
“Optical Measurements”

Master Degree in Engineering
Automation-, Electronics-, Physics-,
Telecommunication- Engineering



Stability Measurements and Active Stabilization of Laser Oscillator

Laser Spectroscopy and Optical Frequency Metrology

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Summary

- Amplitude noise: measurement and suppression
 - Technical noise, drift, and relaxation oscillation
 - Feedback systems for the RIN suppression
- Frequency stability and its characterization
 - Causes for optical frequency instability
 - Measurements with optical frequency discriminator
 - The Allan variance
- Laser spectroscopy
- Optical frequency references and active stabilization
 - Fabry-Perot discriminators and atomic/molecular references
 - Fringe-Side Locking and the Pound-Drever method
- Atomic/molecular optical frequency references
 - Measurements on the closed-loop error signal
or on the beat-note signal
 - References at $1.5 \mu\text{m}$ for the optical communications
- Metrology experiments at the optical frequencies

Amplitude stability

- Time-variable electric field with amplitude fluctuations

$$E(t) = E_0 [1 + a(t)] \exp[-j2\pi\nu_0 t] \quad \text{with } |a(t)| \ll 1$$

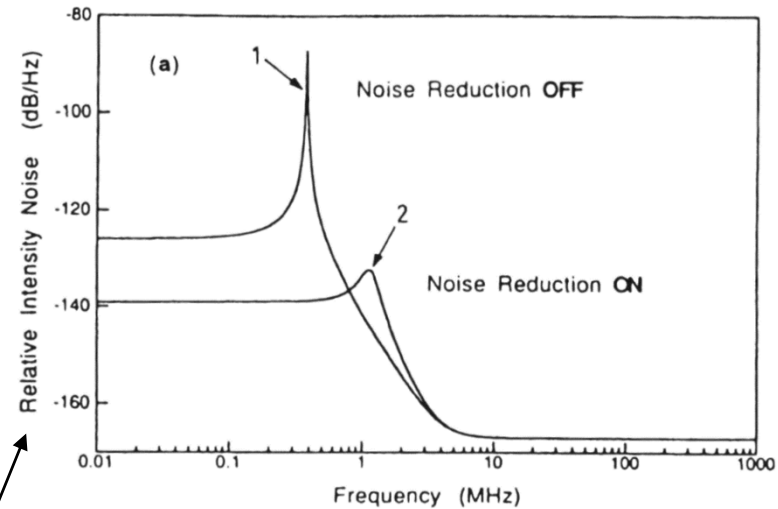
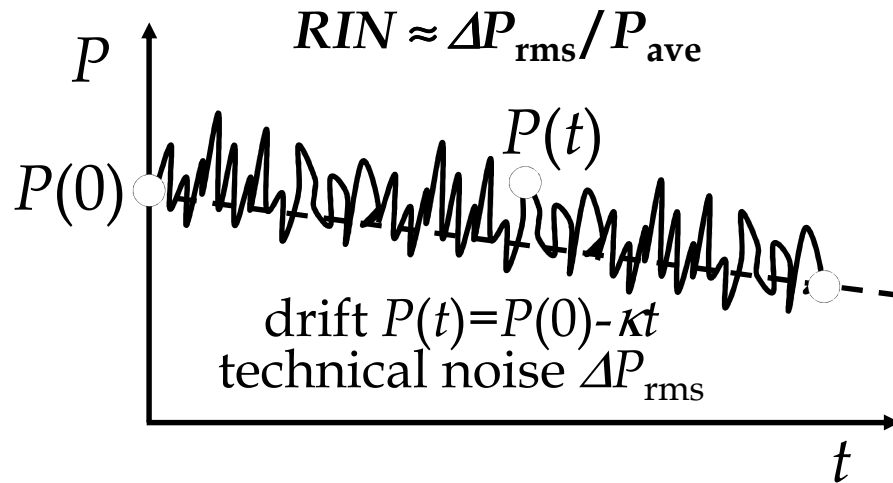
- From a perturbative analysis of the laser system (in response to small fluctuations of the pump rate or intracavity losses) we can point out the phenomenon of **relaxation oscillations**

- oscillation frequency $f_{RIN} = \left[\frac{x-1}{\tau_c \tau_{sp}} \right]^{1/2} \quad (x = P/P_{th})$

- decay time (exponential) $\tau_{RIN} = \frac{2\tau_{sp}}{x}$

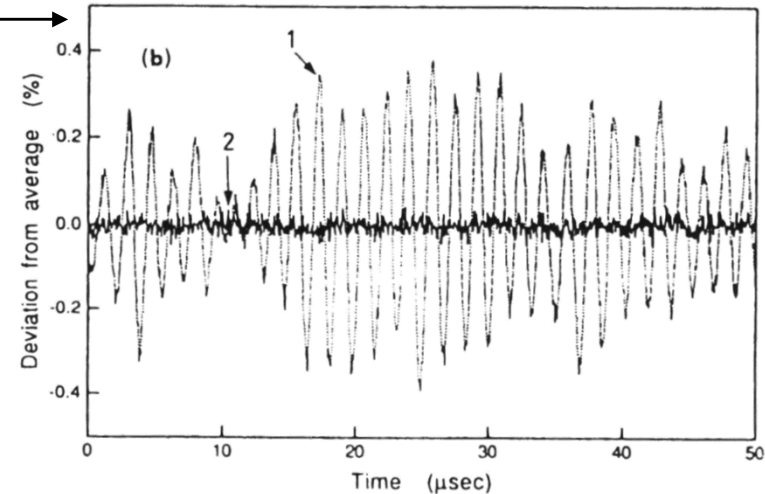
Need for **stabilization systems** (passive/active)

Relative Intensity Noise (RIN)



Power fluctuations $\Delta P(t)$

Spectral density $S_{\Delta P}(f)$



Relative Intensity Noise

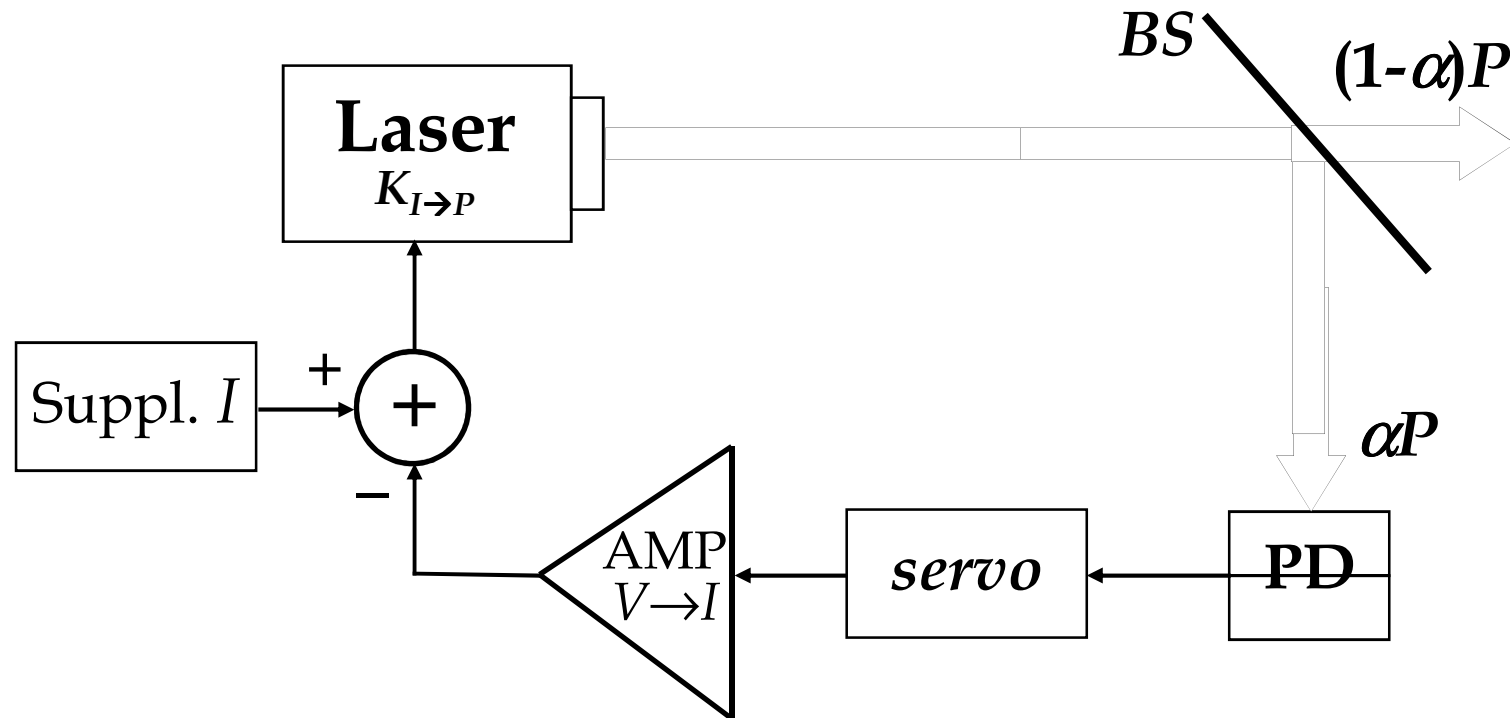
$$RIN(f) = S_{\Delta P}(f) / \langle P \rangle^2 \times 1\text{Hz} \text{ [dB]}$$

$$RIN(f) = \frac{i^2(f) \times 1\text{Hz}}{I_{DC}^2} \text{ [dB]}$$

often stated in dB/Hz to indicate "@1Hz" or "within a 1 Hz bandwidth"

Pb. 2 compl. conj. poles

RIN Suppression



$$G_{loop} = K_{I \rightarrow P} \times \alpha \times \rho_{PD} \cdot G_{I \rightarrow V} \times G_{Servo} \times G_{V \rightarrow I}$$

$$[1] = [W/A] \times [1] \times [A/W] \cdot [V/A] \times [V/V] \times [A/V]$$

Optoelectronic control loop (bandwidth and stability)

Frequency stability

- Time-variable electric field with frequency fluctuations

$$E(t) = E_0 \exp\{-j[2\pi\nu_0 t - \phi(t)]\} \quad \text{with } 1/(2\pi) \times d\phi/dt \ll \nu_0$$

$$\nu_{\text{inst}}(t) = 1/(2\pi) d\phi_{\text{tot}}/dt = \nu_0 - (1/2\pi) d\phi/dt = \nu_0 + \Delta\nu(t)$$

- From the equation of the resonator auto-frequencies:

$$\nu = l \frac{c}{2L} \Rightarrow \Delta\nu = l \frac{c}{2L^2} (-\Delta L) \Rightarrow \boxed{\frac{\Delta\nu}{\nu} = -\frac{\Delta L}{L}}$$

- **Strong dependence of the laser frequency from L**

- e.g. for an Nd:YAG laser ($\lambda=1.064 \mu\text{m}$, $\nu \approx 300 \text{ THz}$)
with $L=30 \text{ cm}$ if $\Delta L=-1 \mu\text{m}$, one gets $\Delta\nu=1 \text{ GHz!!!}$

- e.g. laser diode ($\lambda=1.55 \mu\text{m}$, $\nu \approx 200 \text{ THz}$)
with $L=500 \mu\text{m}$ if $\Delta L=-0.1 \text{ nm}$, one gets $\Delta\nu=40 \text{ MHz!!!}$

Need for **stabilization systems** (passive/active)

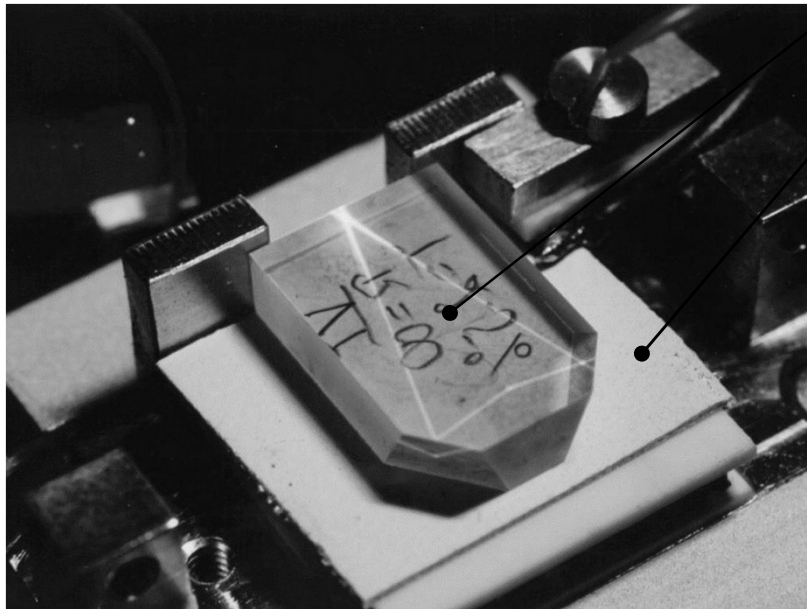
Thermal expansion coefficient: $\alpha = (\Delta L/L) \cdot 1/\Delta T$ " (...) $\cdot 10^{-6} \text{ K}^{-1}$ "

Al (24); steel (12); ord. glass (9); pyrex (4); INVAR (1.6); FS (0.6), ULE 0 ± 0.0001 6/45

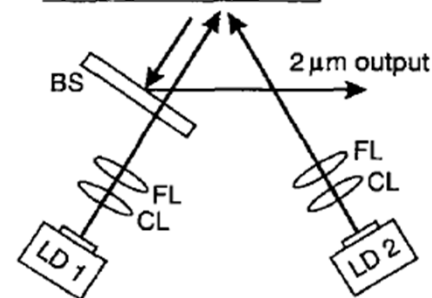
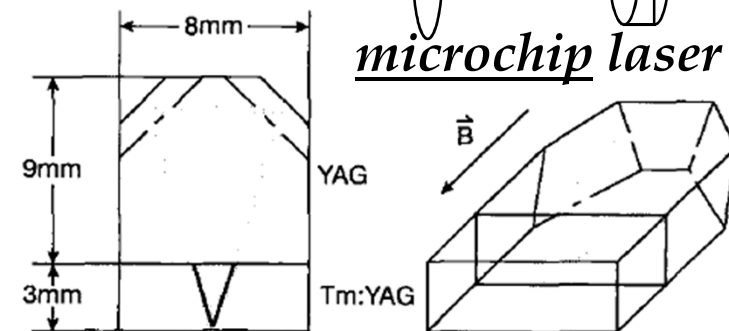
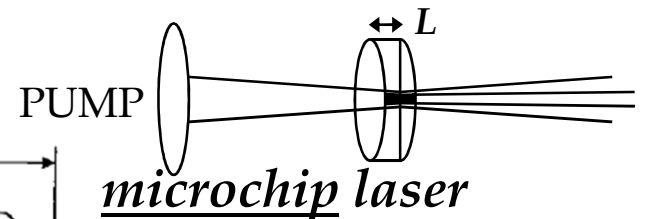
Laser intrinsically stable monolithic laser μ -chip or NPRO Nd(Tm):YAG

Diode-pumped "monolithic" lasers are intrinsically very insensitive to perturbations destabilizing for P and λ

Unidirectional: *spatial hole burning*...



PZT transducer
Peltier cell

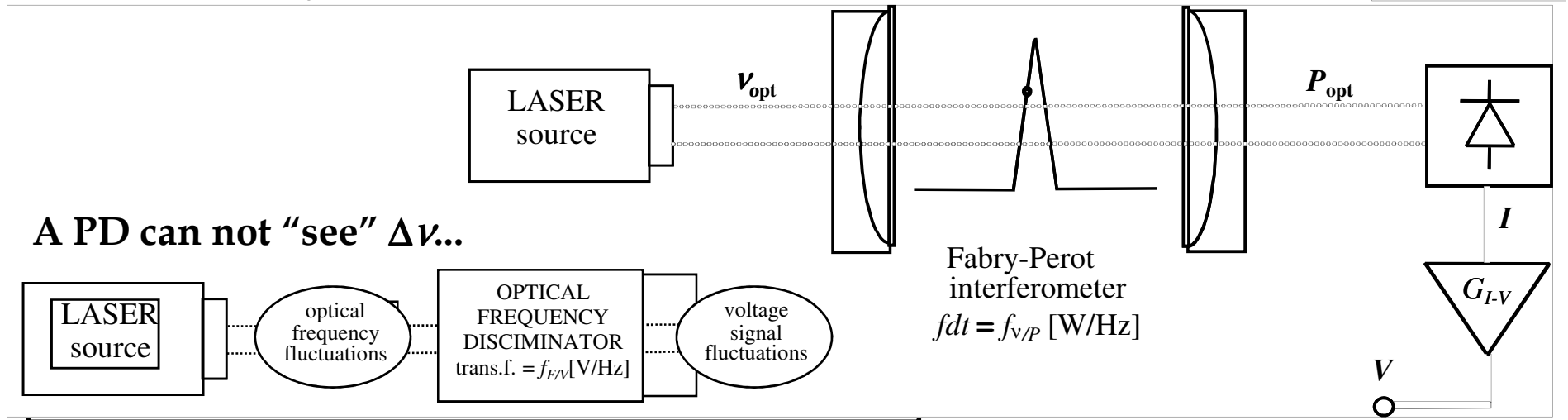


Nd:YAG single-frequency
Non-Planar Ring Oscillator
 $\lambda=1064$ nm; $P=10\text{mW}-2\text{W}$;
 $\nu=282\text{THz}$; $\Delta\nu<5\text{kHz}@1\text{ms}$

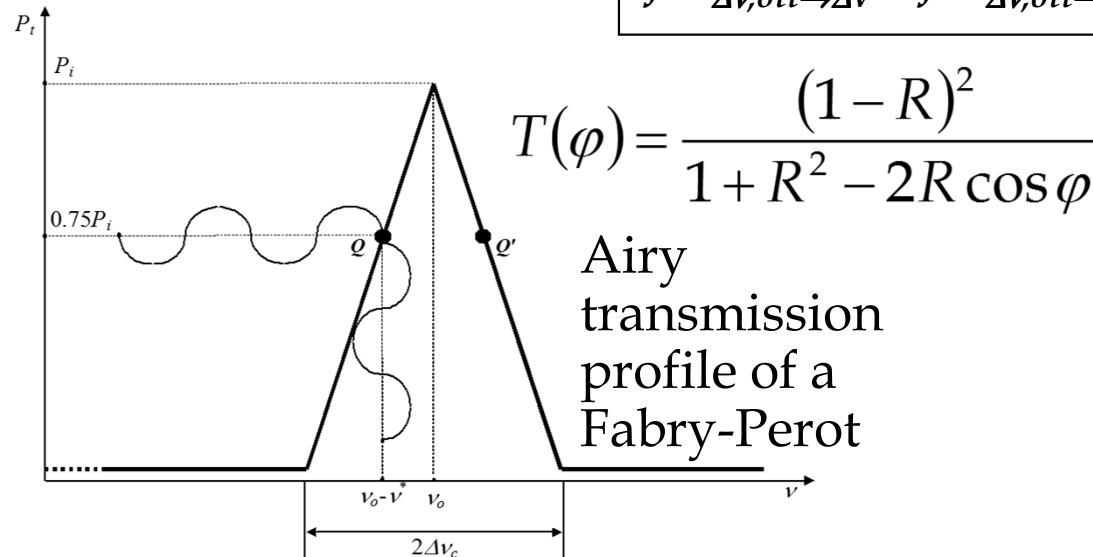
Tm:YAG laser (3-levels)
the active medium must be "all" inverted

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Frequency noise characterization (Fabry-Perot discriminator)

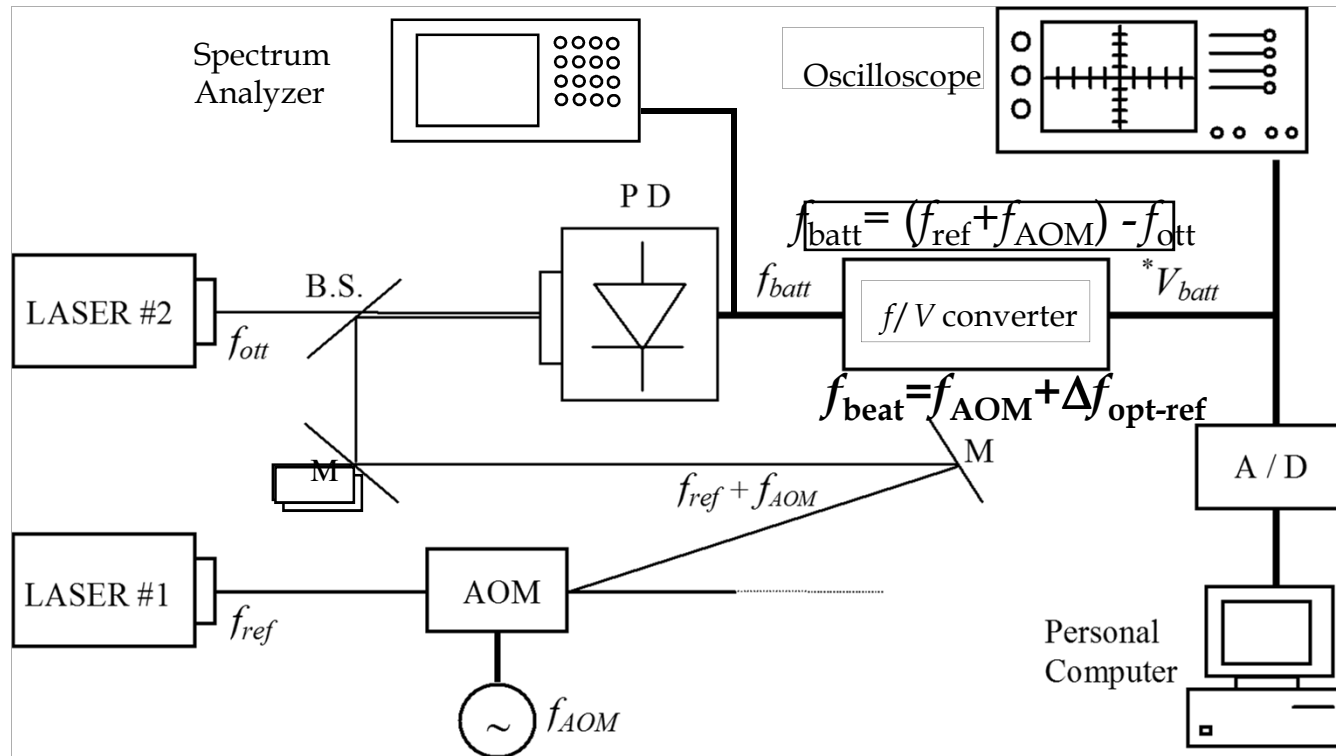


$$f_{dt}_{\Delta\nu,ott \rightarrow \Delta V} = f_{dt}_{\Delta\nu,ott \rightarrow \Delta P,ott} \times f_{dt}_{\Delta P,ott \rightarrow \Delta I} \times f_{dt}_{\Delta I \rightarrow \Delta V}$$



the optical frequency fluctuations $\Delta\nu$ are converted into photovoltage fluctuations ΔV (very important the different "gains" slope \times responsivity \times transimpedance gain found in the discrimination chain)

Frequency noise characterization (optical beat-note signal)



take $f_{ref} \cong f_{opt}$

if $f_{opt} \rightarrow f_{opt} + \Delta f_{opt}$
 \Rightarrow

$f_{beat} = f_{AOM} - \Delta f_{opt}$

if then $f_{ref} \rightarrow f_{ref} + \Delta f_{ref}$
 \Rightarrow

$f_{beat} = f_{AOM} + \Delta f_{opt} + \Delta f_{ref}$
beat-note in AC!!!

$f_{beat} = |f_{ref} - f_{opt}|$ without the (AOM)... f_{batt} is in DC!!!

$$\langle \Delta f_{ref}^2 \rangle^{1/2} \ll \langle \Delta f_{ott}^2 \rangle^{1/2} \Rightarrow \langle \Delta (f_{ref} - f_{ott})^2 \rangle^{1/2} \cong \langle \Delta f_{ott}^2 \rangle^{1/2}$$

$$\langle \Delta f_{ref}^2 \rangle^{1/2} \cong \langle \Delta f_{ott}^2 \rangle^{1/2} \Rightarrow \langle \Delta (f_{ref} - f_{ott})^2 \rangle^{1/2} \cong \sqrt{2} \langle \Delta f_{ott}^2 \rangle^{1/2}$$

Sum of 2 variances of 2 independent random variables

Frequency noise characterization (definition of the Allan variance)

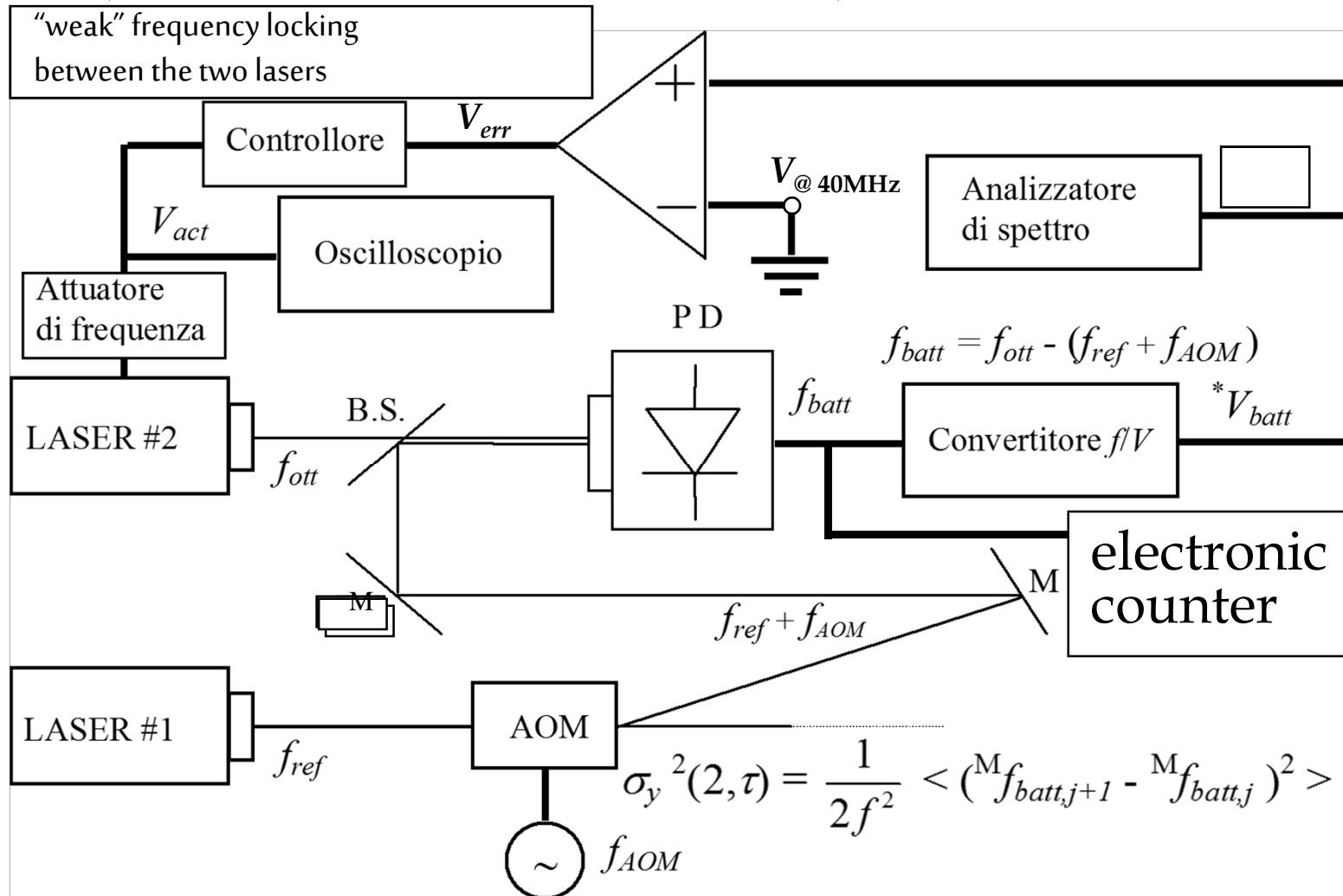
The **Allan variance** is the recommended measurement by several international committees and in particular by CCIR to characterize frequency stability of an oscillator in the time domain. In the case of high frequency oscillators, the Allan variance measurement, is based on the analysis of beatnote signal between two oscillators. In the case of laser sources this technique consists in taking a **measurement of different samples of average beat frequency Mf_{batt} in adjacent time intervals** and calculating the “two sample” variance between adjacent pairs. The instrument usually performing Allan variance measurements is a specific electronic counter but calculation can be performed also by PC on a time series of digitized readings of different values of $Mf_{batt,j}$ evenly spaced by time intervals τ one from the other. Having a series of N samples of the beat frequency [$Mf_{batt,1}, Mf_{batt,2}, \dots, Mf_{batt,N}$], the Allan variance is evaluated as the **average of the squares of differences between adjacent frequency samples** (normalized to the squared working/mean frequency).

$$\sigma_y^2(2, \tau) = \frac{1}{2f^2} \langle (Mf_{batt,j+1} - Mf_{batt,j})^2 \rangle = \frac{1}{2} \left\langle (y_{j+1} - y_j)^2 \right\rangle \quad y_j = \frac{Mf_{batt,j}}{f}$$

σ_y is a function of averaging time τ

relative frequency fluctuation

Frequency noise characterization (Allan variance measurement)



The two stabilized lasers have the same nominal or center frequency and hence we use an acousto-optical modulator to RF shift the beatnote frequency

Spectroscopy

- Atoms and molecules absorption lines
- Linear, FM, and saturated spectroscopy
- Frequency stabilization
 - *fringe-side locking*
 - Pound-Drever
- Optical frequency Metrology
 - recommended wavelengths (most accurate λ values)
 - experiments with stabilized lasers
 - “optical” Atomic Clock
 - second, meter, and accuracy in Measurements

Absorption lines in atoms and molecules

Atoms and molecules have an internal structure allowing electrons to have **pre-defined and discrete energy levels**. Atoms can change from one to another energy state by **absorption** or **emission** of the required energy difference. Such energy leaps are "quantum steps" as:

$$\Delta E = E_2 - E_1 = h \cdot \nu_{21} = hc / \lambda_{21}$$

The **electromagnetic radiation** of wavelength λ (frequency $\nu = c / \lambda$) interacting with an atom/molecule will be more or less **absorbed as a function of how much the wavelength is close to the nominal (central) transition value**

$$\lambda_0 = c / \nu_0 \cong c / \nu_{21} = hc / \Delta E = \lambda_{21}$$

Absorption lines: broadening reasons and types

When dealing with “real atoms always perturbed”, not just a single wavelength λ excites the transition (giving absorption), since the **spectral absorption line** undergoes **different causes of *shift* and *broadening***:

- physical limit due to finite lifetime of the transition:

natural broadening: Lorentzian $\Delta\nu \propto 1/\tau_{sp}$

- external fields (e.g. Stark effect due to e.m. field from other atoms/ions) cause several shifts of line centers and hence an

homogeneous broadening: Lorentzian;

- collisions between atoms/molecules:

collisional broadening: Lorentzian ($\Delta\nu \propto p$);

- velocity of the absorbers:

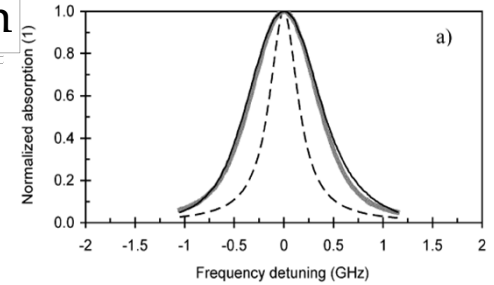
Doppler broadening: Gaussian ($\Delta\nu \propto \sqrt{T/m}$);

The absorption spectrum is in general a **broadened line** with **central value λ_0** and **linewidth $\Delta\lambda_{0,\text{FWHM}}$** depending on the different shifts and broadening mechanisms

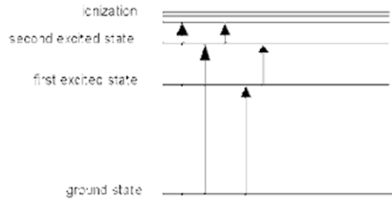
(Voigt profile: convolution of Lorentzian and Gaussian)

CH₃D [10 kPa] R(6,3) line at $\lambda = 1.543.78$ nm

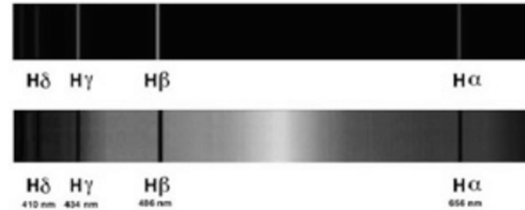
Absorption lines in atoms and molecules (figures)



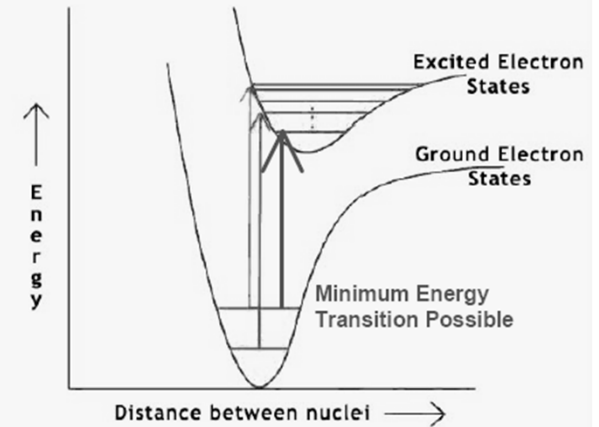
Atoms



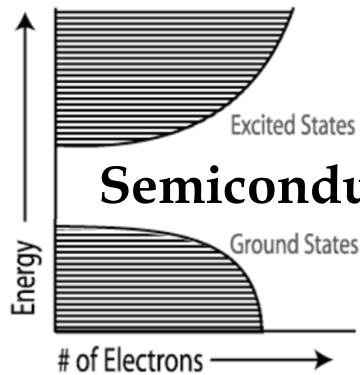
Emission and Absorption Spectra for the Hydrogen Atom



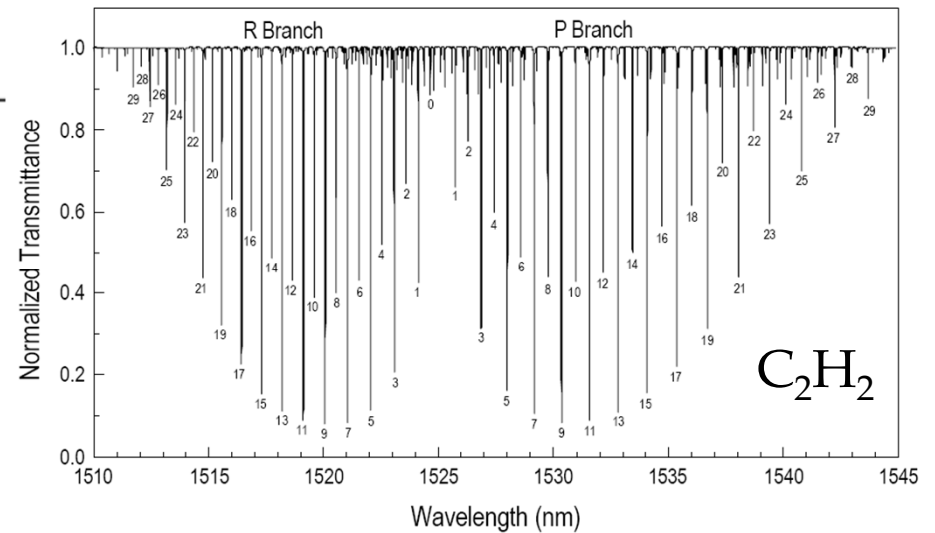
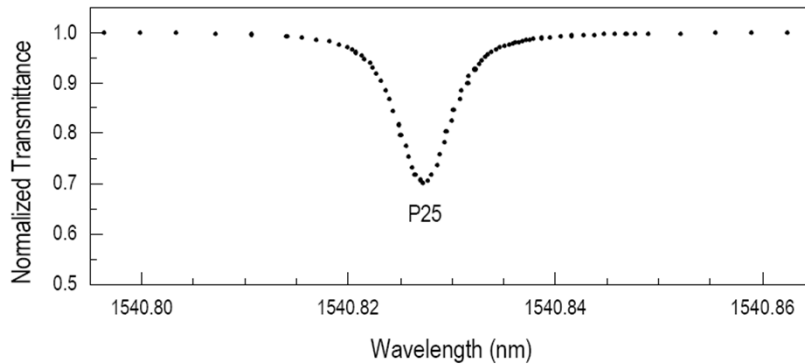
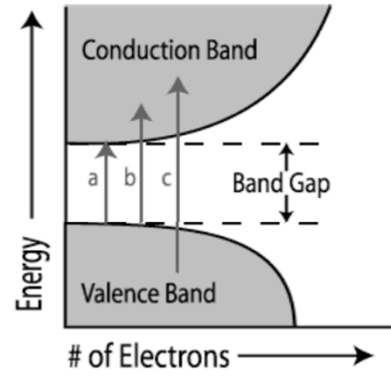
Possible Electronic Transitions



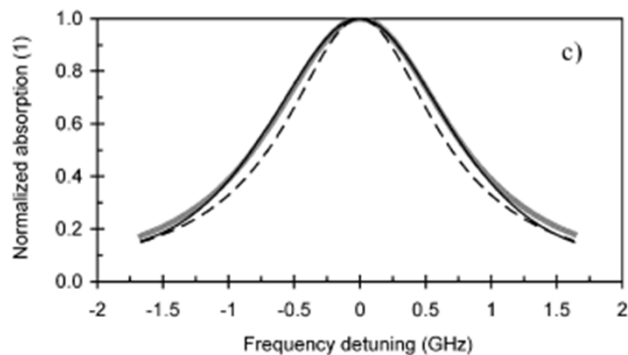
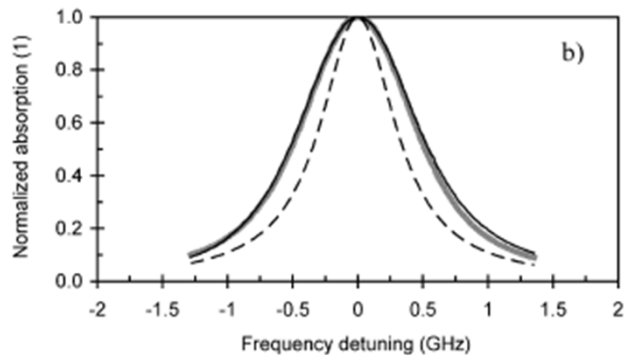
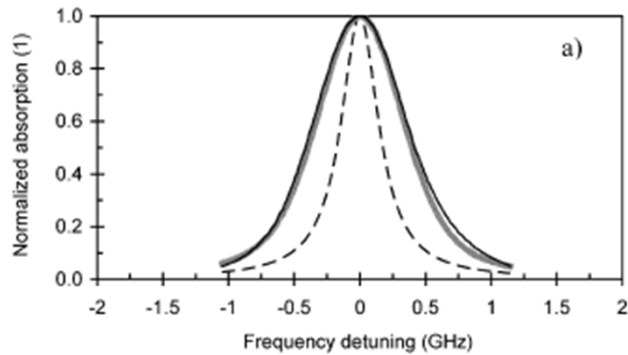
Molecules



Semiconductors



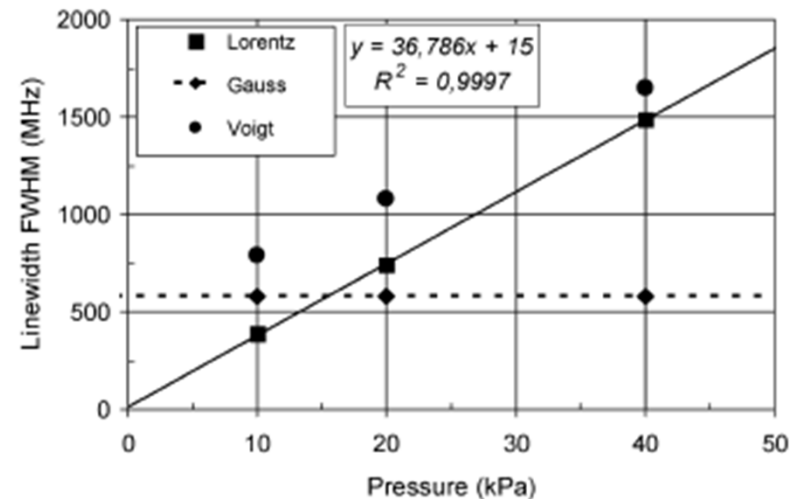
Absorption lines of CH₃D Doppler and collisional broadenings



Doppler: $\sigma_{\text{gauss}} = \Delta f = f_0 \sqrt{\frac{kT}{mc^2}}$ $FWHM = 2\sigma \sqrt{2 \ln(2)}$ ≈ 1.2

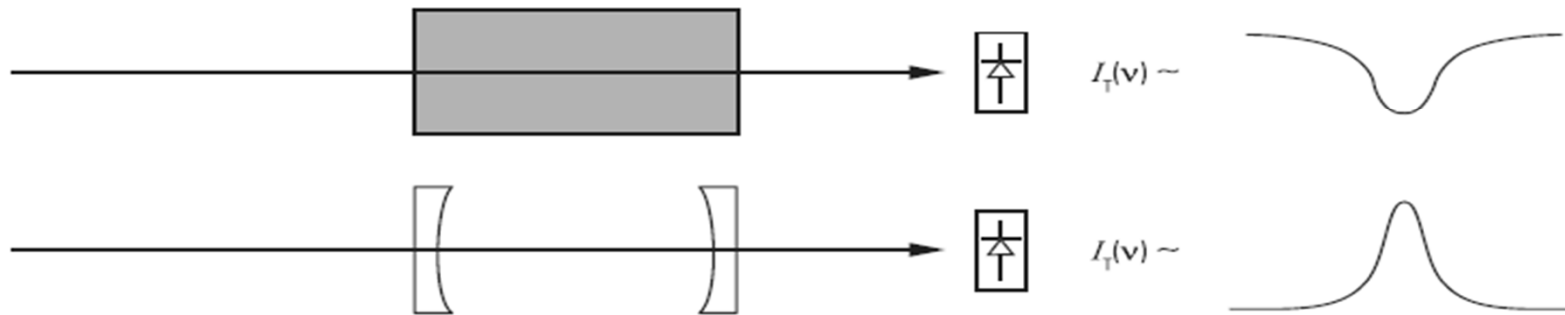
TABLE I
COLLISIONAL (LORENTZ), DOPPLER (GAUSS), AND OVERALL (VOIGT)
LINE BROADENINGS (FWHM) FOR LINE R(6,3) OF CH₃D AT THREE
DIFFERENT GAS PRESSURES

Gas pressure (kPa)	REGR.	CALC.	MEAS.
	Lorentzian FWHM (MHz)	Gaussian FWHM (MHz)	Voigt FWHM (MHz)
10	390	580	790
20	740	580	1080
40	1490	580	1650

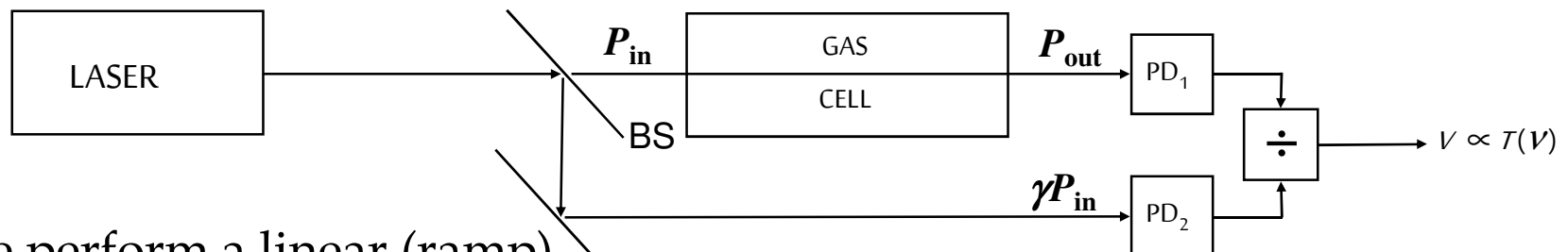


Linear (direct) spectroscopy

TRANSMISSION: *The intensity versus frequency of a laser beam transmitted through a gas cell containing an atomic or molecular vapor (top) and of a laser beam transmitted through a Fabry-Perot interferometer (bottom).*

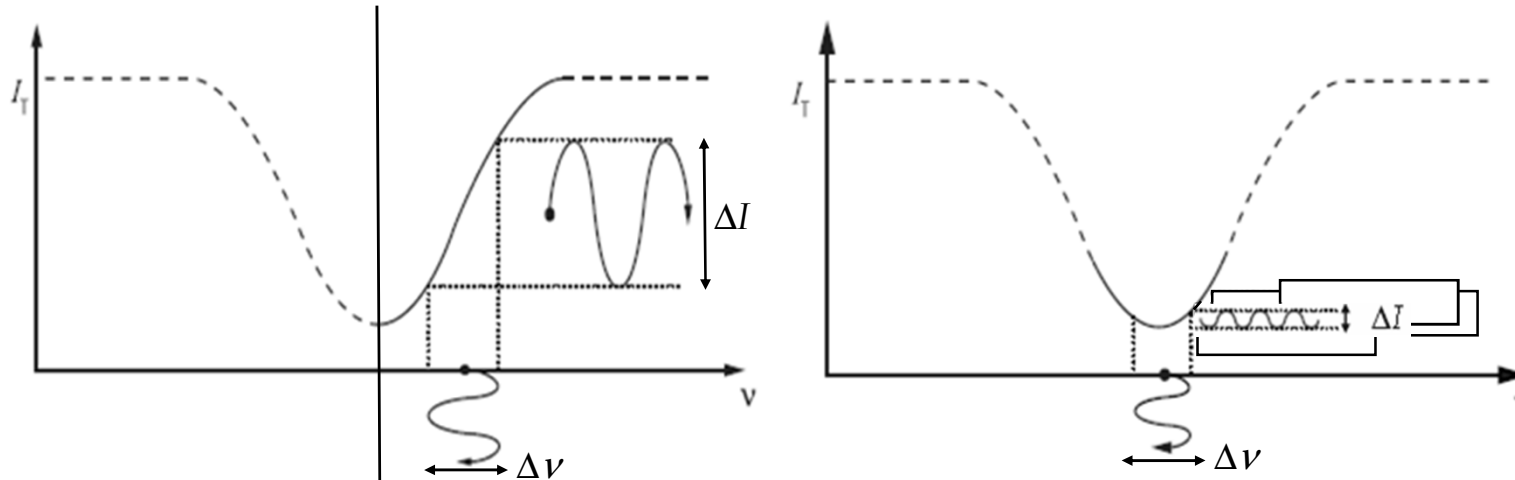


If the laser, during the frequency scan, undergoes amplitude/power variations, it is convenient to **normalize the power transmitted through the cell to the emitted laser power**: in fact, cell transmission is $T = P_{\text{out}}/P_{\text{in}}$, as a ratio of output to input powers

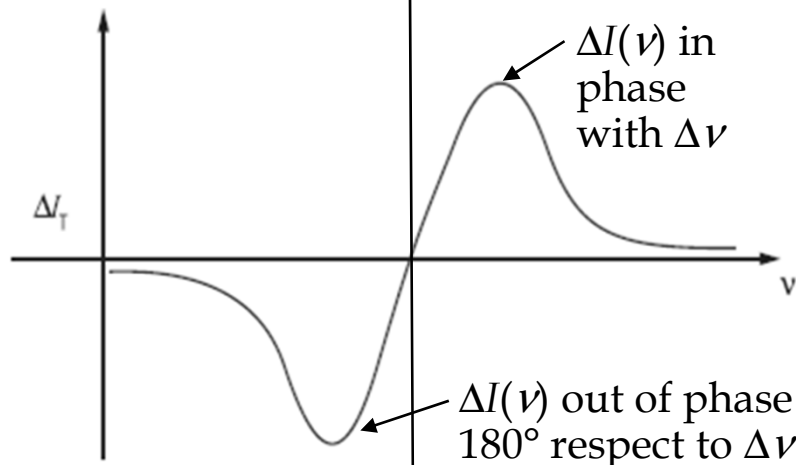


We perform a linear (ramp) frequency scan of ν laser and we measure the spectral profile $T(\nu)$

FM spectroscopy



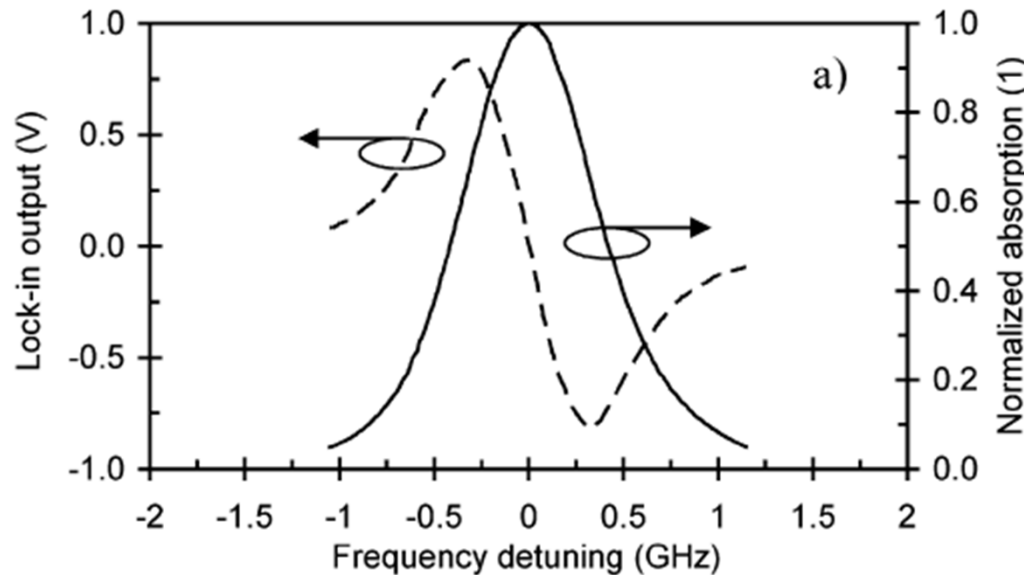
During the “linear” frequency scan the **laser is sine phase/frequency modulated (FM modulation)**. When crossing the intensity profile, the **FM modulation is transformed into AM modulation**.



The **depth of AM modulation $\Delta I(v)$** depends on the **slope of the transmission curve** in the working point of the spectral profile

We get the so called **DISPERSION PROFILE** or “**first derivative**” signal

FM spectroscopy experiments



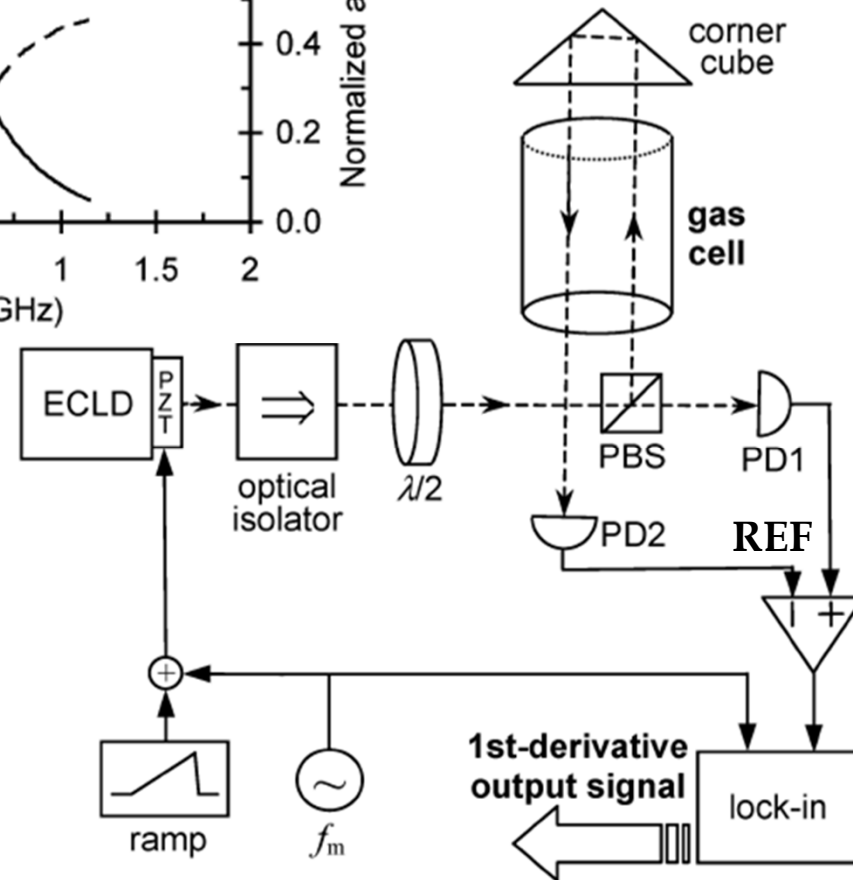
$$T = 1 - \exp(-\alpha L)$$

$$A = \exp(-\alpha L)$$

$$\alpha = \ln(1 - T) / L$$

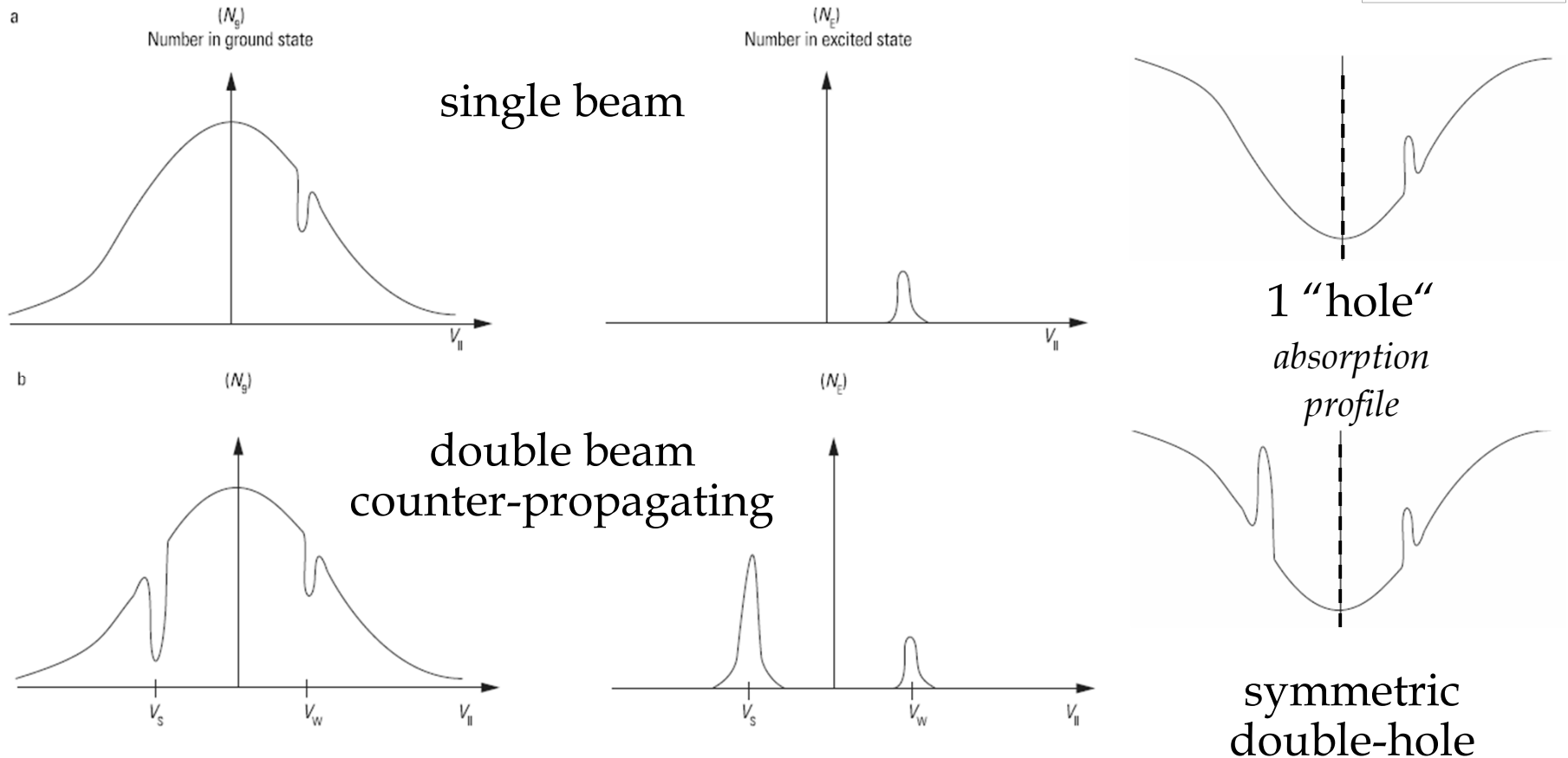
Direct transmission of the cell will exhibit power ups and downs when the optical frequency is scanned through the absorption lines

To detect absorptions smaller than 1% we use the technique of **FM modulation** and coherent detection (*lock-in*) after the cell



$T(\nu)$ is obtained by integration of the "S curve" (1st derivative)

Doppler effect in spectroscopy



Number of atoms in the fundamental state (left) and excited state (right) when varying the velocity component parallel to direction (\mathbf{k}) of the optical beam: **the interaction (absorption) happens at a frequency which is shifted with respect to the center of the transition: $\Delta\nu/\nu = v_{||}/c$**

Linear vs. saturated spectroscopy

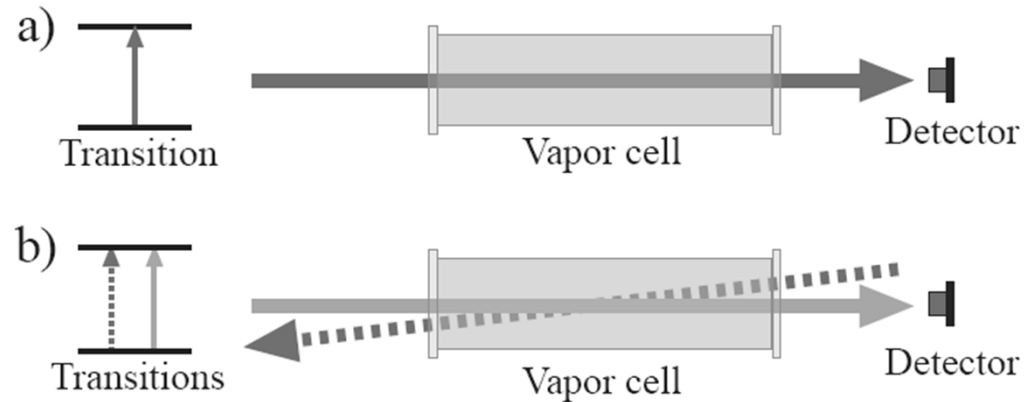
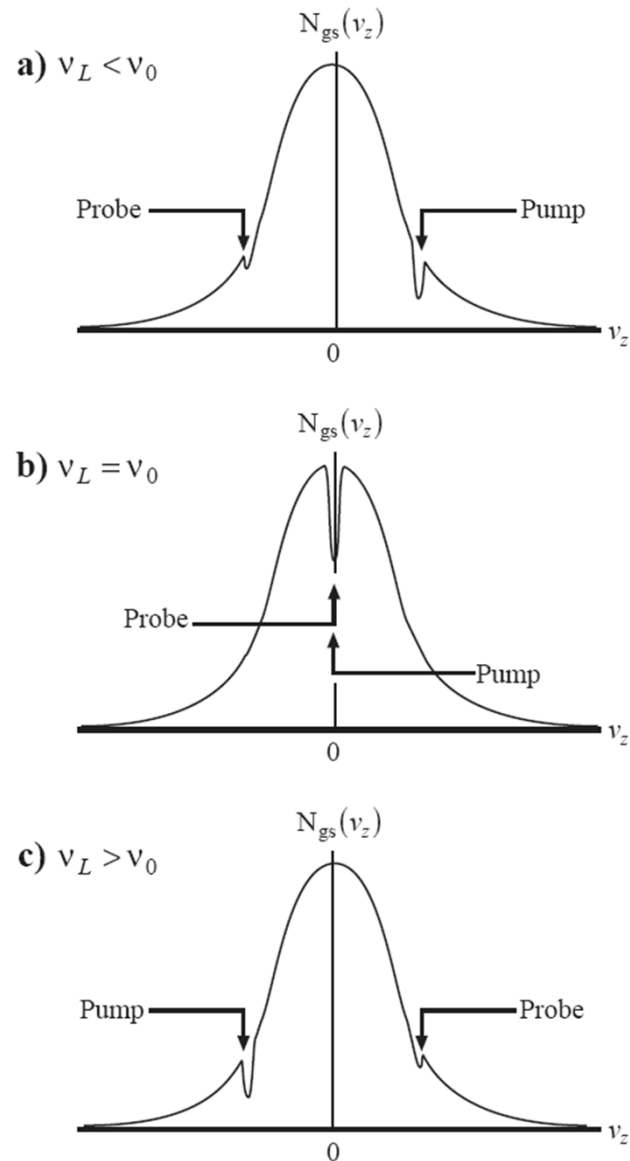


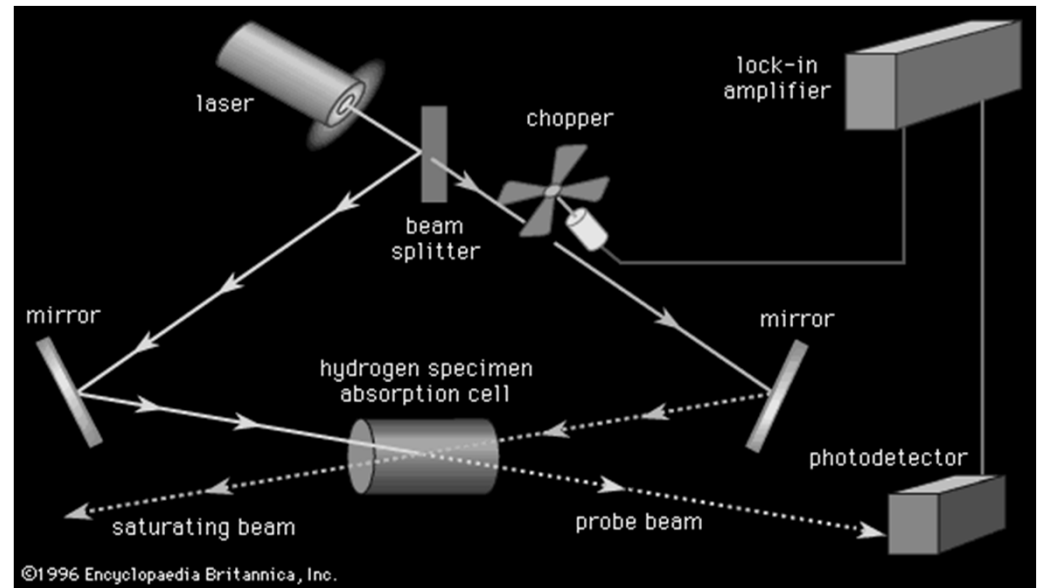
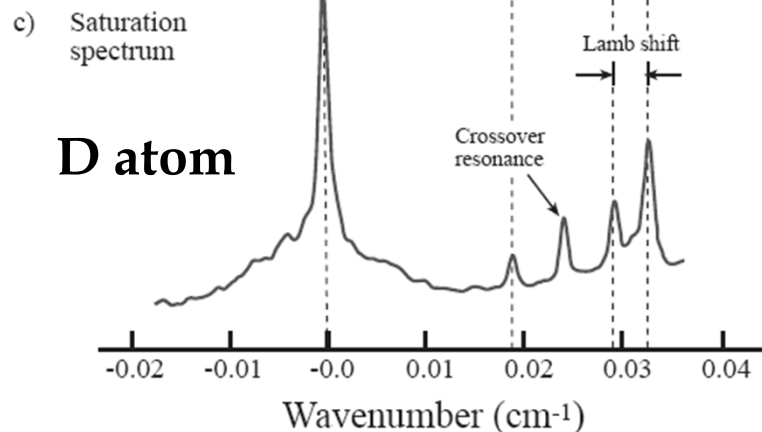
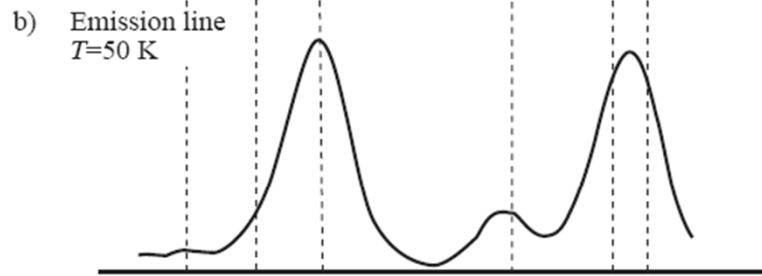
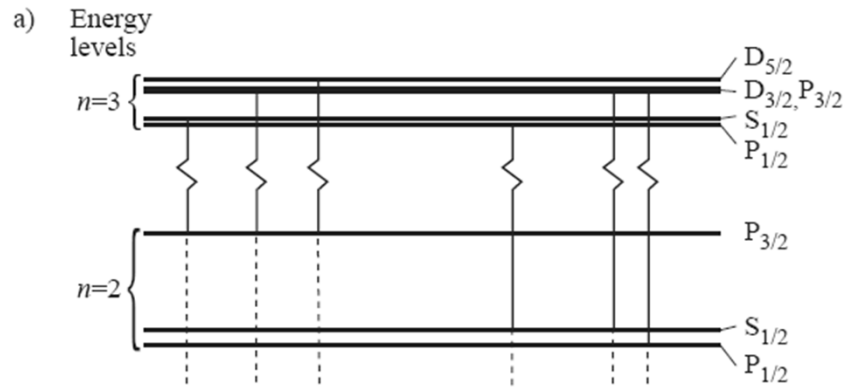
Figure 2. In linear spectroscopy (a) the radiation reaching the detector is proportional to the radiation incident on the sample. In nonlinear spectroscopy (b) the radiation reaching the detector is dependent on both beams.

A first laser beam (powerful: *pump*) saturates the absorption line and a second beam (less powerful: *probe*) sense the **saturated line**

Absorbers population decreases (and hence absorption as well) at the pump laser frequency "seen" by the moving molecules (with a velocity component along the cell axis).

Double-beam interaction with the same molecules at $v=0$ m/s eliminates Doppler broadening and gives a **much narrower saturated line** [figure b)]

Saturated Spectroscopy (Doppler-free)



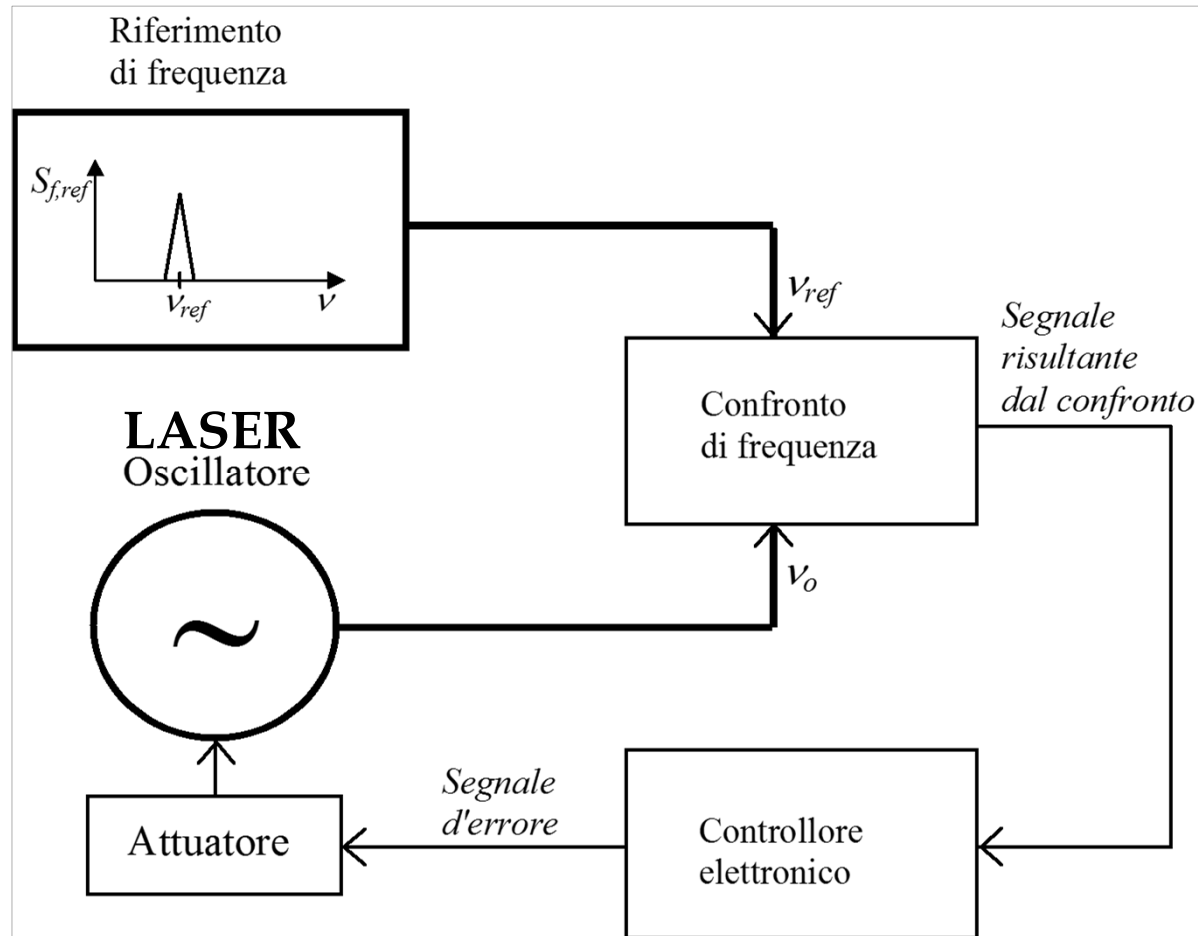
The pump beam is modulated (*e.g.* by chopper) to allow for AC detection (much more "sensitive") of the saturated line transmission profile

In Deuterium at 50 K the saturated line is not much narrower than the Doppler line. **In C_2H_2 at $1.55\ \mu\text{m}$ ($T=T_{\text{amb.}}$) we have a Doppler linewidth of about 500 MHz to a saturated linewidth of 1 MHz**

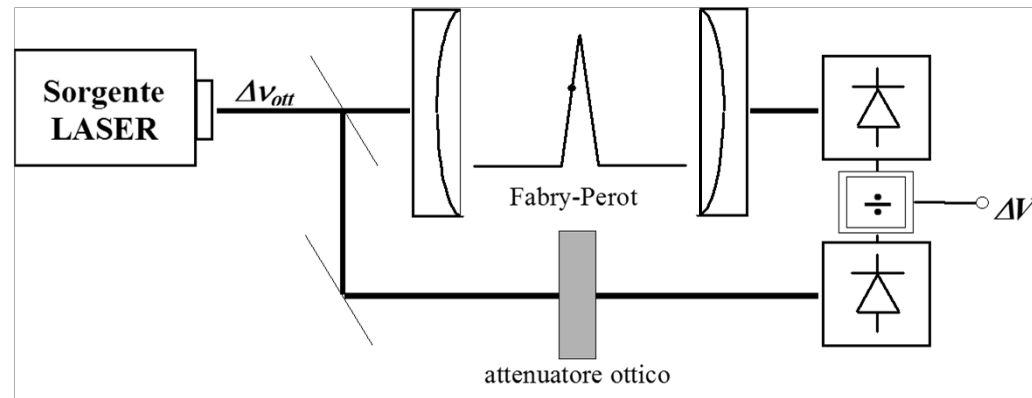
Stabilization

- Atoms and molecules absorption lines
- Linear, FM, and saturated spectroscopy
- Frequency stabilization
 - *fringe-side locking*
 - Pound-Drever
- Optical frequency Metrology
 - recommended wavelengths (most accurate λ values)
 - experiments with stabilized lasers
 - “optical” Atomic Clock
 - second, meter, and accuracy in Measurements

Frequency Stabilization (general principle scheme)



Frequency Stabilization (fringe-side locking method)



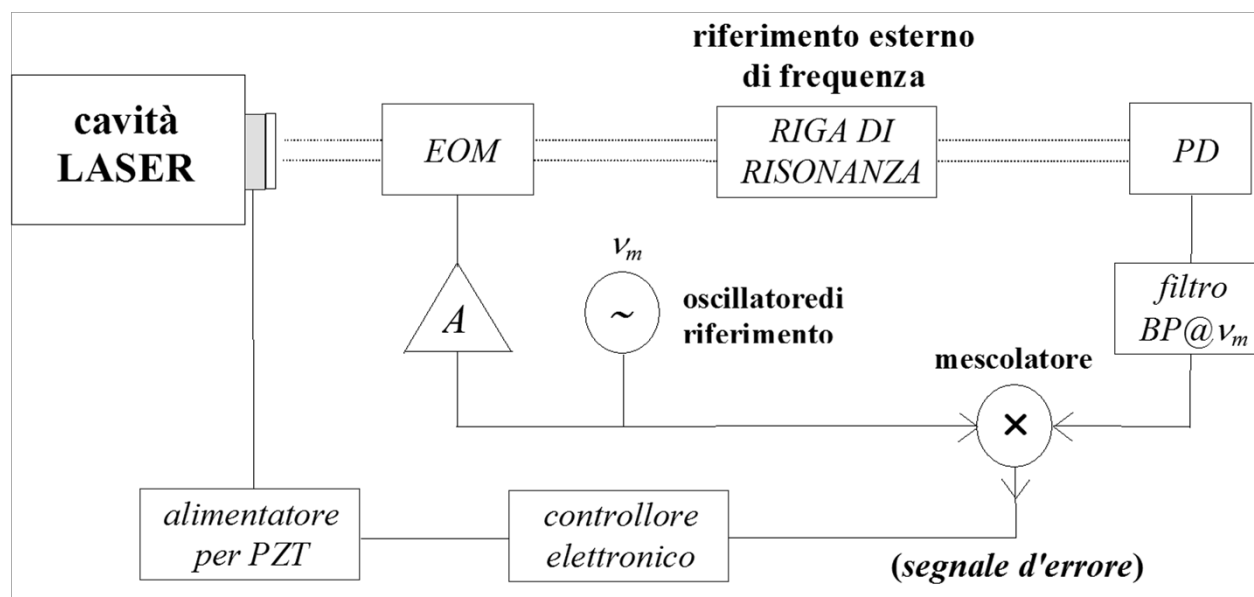
ΔV is the **error signal**
(ratio or difference)

$$\overline{\Delta V} = \frac{\Delta V}{f dt_{\Delta V, ott \rightarrow \Delta V} \times [1 + G_{loop}(f)]}$$

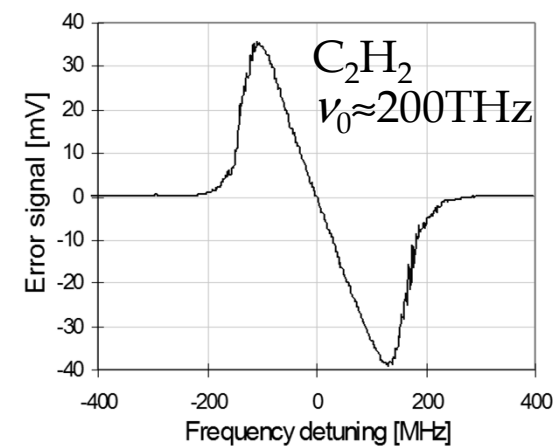
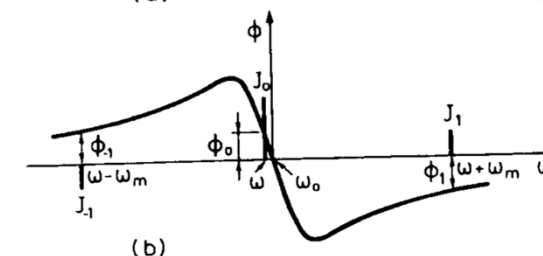
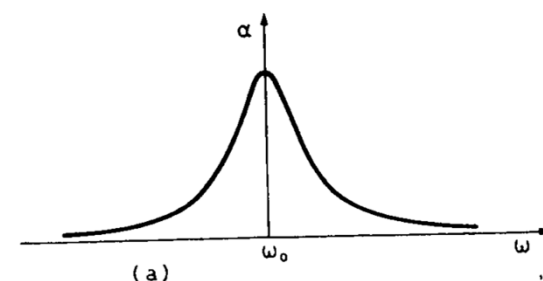
$$\overline{\Delta V} \propto \text{frequency signal } \Delta f = \nu - \nu^*$$

with "moderate" locking

Frequency Stabilization (Pound-Drever method)



modulation phase/frequency \Rightarrow
 addition of "two" side bands
 interrogation of **dispersion (phase) profile** \Rightarrow
 different phase shifts \Rightarrow
 unbalance of lock-in detected signal \Rightarrow
error signal ("odd" simetry vs. detuning)



Frequency Stabilization (Pound-Drever method - EQUATIONS)

Optical electric field phase modulated (sine mod.):

$$e^{i(\omega t + \beta \sin \Omega t)} = \sum_{n=-\infty}^{+\infty} J_n(\beta) e^{i(\omega + n\Omega)t}$$

$$J_n(\beta) = \sum_{m=0}^{+\infty} \frac{(-1)^k}{m! \Gamma(m+n+1)} \left(\frac{\beta}{2}\right)^{2m+n}$$

$$J_n(-\beta) = (-1)^n J_n(\beta)$$

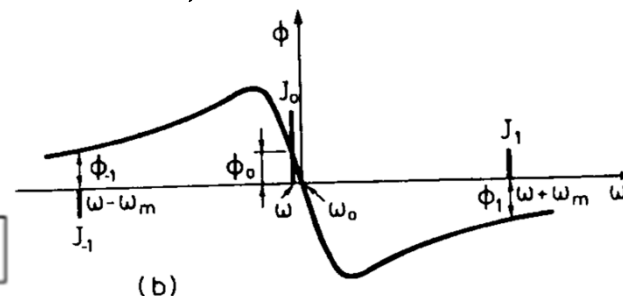
Ω or ω_m is the angular **frequency of phase modulation**
 β is the **modulation depth**

When the phase-modulated optical field senses the resonance,

$$E_{inc} = E_0 e^{i(\omega t + \beta \sin \Omega t)}$$

$$\approx E_0 [J_0(\beta) + 2iJ_1(\beta) \sin \Omega t] e^{i\omega t}$$

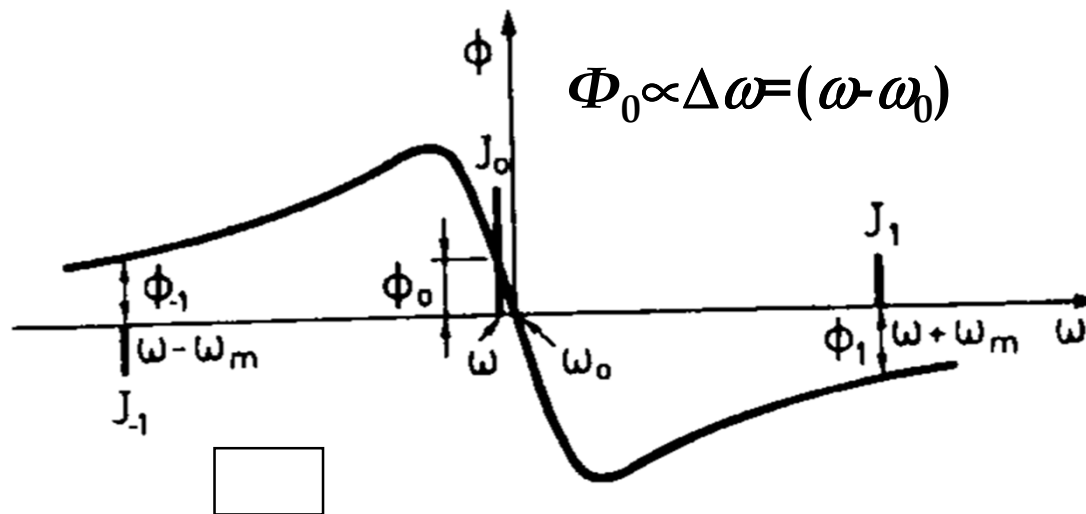
$$= E_0 [J_0(\beta) e^{i\omega t} + J_1(\beta) e^{i(\omega + \Omega)t} - J_1(\beta) e^{i(\omega - \Omega)t}]$$



we get **two optical beat signals at the modulation frequency $f_m = \Omega/2\pi = \omega_m/2\pi$** :
 a first one between carrier (at frequency $\nu = \omega/2\pi$) and left sideband and
 a first one between carrier and right sideband

(without the optical resonance sidebands J_1 and J_{-1} beating with carrier J_0 do not produce any amplitude modulation at frequency f_m , as correct for a pure FM)

Frequency Stabilization (Pound-Drever method - DISCRIMINATION)



When the **optical carrier is at the line center**, amplitudes of the two beat signals are equal but with opposite phases: the **resulting amplitude at frequency f_m is null**

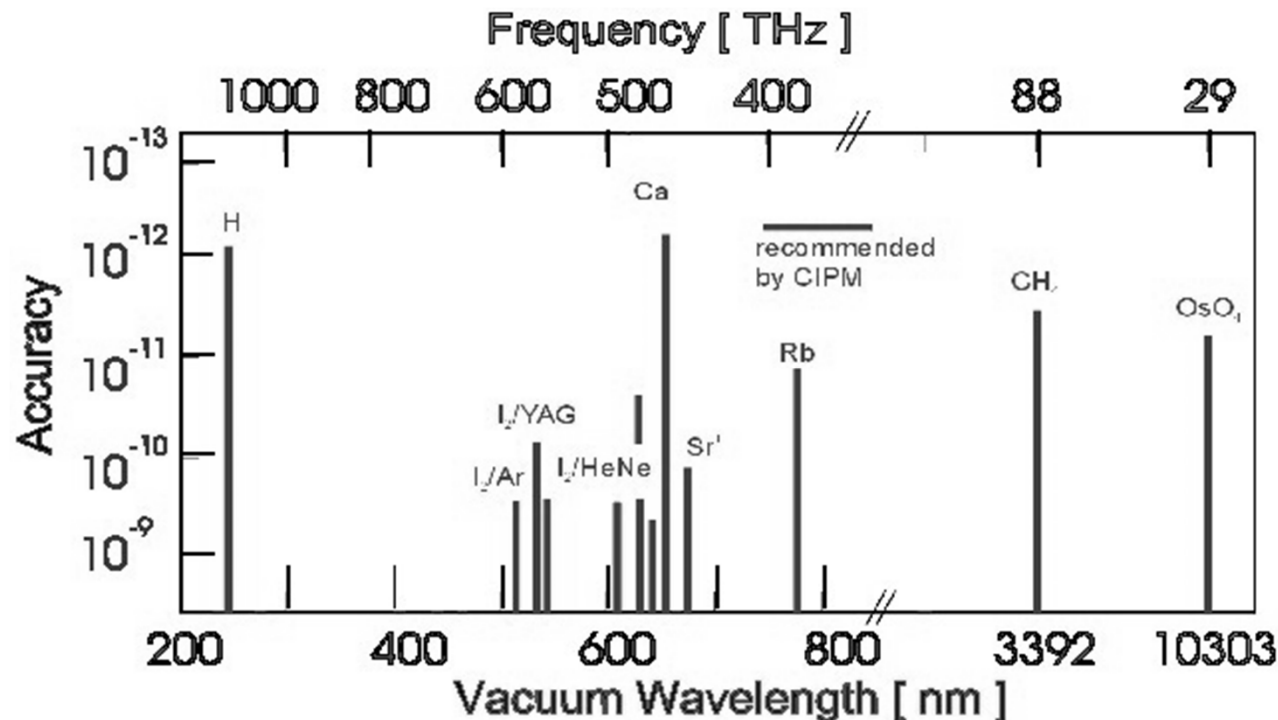
When the **optical carrier is shifted from line center**, amplitudes of the two beat signal are still almost equal but **the two phases**, $(\Phi_0 - \Phi_{-1})$ and $(\Phi_0 - \Phi_1)$, **differ by a quantity $2\Phi_0$** , beeing still $\Phi_1 \cong -\Phi_{-1}$ but now $\Phi_0 \neq 0$, and hence **the resulting amplitude at frequency f_m is proportional to $\sin(2\Phi_0) \approx (2\Phi_0) \propto \Delta\omega = (\omega - \omega_0)$**

\Rightarrow ***error signal \propto frequency detuning (with sign!)***

Metrology

- Atoms and molecules absorption lines
- Linear, FM, and saturated spectroscopy
- Frequency stabilization
 - *fringe-side locking*
 - Pound-Drever
- Optical frequency Metrology
 - recommended wavelengths (most accurate λ values)
 - experiments with stabilized lasers
 - “optical” Atomic Clock
 - second, meter, and accuracy in Measurements

Metrologia alle frequenze ottiche: lunghezze d'onda "raccomandate"



12 lunghezze d'onda "ottiche" raccomandate, dal CIPM (1997) e da lavori del CCL, per la realizzazione del "metro" (def. 1983)

2 note con accuratezza $\approx 10^{-12}$

3 note con accuratezza $\approx 10^{-11}$

- nel 2003 le λ diventano 13 e le accuratezze migliorano...

Optical Frequency Metrology: “most accurate” wavelengths

- in 2007 recommended λ became 20 with accuracy up to 3×10^{-15}

Absorbing ion $^{199}\text{Hg}^+$, $5d^{10}6s^2S_{1/2}$ ($F=0$) – $5d^96s^2D_{5/2}$ ($F=2$) $\Delta m_F=0$

CIPM recommended values

– $f = 1\,064\,721\,609\,899\,145$ Hz (≈ 1000 THz)

– $\lambda = 281\,568\,867.591\,968\,6$ fm (≈ 280 nm)

with a relative standard uncertainty of 3×10^{-15} , apply to the unperturbed quadrupole transition of a **trapped and cooled mercury ion**

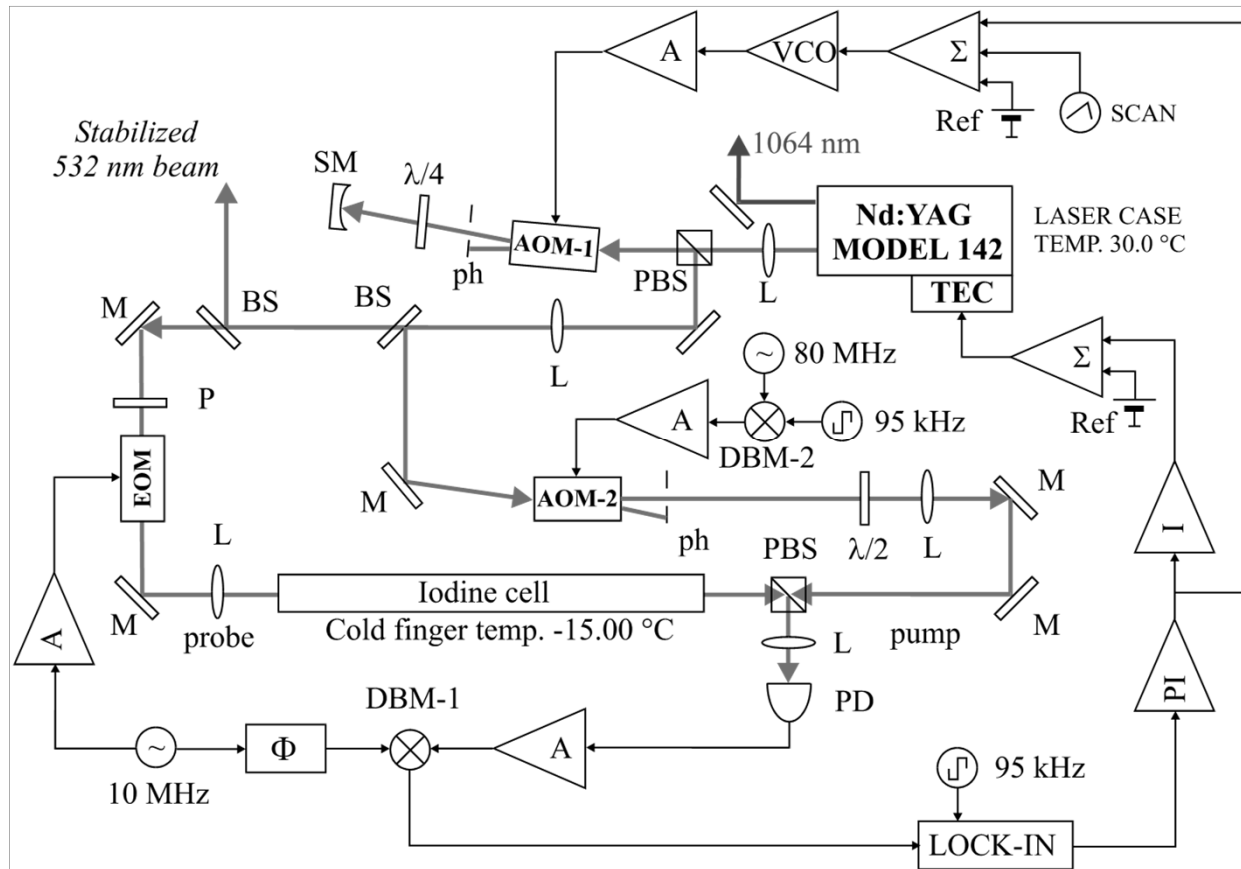
$^{88}\text{Sr}^+$ uncertainty of 7×10^{-15} ; $^{171}\text{Yb}^+$ uncertainty of 9×10^{-15}

^{40}Ca uncertainty of 1.8×10^{-14} ; OsO_4 uncertainty of 1.4×10^{-13}

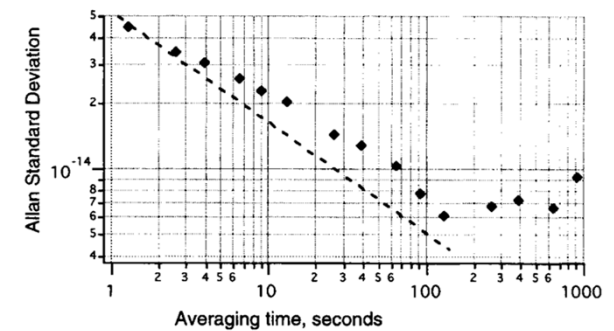
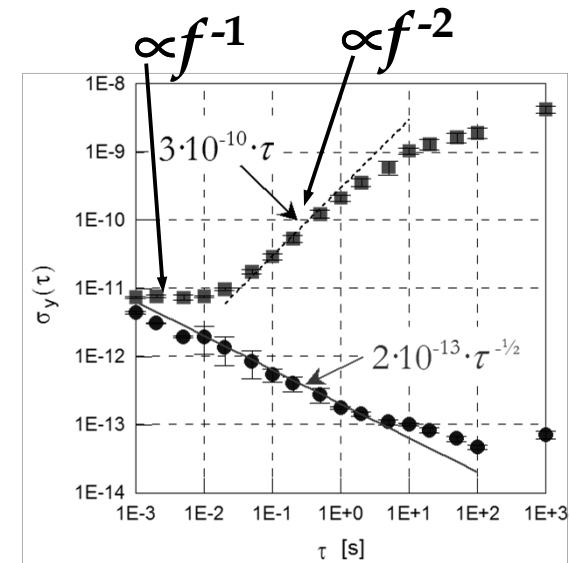
^1H uncertainty of 2.0×10^{-13} ; $^{115}\text{In}^+$ uncertainty of 3.6×10^{-13}

$^{127}\text{I}_2$ uncertainty of 8.9×10^{-12} ; $^{13}\text{C}_2\text{H}_2$ uncertainty of 2.6×10^{-11}

Experiments with Nd:YAG Lasers freq. doubled and stabilized to I₂



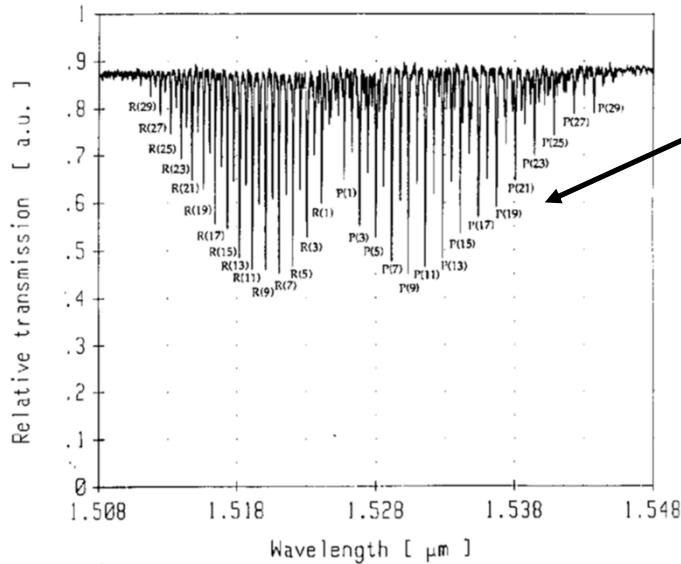
G. Galzerano, C. Svelto, E. Bava, F. Bertinetto, Appl. Opt. (1999)



J. Hall, ... (2000)

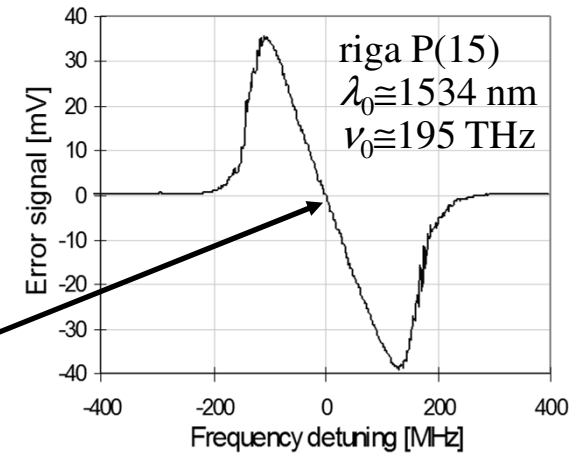
Nobel Prize!!! (2005)

The Acetylene Molecule (C_2H_2) and Stabilized Er-Yb Lasers

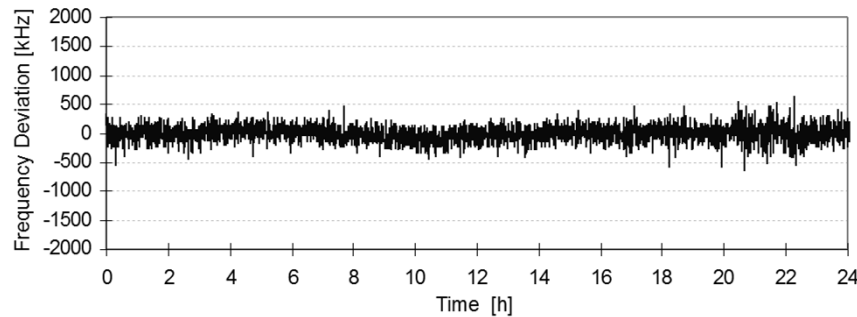


~50 resolved lines
spaced by ~100GHz
and large ~0.5GHz

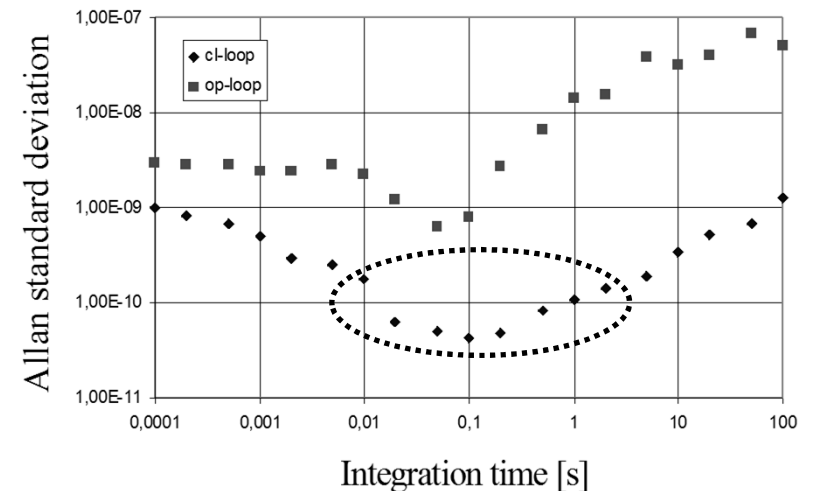
$p=1\text{kPa}$, $L=10\text{cm}$
 $\text{slope}=0.4\text{mV/MHz}$



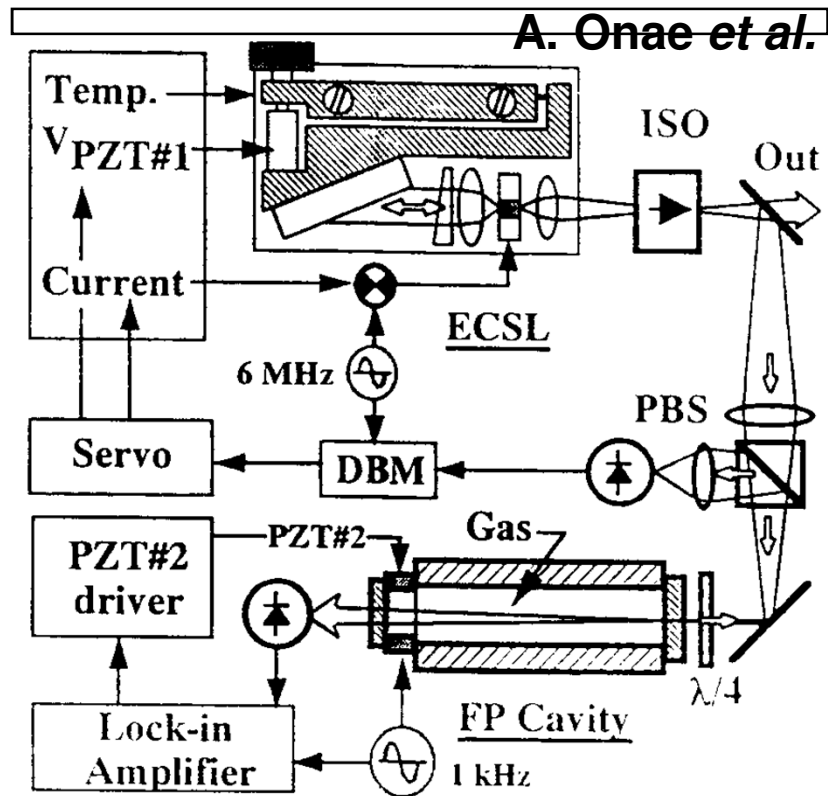
Analysis of beat-note frequency



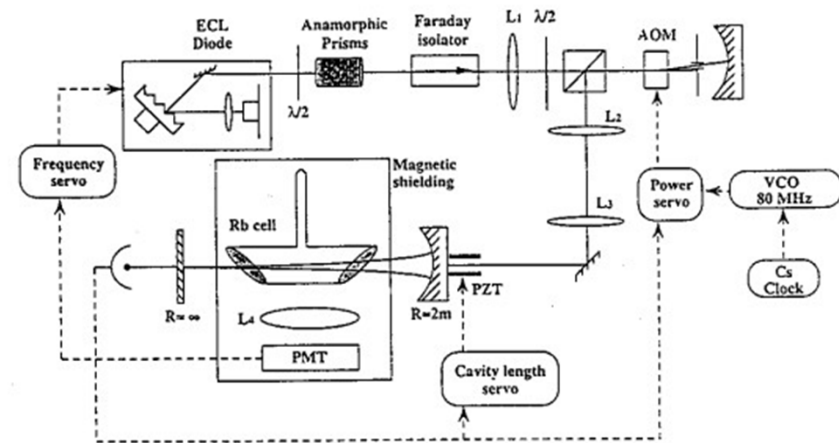
$$\Delta\nu_{\text{rms}} = 160\text{kHz} < 10^{-9} \nu$$



International Experiments at $1.5 \mu\text{m}$



saturated $^{13}\text{C}_2\text{H}_2$ ($\lambda=1550.18\text{nm}$)
 $(\lambda_{\text{laser}}=1.55\mu\text{m})$
 stability = $\pm 2\text{kHz}$ (10^{-11})
 accuracy = 200kHz



D. Touahri *et al.*, Opt. Comm. 133, 471 (1997).

2-photon Rb ($\lambda=778\text{nm}$)
 $(\lambda_{\text{laser}}=1.556\mu\text{m})$
 stability = $\pm 2\text{kHz}$ (10^{-11})
 accuracy = 20kHz

Experiments PoliMI *et al.* at 1.5 μm

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IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT, VOL. 51, NO. 4, AUGUST 2002

High-Resolution Spectroscopy of the ^{39}K Transitions at 770 nm and $^{13}\text{C}_2\text{H}_2$ Saturated Lines by a Solid-State Laser at 1.54 μm : Toward an Accurate Frequency Standard in the Optical Communication Band

Cesare Svelto, *Member, IEEE*, Gianluca Galzerano, Elio Bava, *Associate Member, IEEE*, Fabio Ferrario, Ady Arie, *Senior Member, IEEE*, R. Klein, M. A. Arbore, M. A. Fejer, Atsushi Onae, and Marcello Marano

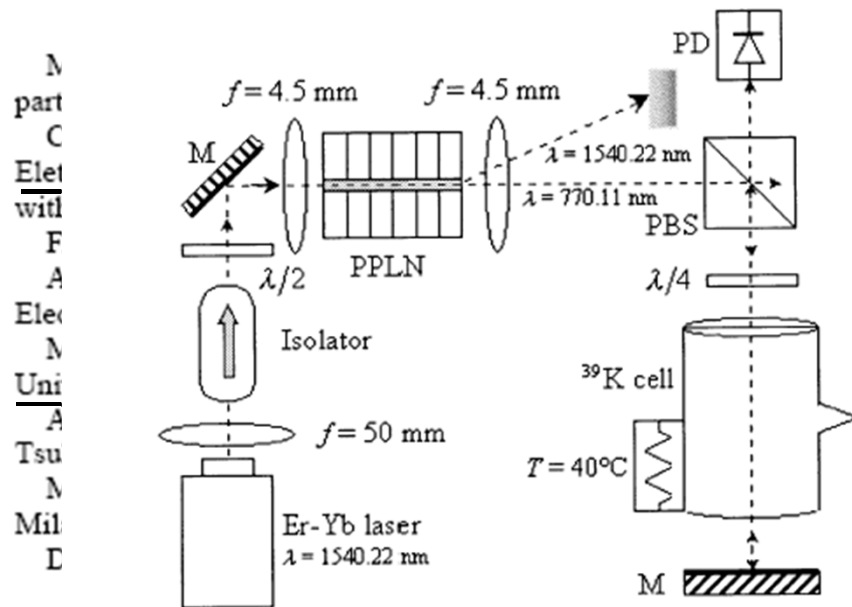


Fig. 3. Schematics of the experimental setup for frequency doubling of the Er-Yb: glass laser and high-resolution spectroscopy of the ^{39}K line; M: Mirror; PD: silicon photo detector; PBS: polarizing beam splitter.

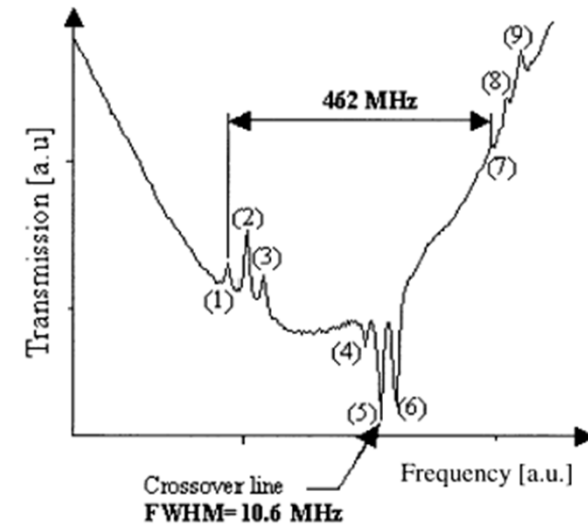


Fig. 5. Saturation spectroscopy of the ^{39}K sample as obtained by scanning the Er-Yb laser frequency through the Doppler-broadened resonance.

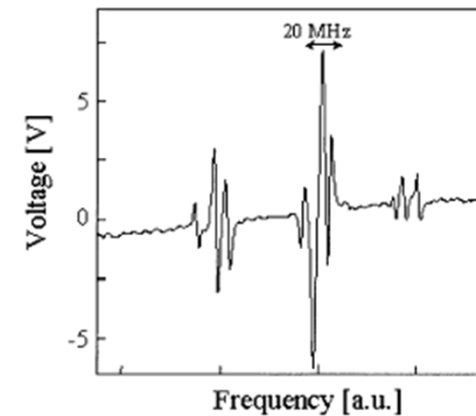


Fig. 6. First derivative voltage signal as obtained at the lock-in output after amplification of 50 V/V.

freq. stab. $\Delta\nu/\nu \approx 10^{-12}$ 35/45

Experiments PoliMI *et al.* at $1.5 \mu\text{m}$

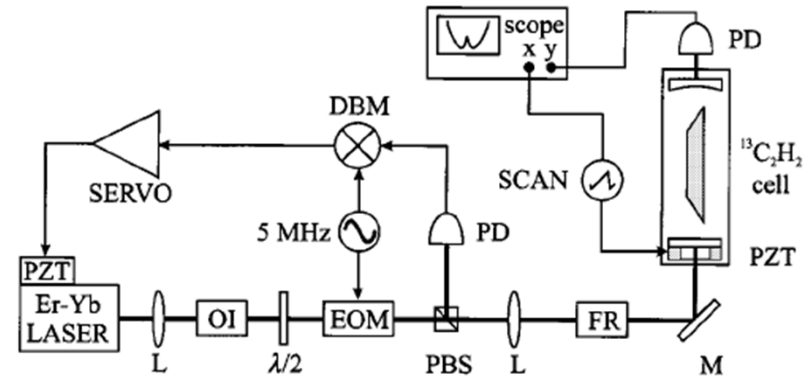
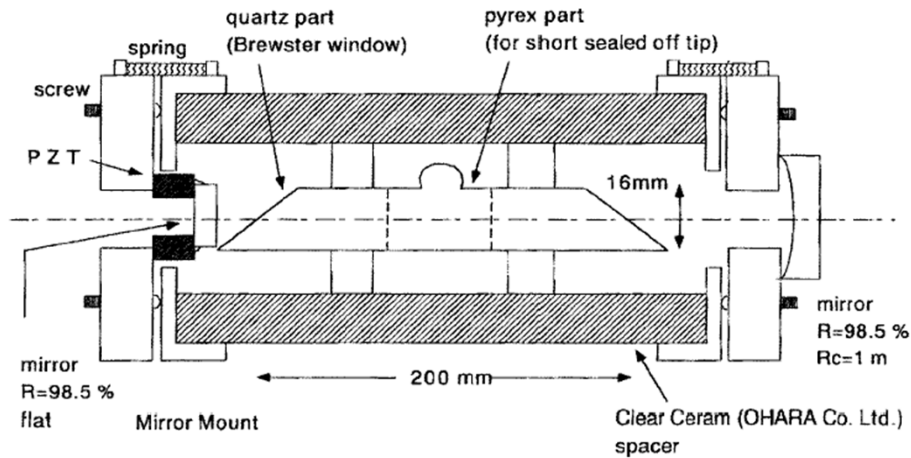


Fig. 4. Experimental setup. PZT: piezoelectric transducer; L: lens; OI: optical isolator; $\lambda/2$: half-wave plate; EOM: electro-optic modulator; PBS: polarizer beam splitter; PD: photodiode; FR: Faraday rotator; M: mirror; DBM: doubled balanced mixer.

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IEEE TRANSACTIONS ON INSTRUMENTATION AND MEASUREMENT, VOL. 51, NO. 4, AUGUST 2002

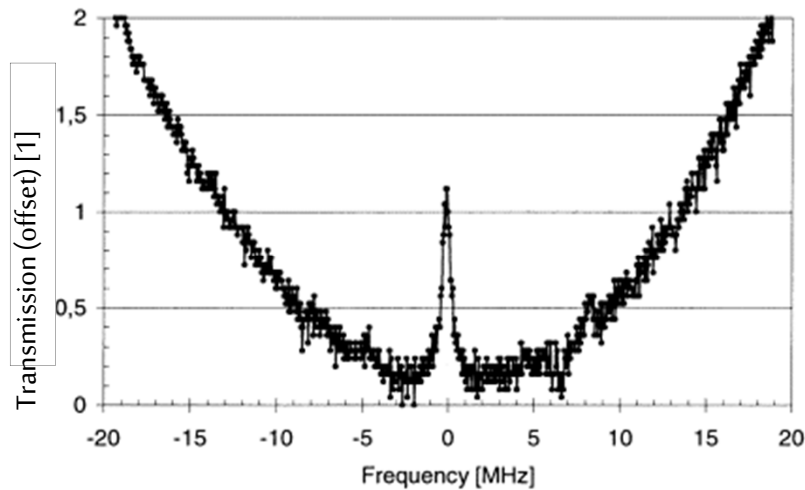


Fig. 7. Saturation spectroscopy of the $^{13}\text{C}_2\text{H}_2$ P(16) line at $\lambda = 1542.384 \text{ nm}$. The saturation line contrast is $\sim 3\%$ and its linewidth is 1.2 MHz.

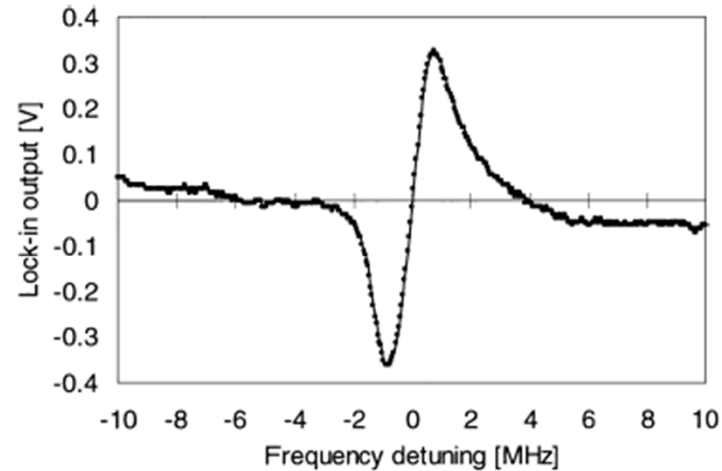
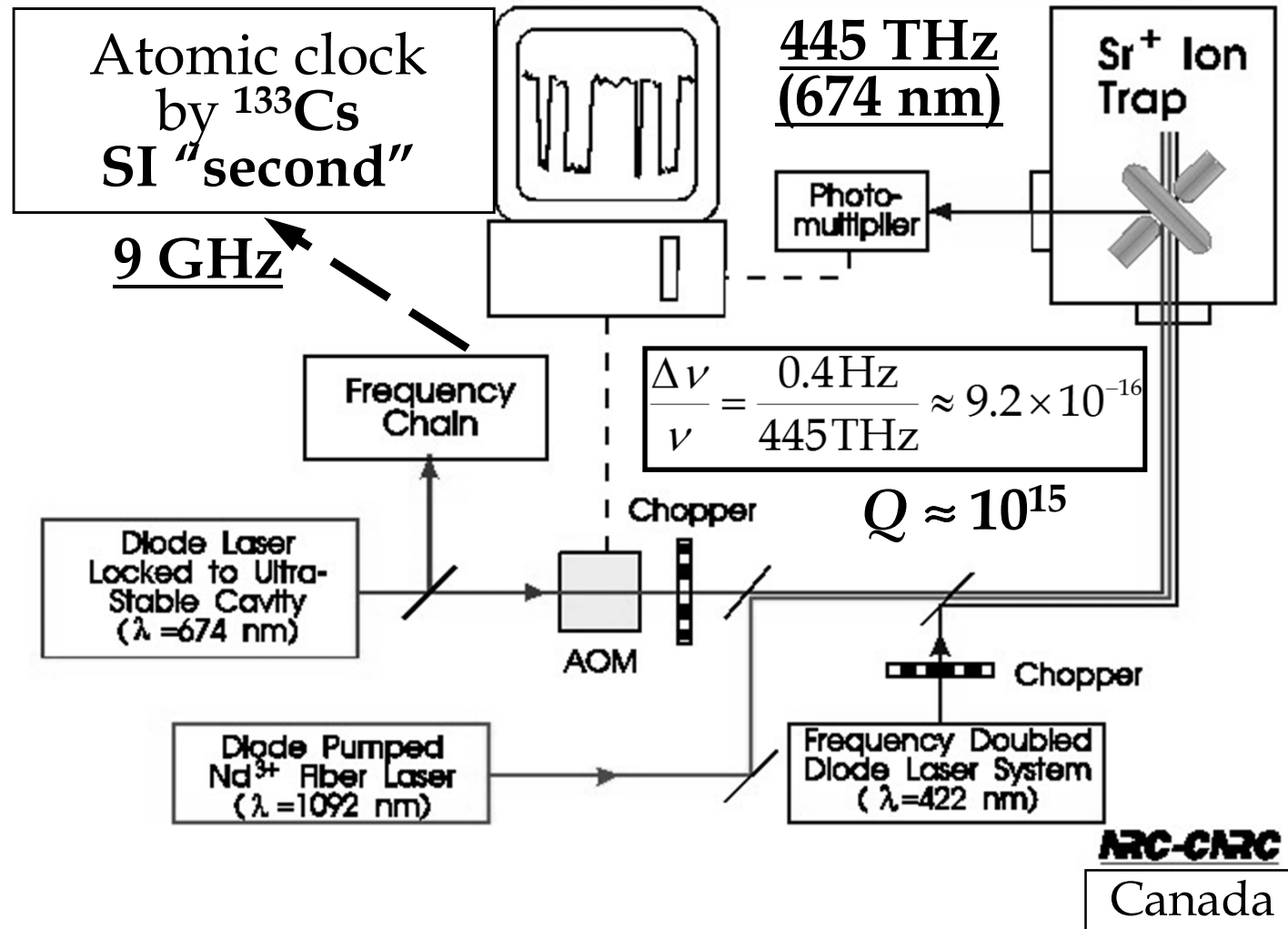


Fig. 8. First derivative of the saturated absorption dip in line P(16) of $^{13}\text{C}_2\text{H}_2$.

freq. stab. $\Delta\nu/\nu \approx 2 \times 10^{-12}$

Optical Atomic Clock - single Sr⁺ ion (yr. 2002)

The Sr⁺ Trap Experimental Setup



Optical Atomic Clock - Sr⁺ multi-ion (1/3)

<http://www.newscientist.com/> New Scientist Breaking News

The most accurate clock of all time

18:00 18 May 2005

Related Articles

13 July 2001 World's most accurate clock created

- Max Plank Institute for Quantum Optics
- Department of Quantum Engineering and Systems Science, University of Tokyo

If "TIME waits for no man", then neither does human ingenuity in measuring its passing. Throughout history, more stable and accurate clocks have led to advances in communications and navigation. Now a clock 1000 times more accurate than any of its predecessors has set another benchmark, and could even be used to create a more precise definition of how long a second is. The new clock is a variant on the atomic clocks that appeared in the 1950s. **Atomic clocks usually work by measuring the frequency at which atoms resonate.** For instance, the outer electrons of a caesium-133 atom resonate between two energy states exactly 9,192,631,770 times each second, emitting microwaves of exactly that frequency as they do so. This property has been used since 1967 to define what we mean by 1 second - it is officially the time it takes for a caesium atom to resonate 9,192,631,770 times.

Optical Atomic Clock - Sr⁺ multi-ion (2/3)

“One way to create a more accurate clock is to increase the rate at which it ticks”.

The first atomic clocks could pin this down to an accuracy of 1 part in 10¹⁰.

Today's caesium clocks can measure time to an accuracy of 1 in 10¹⁵, or 1 second in about 30 million years. But the search is on for ever more precise timepieces.

"A clock has a counter that counts something that is periodic. The shorter that period is, the more accurate the clock. That is why people went from sundials, with one period per day, to pendulum clocks, with one period per second, to quartz clocks with 10,000 oscillations per second. Now we have the caesium clock counting with 9 billion oscillations per second."

So **what ticks faster than a caesium atom?** Elements being scrutinized include ytterbium, mercury and **strontium** (which **resonates 429,228,004,229,952 times each second**). But until now it has proved impossible to create a useful strontium atomic clock. In principle, there are **two ways to create a strontium clock: using the oscillations of a single atom, or doing the same with many atoms at the same time.**

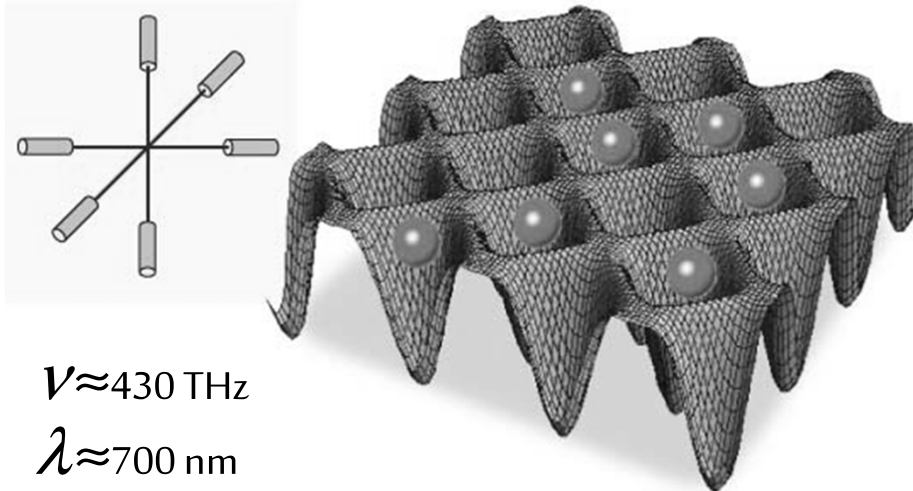
The advantage of using a **single atom** is that it is relatively **easy to shield** it from external electromagnetic fields, which interfere with its oscillating frequency. The disadvantage is that it is extremely **difficult to accurately measure** a single atom vibrating at such a high frequency. **A multi-atom clock produces a much clearer signal but is less accurate, because the electromagnetic fields of the atoms interfere with each other.**

Optical Atomic Clock - Sr⁺ multi-ion (3/3)

Now **Hidetoshi Katori** and his colleagues at the University of Tokyo have come up with an elegant solution that combines the **advantages of both systems** (*Nature*, vol 435, p 321). Katori uses six laser beams to create a pattern of standing electromagnetic waves. This creates a series of energy wells, each of which supports one strontium atom, in much the same way as each dimple in an egg box holds an egg (see Diagram). This prevents the electromagnetic fields of individual atoms interfering with those of their neighbors, and allows the oscillating signals of many atoms to be measured at once. **Previous attempts to make clocks this way failed because the trapping lasers themselves interfered with the atoms' oscillation frequency.**

OPTICAL LATTICE CLOCK

Six laser beams create a pattern of standing waves that traps strontium atoms in energy wells. The trapping laser frequency is one that does not interfere with the atoms, which tick at about 429 terahertz, providing unsurpassed timekeeping accuracy



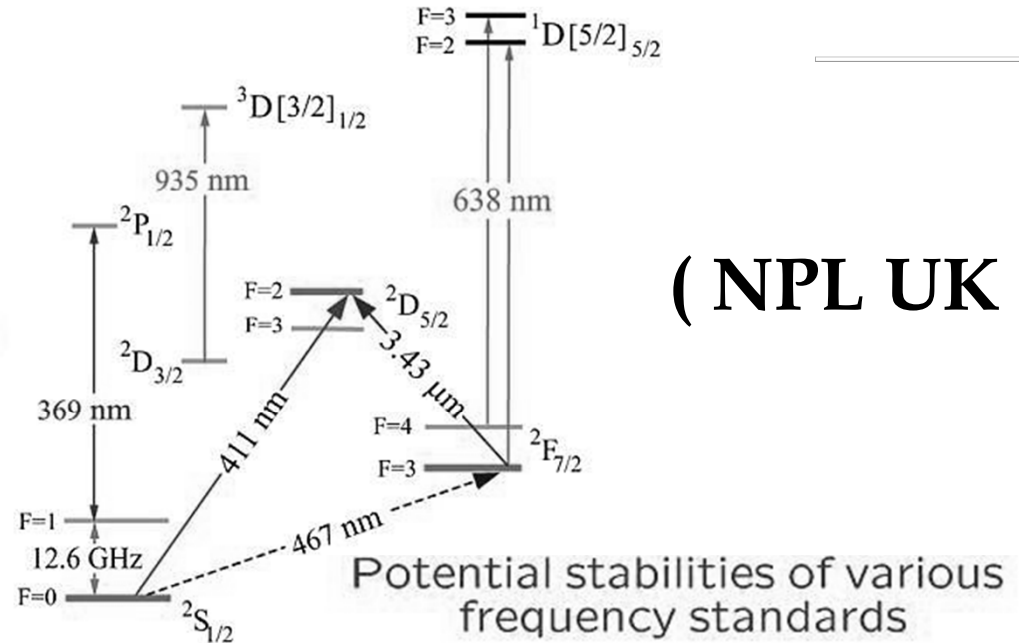
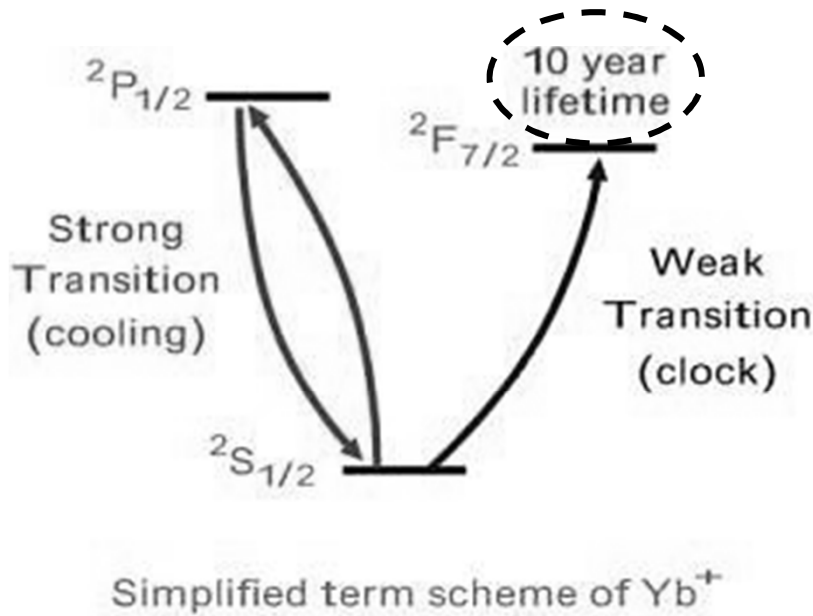
$$\nu \approx 430 \text{ THz}$$

$$\lambda \approx 700 \text{ nm}$$

SOURCE: NATURE

Katori's group has got round this by **tuning the frequencies of the lasers so they alter the upper and lower transition energy levels of strontium by exactly the same amount**, so the oscillation frequency remains unaltered. Katori claims that this "optical lattice clock" will keep time with an accuracy of 1 part in 10¹⁸.

Optical Atomic Clock - single Yb⁺ ion

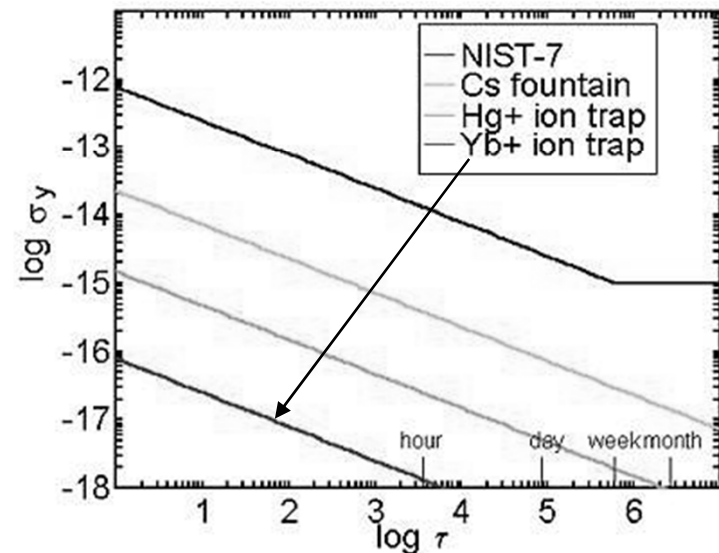


$$\lambda = 467 \text{ nm}$$

$$\nu = 642 \text{ THz}$$

$$\Delta \nu \approx 1 \text{ nHz}$$

$$Q = \nu / \Delta \nu \approx 6 \cdot 10^{23}$$



Optical Spectroscopy and the “SI meter”

High-resolution spectroscopy of atoms and molecules has been, and continues to be an essential experimental tool in the formulation and development of quantum mechanics, the measurement of atomic properties, the determination of fundamental constants, and in the realization of standards for time and length. Not only are we able to directly probe the detailed structure of atoms and molecules but also **test the invariance of the velocity of light** as postulated by the theory of special relativity, which has allowed us to redefine the SI unit of length and realize it in terms of experimental observables of quantized systems rather than physical artifacts.

The SI unit of length was originally intended to be one ten millionth part of the quadrant of the **Earth**, and the original 1799 metre was based on measurements made along a **meridian** from Dunkerque to Barcelona represented an attempt to realize this length. Such a physical artifact remained in the definition of the metre until **1960** when the definition was replaced by one based on a measurement of the **wavelength of orange light emitted by excited atoms of an isotope of krypton**. This definition was eventually superseded in 1983 by the present one, which is derived by assigning a defined value to the speed of light in free space. Today, the **SI** unit of the **metre** is ‘the length of the **path traveled by light in vacuum during a time interval** of $1/299\,792\,458$ of a second’, thus fixing the velocity of light at $299\,792\,458\text{ m s}^{-1}$.

Accuracy Limits in MEASUREMENTS

With a **fixed velocity of light relating distance and time**, and hence **frequency** and **wavelength**, the ability to measure time more accurately than other physical quantities provides a **much improved realization of the metre**. In principle, we are limited only by the accuracy of the time reference.

At present the **best microwave time standard** based on transitions within caesium atoms has an accuracy of about **one part in 10^{15}** , whereas **the best optical frequency standards** (spectroscopy of laser cooled single atoms and ions) are expected to approach a level of one part in 10^{18} .

The developments in **optical comb technology** represented by the measurements of Long-Sheng Ma et al. (a collaboration involving the Bureau International des Poids et Mesures, Sèvres, France; Physics Department, East China Normal University, Shanghai, China; National Institute of Standards and Technology, Boulder, CO, USA; and OFS Laboratories, Murray Hill, New Jersey, USA), recently reported in Science (19th March, 2004), **demonstrate the potential to connect the frequency of radiation in the optical range of the electromagnetic spectrum to the frequency of radiation in the radiofrequency domain with an accuracy approaching one part in 10^{19}** .

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