surface S (m^2), the collected optical power is

$$\boldsymbol{P = I \cdot S} \quad \textbf{(W)}$$

If the photo detector is a photodiode, the output is a **current signal directly proportion to the optical power impinging on the photo detector**:

$$\boldsymbol{i = \rho \cdot P} \quad \textbf{(A)}$$

where the conversion factor ρ between photocurrent I and detected optical power P is the **sensitivity ("responsivity")** $\boldsymbol{\rho}$ **(A/W)** of the material by which the photodiode is made (ρ_{Si}~0.3-0.4 A/W in the VIS and ρ_{InGaAs}~0.8 A/W in the NIR)

Photodetectors (voltage output)

The photodiode **current** is typically **amplified** by a resistance gain **(transimpedance)**, let's say with gain $G_{i\to v}$ (V/A) = R (Ω), in order to produce an output voltage

$$v = G_{i\to v} \cdot i = G_{i\to v} \cdot \rho \cdot P \qquad \textbf{(V)}$$

Thus the voltage is directly proportional to the optical power (or intensity) on the detector, i.e. the squared amplitude of the electric field (at optical frequency) reaching the detector:

$$v \propto P \propto I \propto EE^* = |E|^2$$

Among the time fluctuations of the electric field in the optical signal, **we can observe/measure** in the voltage signal only those **fluctuations** whose frequencies are **in the passband of the photodetector** (photodiode + trans.amp.)

Direct optical detection

Let us consider the LASER beam electric field

$$E(t)=\overset{\boxed{\text{amplitude (V/m)}}}{\overset{\downarrow}{E_0}}[1+\underbrace{a(t)}_{\substack{\text{amplitude mod.}\\\text{(eventually)}}}]\exp[-j(2\pi\,\overset{\boxed{\text{frequency (THz)}}}{\overset{\downarrow}{\nu_0}}t+\underbrace{\phi(t)}_{\substack{\text{phase\&freq. mod.}\\\text{(eventually)}}})]$$

<u>depend on measurement signal</u>

The “direct” photodetected voltage is

$$v(t) \propto EE^* = (E_0)^2[1+a(t)]^2 \propto P(t)=P_0\alpha_{\text{mod}}(t)$$

All the information about **phase&frequency variations** of the optical signal **gets lost**, while the electric signal shows a **<u>sensitivity</u>** to variations (attenuation) of the **<u>optical power</u>**

Beating of two optical signals

Take two laser beams impinging on the photodetector and, for the sake of simplicity, neglect amplitude fluctuations

$$E_{\mathrm{R}}(t)=E_{\mathrm{D0}}[1+\cancel{\underbrace{a(t)}_{\text{amplitude mod.}}}]\exp[-\mathrm{j}(2\pi\nu_0 t+\underbrace{\phi(t)}_{\text{phase/freq. mod.}})]$$

RECEIVED SIGNAL

$$E_{\mathrm{L}}(t)=E_{\mathrm{L0}}\exp[-\mathrm{j}(2\pi\nu_{\mathrm{L}} t)]$$

For simplicity choose $\phi_{\mathrm{L}}=0$

LOCAL OSCILLATOR

Considering two electric fields, linearly-polarized (in the same direction), the resulting field by superposition in space (hence “sum”) of the two fields is

$$E(t)=E_{\mathrm{R}}(t)+E_{\mathrm{L}}(t)$$

Coherent detection (heterodyne)

The corresponding **optical power** is

$$\boxed{P(t)=\frac{EE^*}{\eta_0}\cdot S=\frac{S}{\eta_0}\left\{\left(E_R E_R^*\right)+\left(E_L E_L^*\right)+\left(E_R E_L^*\right)+\left(E_L E_R^*\right)\right\}= \quad |E|\cdot\sqrt{\frac{S}{\eta_0}}=\sqrt{P}}$$

$$=P_R+P_L+\frac{S}{\eta_0}\left(E_{R0}E_{L0}\right)\exp\{-j[2\pi(\nu_0-\nu_L)t+\phi(t)]\}+\text{c.c.}=$$

interference

beat note frequency

$$=P_R+P_L+\boxed{2\sqrt{P_R P_L}\cos[2\pi(\nu_0-\nu_L)t+\phi(t)]}$$

and **depends on the phase ϕ of the detected optical signal**

Changing the phase $\phi(t)$ the power oscillates between $P_{max}=[(P_R)^{1/2}+(P_L)^{1/2}]^2$ e $P_{min}=\{\text{abs}[(P_R)^{1/2}-(P_L)^{1/2}]\}^2$
Practically we **sum the fields** and hence $P^{1/2}$

$$\boxed{|E|\propto\sqrt{P}}$$

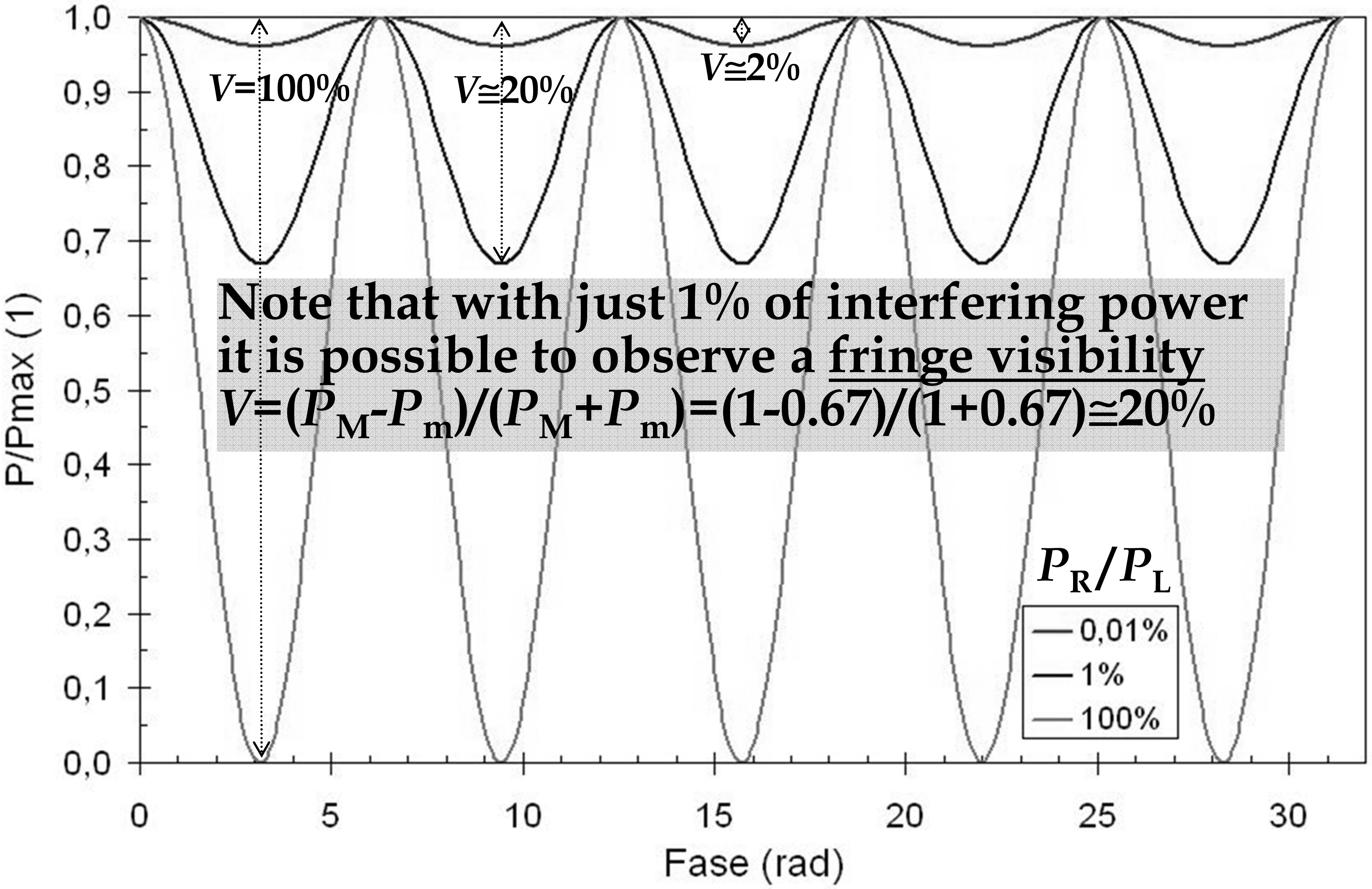
In the particular case where $E_{R0}=E_{L0}=E_0$ and hence $\boldsymbol{P_R=P_L=P_0}$ we obtain **"complete interference between the two signals":**
$\boldsymbol{P_{max}=4P_0}$ **e** $\boldsymbol{P_{min}=0}$ with 2 BS at 50% we get $P_R=P_L=P_0=P_{laser}/4$

Diapositiva 21

CS11 incluso questo lucido è terminata la 4a lez 05-06 in aula N.1.3

Cesare Svelto; 27/03/2006

Interference for different ratios P_R / P_L



Note that with just 1% of interfering power it is possible to observe a fringe visibility $V=(P_M-P_m)/(P_M+P_m)=(1-0.67)/(1+0.67)\cong 20\%$

Measurements with coherent detection

The **variable optical power**, and hence photodetected voltage, is a **function of the phase $\phi(t)$** of the detected signal $E_R(t)$

A **misurand** ***M*** (physical parameter of interest) changing the phase $\phi(t)$ or the frequency $\nu=(1/2\pi)(d\phi/dt)$ of the detected signal can be **coherently detected** and measured **observing variations of phase/frequency of the beat signal** at the intermediate (electronic) frequency $\nu_{IF}=(\nu_R - \nu_L)$, that can easily fall in a radio-frequency bandwidth (tes or hundreds of megahertz) or even at lower electrical frequencies

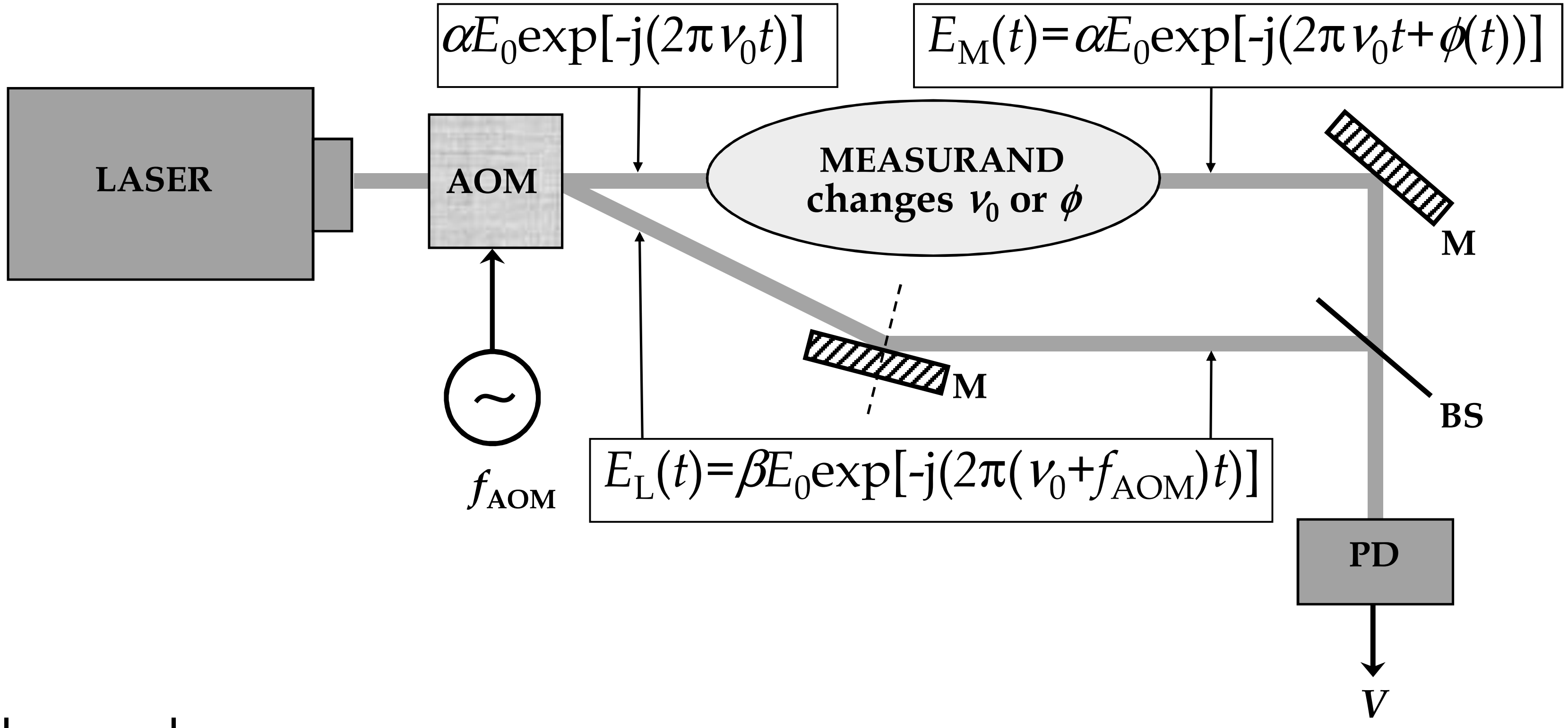
Performing a coherent detection and observing the phase of the intermediate frequency signal, we can also obtain a **measurement proportional to sin[$\phi(t)$]** that, for "small variations" of the measurand (dM/dt such as $d\phi/dt<<1$), is **directly proportional to $\phi(t)$ and hence to variations of M** (in this case we say that the **interferometer** is **"in quadrature"**)

Diapositiva 23

CS12 incluso questo lucido è terminata la 5a lez 04-05 in aula N.1.6

Cesare Svelto; 03/04/2005

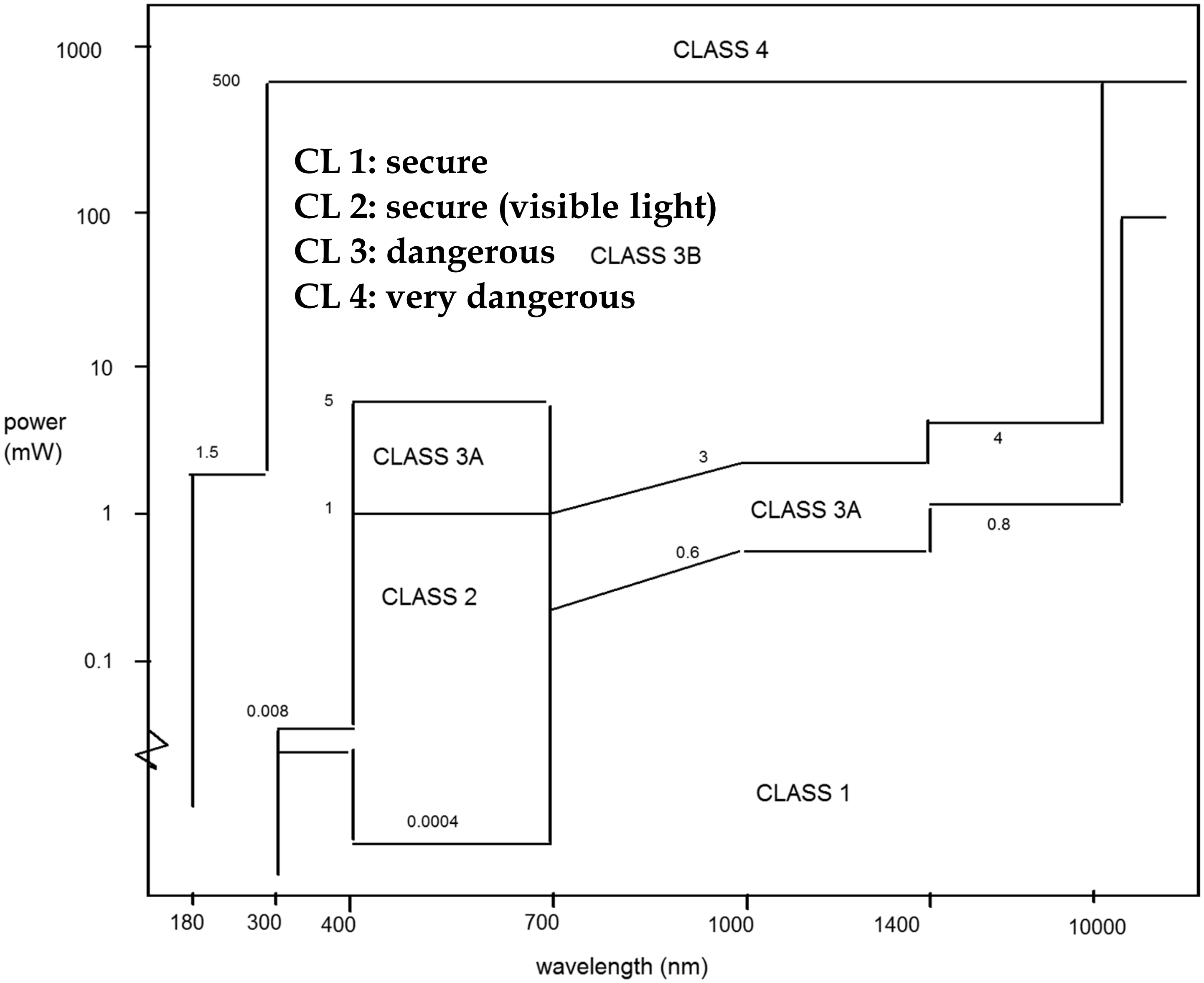
Example of beat measurement



$$V \propto |E_{RIV}|^2 = (\alpha E_0)^2 + (\beta E_0)^2 + 2\alpha\beta(E_0)^2 \cos[\,2\pi f_{AOM} t + \phi(t)\,]$$

The photosignal V oscillates at about the frequency f_{AOM}
still depending, in its exact electrical phase/frequency, on the optical phase/frequency changes driven by the measurand

LASER safety diagram



CL 1: secure
CL 2: secure (visible light)
CL 3: dangerous
CL 4: very dangerous

LASER safety (1/2)

Class 1

- a) **Use without prescriptions**

Class 2 [[only for visible λ]] ***(direct vision practically safe)***

- a) Avoid protracted vision of direct beam **(shutting-eye reflex!)**
- b) Do not aim the beam directly at people

Class 3A ***(direct vision little dangerous)***

- a) Avoid use of optical focusing systems *e.g*. ocular, theodolite
- b) Post a LASER advice note
- c) Laser alignment by mechanical or electronics means
- d) Terminate the beam in an zone external to the working area or enclose the working area
- e) Chose an height of the laser beam well above the eye-sight
- f) Avoid directing the laser beam onto reflecting surfaces
- g) Store the portable laser, when not used, out of reach from unauthorized persons

Laser safety (2/2)

***Class 3B** (direct vision always dangerous; diffused light vision potentially dangerous)*

Can cause damage to unprotected eye. Same prescriptions as for class 3A and

- a) Use only in areas controlled by instructed operators
- b) Absolutely **avoid direct reflections**
- c) **Terminate the beam** on absorbing material designed for heat dissipation
- d) **Were eye-protection (laser-safety goggles!)**

***Class 4** (direct vision extremely dangerous; diffused light vision dangerous)*

Causes damage to the eye with direct or reflected or diffused beam. Potential danger of skin burning and fire onsets. Same prescriptions as for class 3B and

- a) **Beam paths protected** by physical enclosures that can stop the beam
- b) During use, only technical personnel can be present and must be protected by proper **eye-protecting-goggles** and wearing special **protecting dresses**
- c) To avoid the presence of personnel it would be preferable a **remote control** of the laser devices and apparatuses
- d) Use non-flat **metallic targets** properly cooled *e.g*. absorbing cones
- e) To avoid undesirable reflections in the invisible part of the spectrum (for FIR light) the beam path and the impact area should be enclosed inside an opaque material to the laser wavelength

LASER protections

Eye protections

- A **prescribed eye protection must be used** (**LASER goggles: $\mathbf{OD=\log_{10}[1/}\boldsymbol{T}\mathbf{]}$** if T=10^{-x} then OD=x) to ensure adequate protection against specific laser wavelengths in all **dangerous areas where class 3 and class 4 lasers are in use**.

Protective dresses

- To be used where operating personnel is exposed to radiation levels above the MPE (Maximum Permitted Exposure) for the skin (class 4 lasers are a potential risk of fire onset and the protecting dresses must be fabricated with proper anti-fire materials).

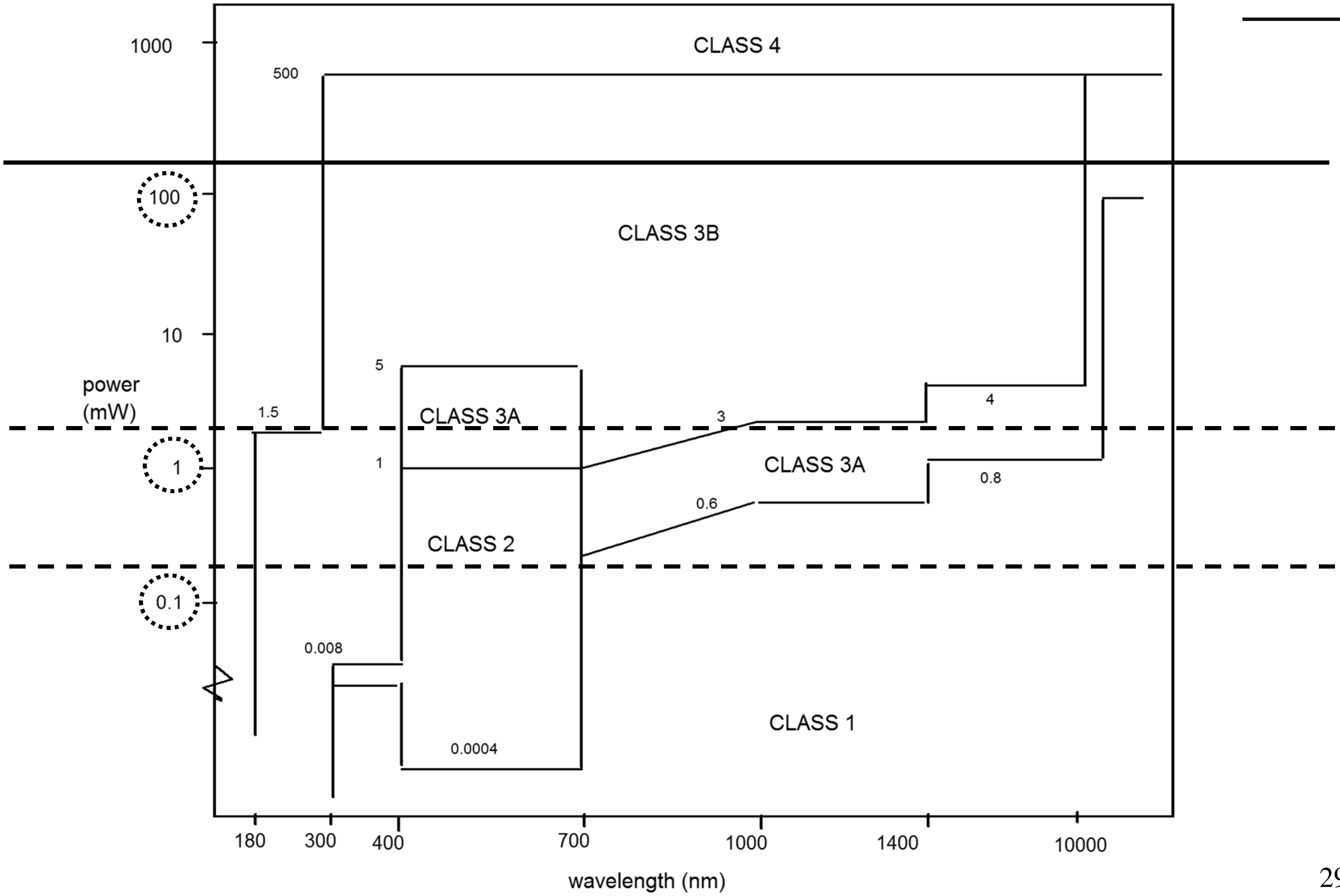
Formation

- Security classes 3 and 4 lasers can be a danger not only for the operator bu also for other people, even at large distance.
  The **personnel** operating these lasers must have **adequate preparation** in order to minimize the professional risks.

Medical surveillance

- **Oculist exams**, before work start and during work period, should be performed on personnel operating with lasers in the laser security classes 3 and 4.

LASER safety diagram



Industrial LASER applications

Material processing

- Drilling, cutting, soldering, thermal hardening, *etc*.

Industrial measurements, civil and envirromental

- ***Industrial compartment***: interferometers for dimensional laser metrology, wire diameter measurements, granulometers, roughness meters, deformation fields detectors.
- ***Civil compartment***: alignment laser systems, laser levels, topographic telemeters and geodimeters.
- ***Environmental compart***: LIDAR for pollutant remote monitoring.
- ***Presentation compartment***: lasers for holograms visualization, laser pointers for conferences, didactical usage laser systems.
- ***Light show compartment***: laser for special effects in discotheques, exhibitions, open-air shows and similar.
- ***Durable goods compartment***: laser bar-code readers, compact disk players, **laser printers** and similar.

Applications of LASERs for TLC, Medicine and Basic Research

Telecommunications and optical fibers

- Semiconductor lasers for usage, with optical fibers, in data transmission and processing and networking

Medical fields

- Laser applications in ***Ophthalmology***
- Clinical applications of lasers in ***General Surgery***
- Clinical applications in ***Surgery with operatory microscopy***
- Clinical applications in ***Endoscopic Surgery***

Research laboratories and experiments

- Nonlinear optics
- Spectroscopy (linear and nonlinear)
- Interaction of radiation with mattera
- Ultrafast optical pulses
- *Precision Measurements*

Conclusions

- Physical principles and devices for LASER action
- Solid-state and semiconductor laser structures
- Principal properties of lasers
- Free-space and guided propagation
- Properties of laser beams (profile, noise amp. / freq.)
- Optical power and photodetection
- Laser safety and applications

LASER “bright solution in search for a problem”

laser instrumentation: now widely used in **research, technology , R&D, production, optical measurements**

Bibliography

- J. M. Vaughan, *The Fabry Perot Interferometer*, Hilger, Bristol, 1989
- A. Yariv, *Quantum Electronics*, 3th ed., Wiley, New York, 1989
- A. E. Siegman, *Lasers,* Oxford University, Cambridge, 1986
- O. Svelto, *Principles of Lasers*, 5th ed., Plenum, New York, 2010 (from which some figures of this Chapter are reproduced)
- W. Koechner, *Solid-State Laser Engineering*, 6th ed., Springer, Berlin, 2006