
“Optical Measurements”

Master Degree in Engineering
**Automation-, Electronics-, Physics-,
Telecommunication- Engineering**



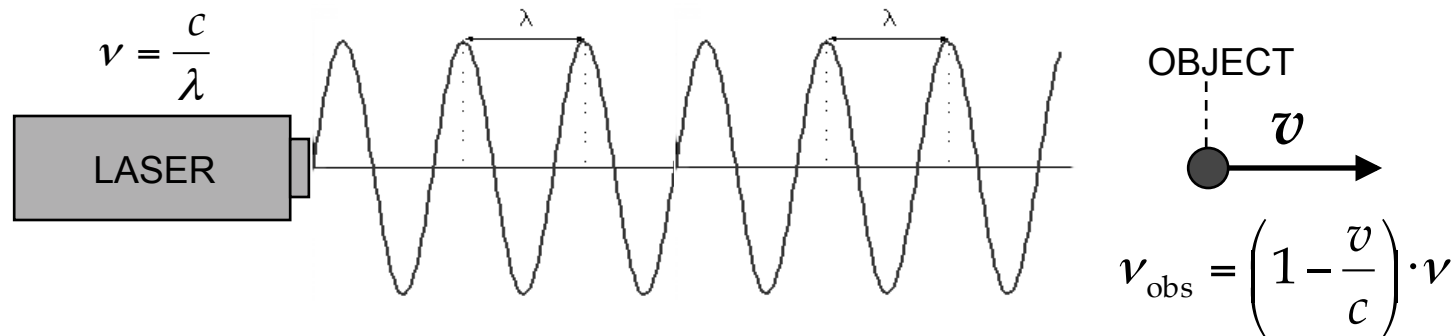
Optical Velocimeters

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Summary

- **Measurement principles and application**
Doppler effect (on laser beams)
Heterodyne method for velocimetry (laser Doppler)
- **Velocity measurements in fluids**
LDV (Laser Doppler Velocimetry)
PIV (Particle Image Velocimetry)
- **Autovelox**
 - optical barrier
 - tele-laser (time of flight and Doppler)

Doppler effect (on laser beams)



The observed optical frequency (ν_{obs}) is smaller than the one emitted by the laser (ν) when the object moves farther ($\nu_{\text{obs}} < \nu$ when object gets closer)

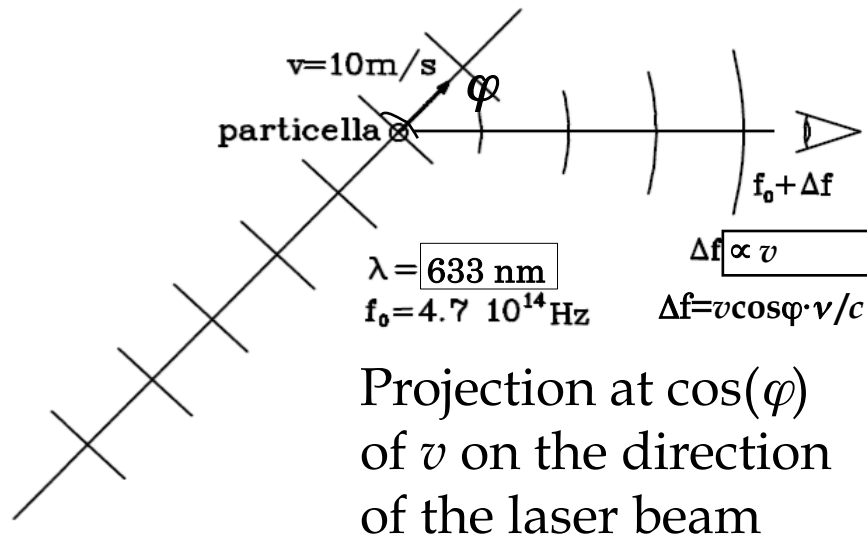
Considering a velocity $v \ll c$, the **Doppler shift**

$$\Delta \nu = (\nu - \nu_{\text{obs}}) = (v/c) \cdot \nu \ll \nu$$

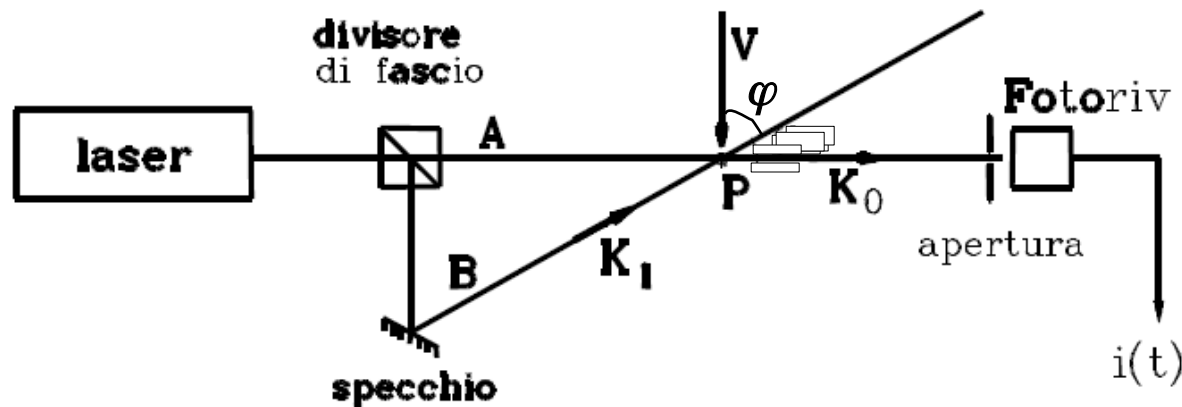
is much smaller than the optical frequency

[for $\lambda = 0.6 \mu\text{m}$ ($\nu = 500 \text{ THz}$), $v = 1 \text{ m/s} \rightarrow \Delta \nu \approx 1.7 \text{ MHz}$, $\Delta \nu / \nu \approx 3.3 \times 10^{-9} \approx 3 \text{ ppb!}$]

Heterodyne method for velocimetry



To detect Δf of a few MHz on the optical beam at $\approx 500 \text{ THz}$ we can not use a monochromator or OSA but much better (being $\Delta f / f_0 = \Delta \nu / \nu \ll 1$) to measure the “heterodyne” beat signal with a reference laser beam



Laser Doppler Velocimetry (LDV)

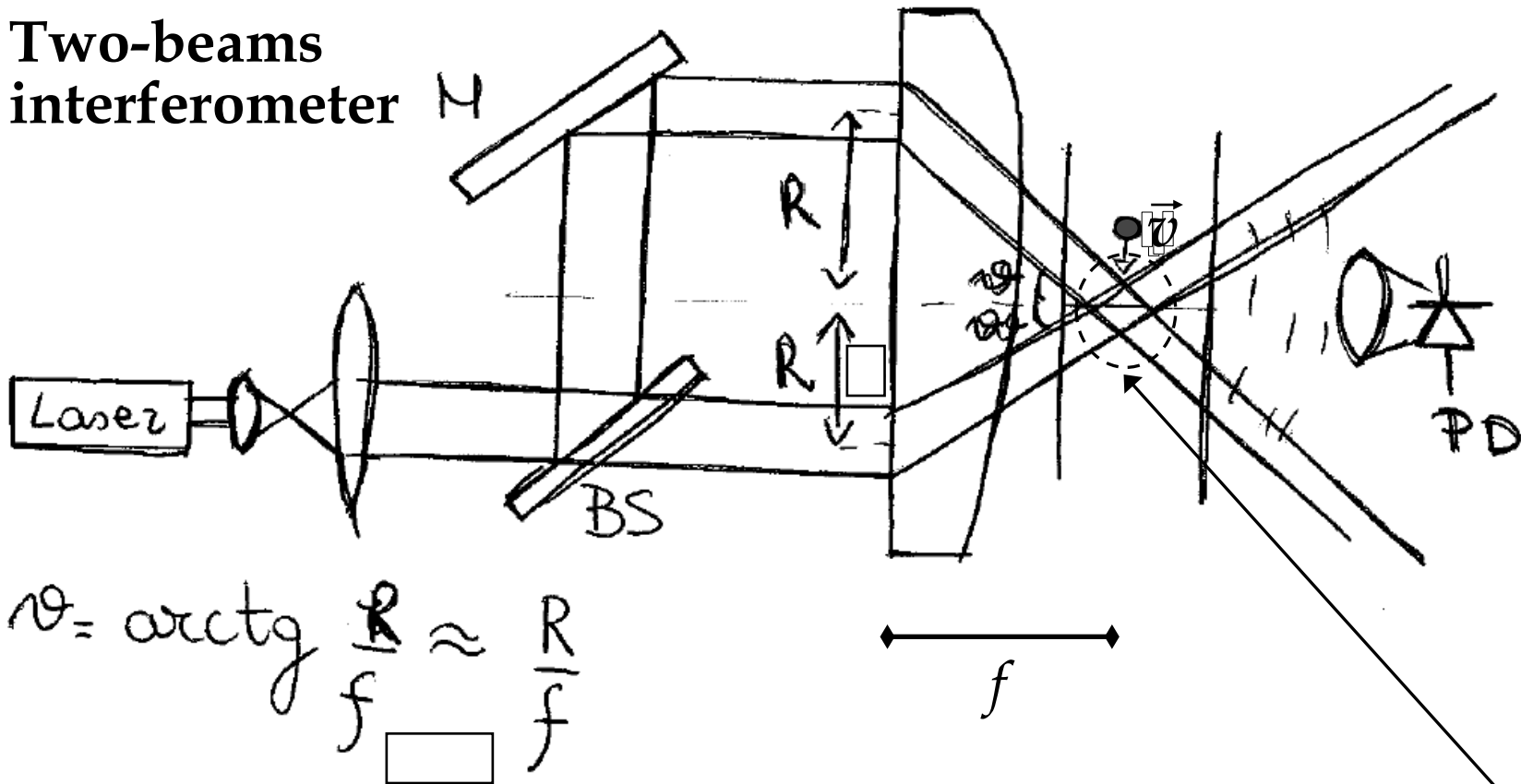
- **LDV was invented in 1964** and is still today a great success of optical measurements, to *contactless* detect a **wide range of velocities in moving fluids**

The **LDV techniques** are used to measure velocity of fluids (liquids / gases) **carrying diffusing (scattering) particles**, naturally present or artificially “inseminated” in the fluid

- The diffusion, or *scattering*, due to interaction of the e.m. field and the particle can be divided into two types:
 - **Rayleigh** ($r \ll \lambda$) $\alpha_s \propto r/\lambda^4$ $f(\theta)$ constant with the angle
 - **Mie** ($r \approx \lambda$) $\alpha_s \approx \text{const.}$ $f(\theta)$ max. for $\theta = 0$ (*forward scatt.*)
- The **measurement signal** can be seen as due to:
 - **fringe crossings** ←
 - **Doppler effect**
 - **interferometric phase shift**

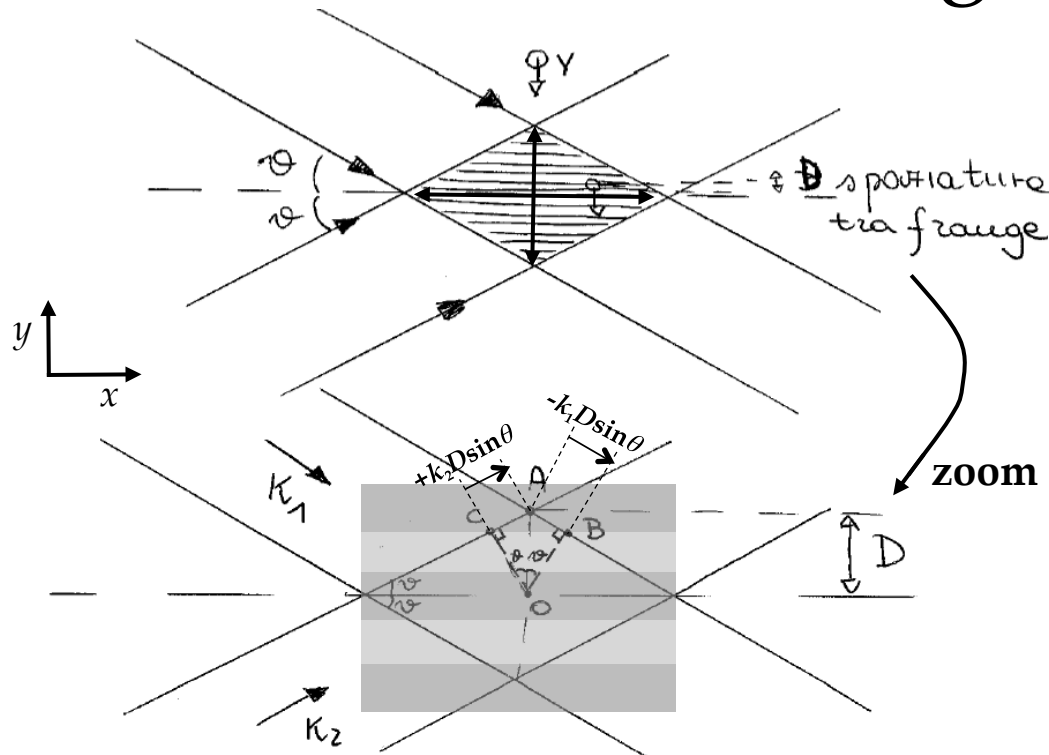
LDV: measurement set-up

Two-beams
interferometer



The two collimated beams impinge off-axis on the lens and are focused at distance f from the lens in an **interference region**
example: $R=0.5\text{cm}$ and $f=250\text{mm} \rightarrow \theta \approx 20\text{mrad} = 1.15^\circ \approx 1^\circ$

LDV: interference fringes



collimated beams
 \rightarrow wavefronts
 approx. plane
 with width $\pm w_0$

the interaction
 zone is wide
 $\Delta X = \pm 2w_0 \cdot \cos \theta$
 $\Delta Y = \pm 2w_0 \cdot \sin \theta$

$$\Delta \Phi_{(O \rightarrow A)} = [\Phi_2 - \Phi_1]_{(O)} - [\Phi_2 - \Phi_1]_{(A)} = 2\pi \quad \text{fringe spacing } D \text{ dark/dark}$$

$$\text{From O to A: } \Delta \Phi_2 = \Phi_{2(O \rightarrow C)} + \Phi_{2(C \rightarrow A)} \quad \text{and} \quad \Delta \Phi_1 = \Phi_{1(O \rightarrow B)} + \Phi_{1(B \rightarrow A)}$$

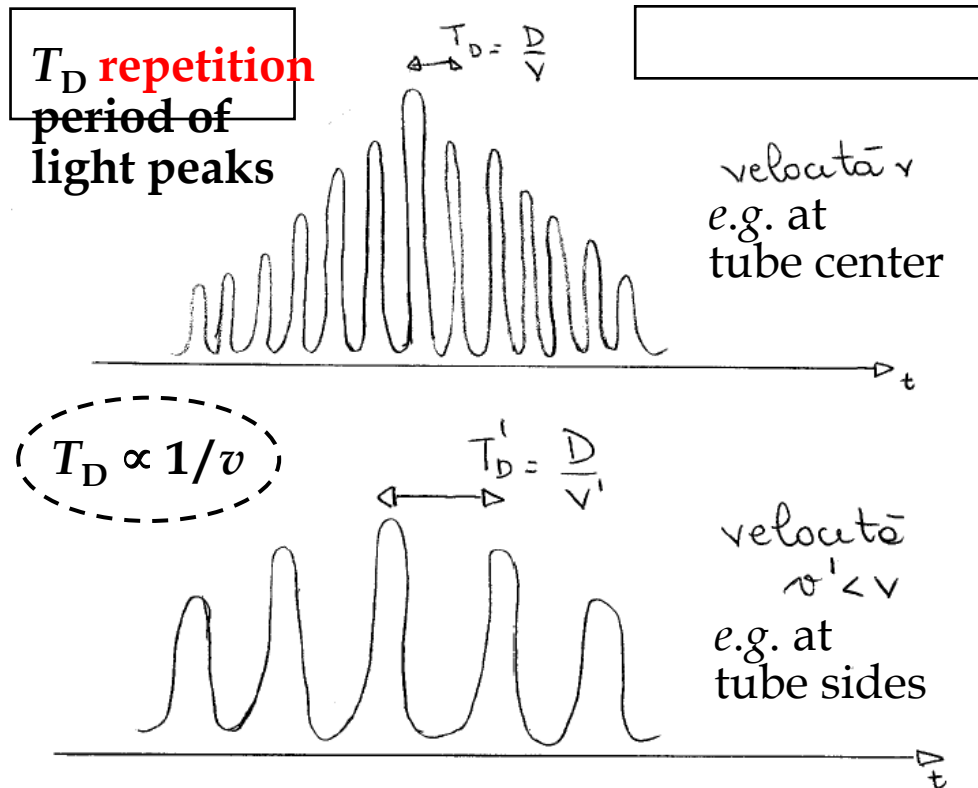
$$\Delta \Phi_2 = (0 + k_2 \underline{CA}) = +(2\pi/\lambda) D \sin \theta \quad \text{and} \quad \Delta \Phi_1 = (0 - k_1 \underline{BA}) = -(2\pi/\lambda) D \sin \theta$$

$$\Delta \Phi_{(O \rightarrow A)} = \Delta \Phi_2 - \Delta \Phi_1 = 2 \cdot (2\pi/\lambda) D \sin \theta = 2\pi \Rightarrow D \sin \theta = \lambda/2$$

$$D = \frac{\lambda}{2 \sin \theta}$$

Light peaks repeated at distance D (\perp a v)

LDV: relation velocity \rightarrow frequency

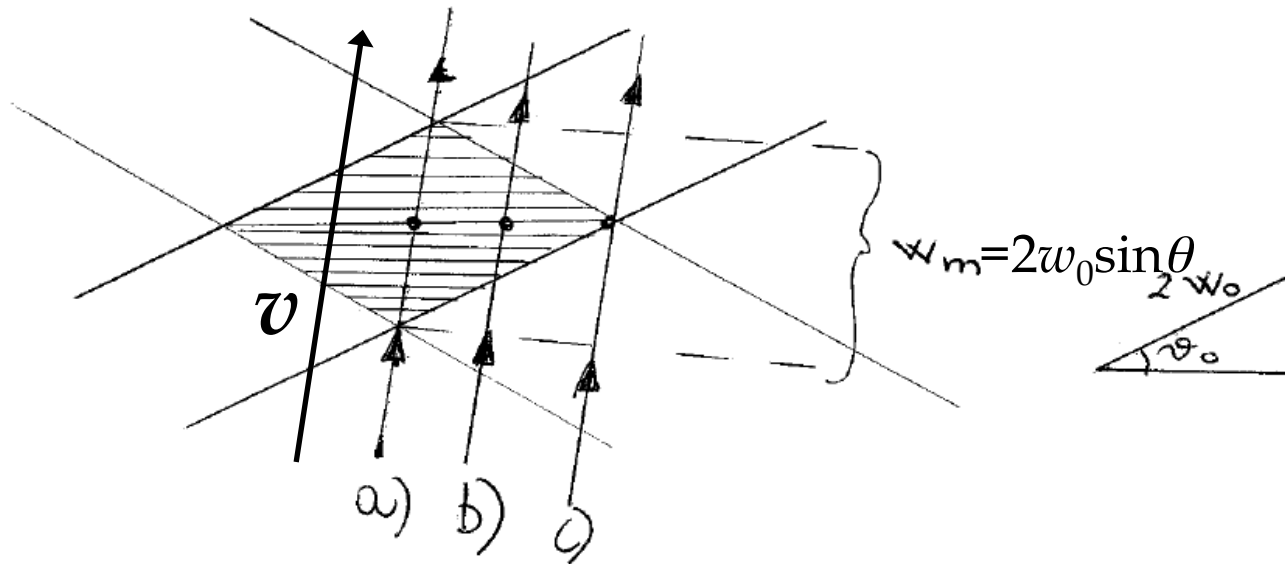


Thickness of crossing zone changes with position of crossing line (in the lit zone) but **light-light distance** remains constantly D in orthogonal direction to the interference zone.

On a time axis the distance T_D between light peaks depends on velocity v

The optical beam profile, crossed by particles, is Gaussian and hence diffused light intensity profile varies with position. The frequency of alternating light-dark-light depends on the fringe distance and particle/fluid velocity $f_D = 1/T_D \propto v$

LDV: signal period and frequency



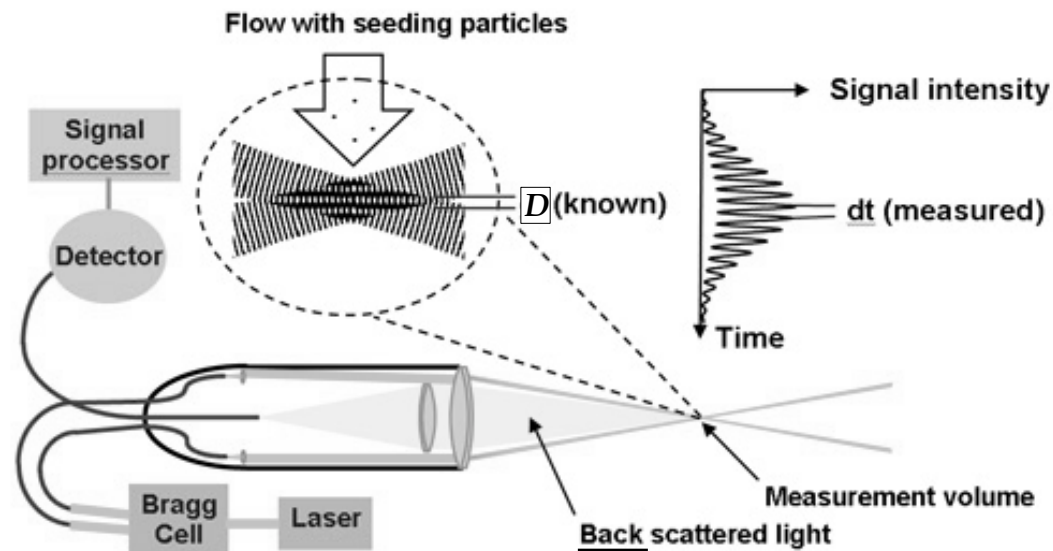
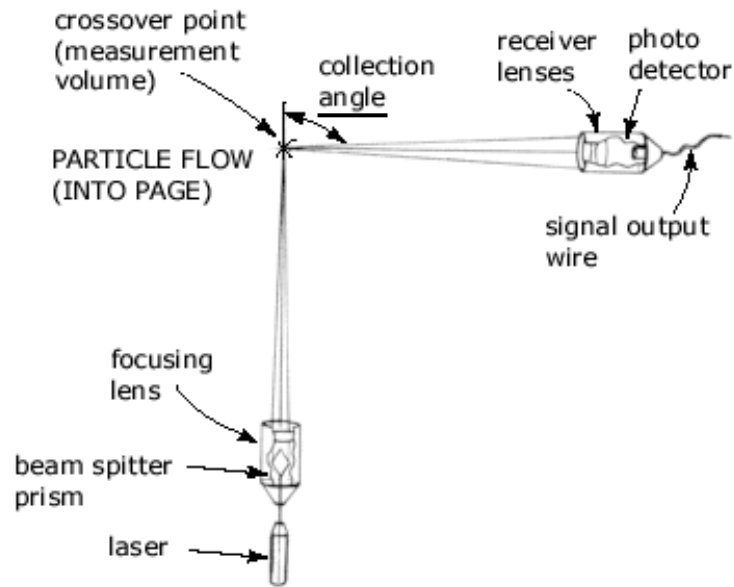
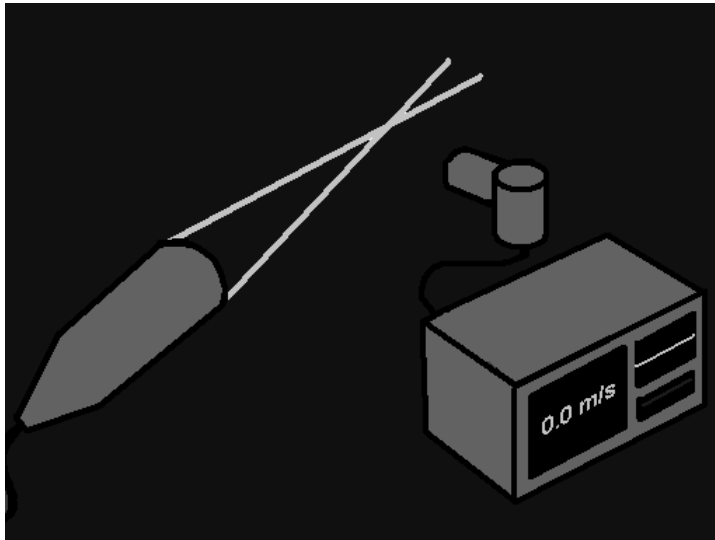
$T_D = D / v$ period of diffused light signal (light-dark-light)

$$\Rightarrow \boxed{f_D = \frac{1}{T_D} = \frac{v}{D} = \frac{2 \sin \theta}{\lambda} v \propto v} \Leftarrow$$

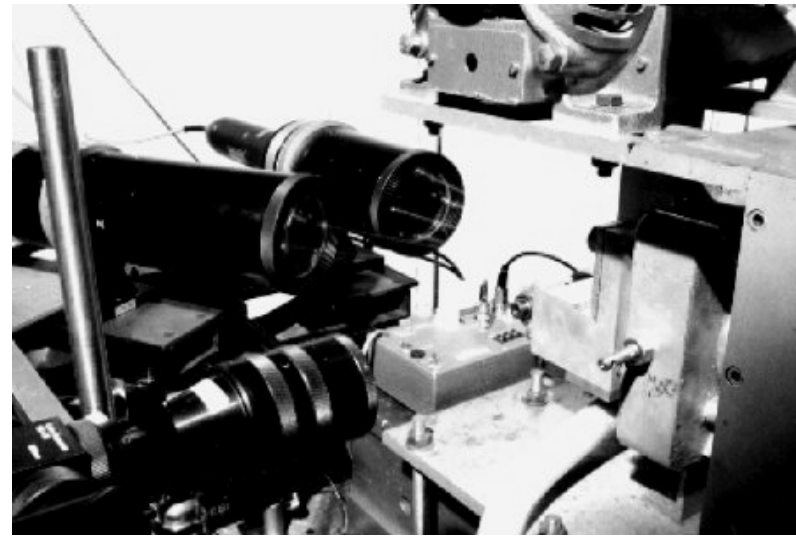
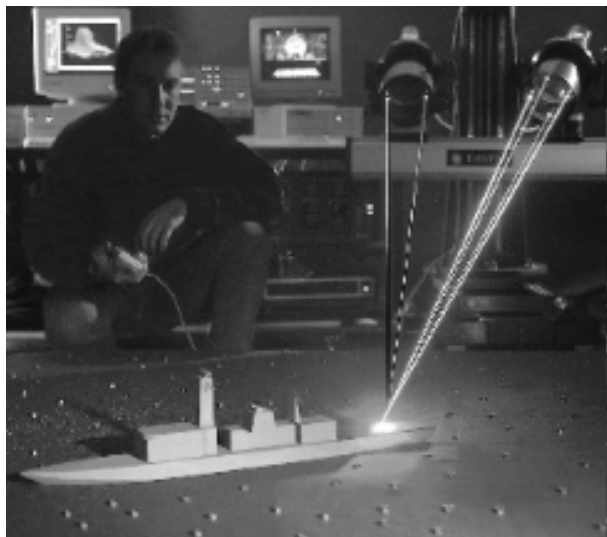
MEASURE SENSITIVITY: $S_{f \leftarrow v} = \Delta f / \Delta v = 2 \sin \theta / \lambda$ (Hz/ms⁻¹)

Measure of f_D : 1) counter; 2) electron. AS; 3) correlation 9/20

LDV: schemes (*angle_scatt./back_scatt.*)



LDV: photos of 3D LDVs



LDV: practical example

$\lambda = 532 \text{ nm}$ (Nd:YAG duplicato-)

$\theta_1 = 3^\circ$ ($\approx 0.05 \text{ rad}$) $\Rightarrow S_{1, v \rightarrow f} \approx 200 \frac{\text{MHz}}{\text{m/s}}$

$\theta_2 = 20^\circ$ ($\approx 0.35 \text{ rad}$) $\Rightarrow S_{2, v \rightarrow f} \approx 1.3 \frac{\text{MHz}}{\text{m/s}}$

Se mi misura una velocità $v = 500 \frac{\text{m}}{\text{s}}$

$f_{D,1} = 100 \text{ MHz}$

$f_{D,2} = 650 \text{ MHz}$

$$S = \frac{\Delta f_D}{\Delta v} = \frac{2 \sin \theta}{\lambda}$$

La misura diviene critica per alte velocità del fluido ($v > 100 \text{ m/s}$) mentre risulta agevole e molto sensibile per misure di basse velocità.

con $\theta = 0.5 \text{ rad}$ ($\approx 32^\circ$) $\Rightarrow S_{v \rightarrow f} \approx 2 \frac{\text{MHz}}{\text{m/s}}$

e f_D è per $v = 1 \text{ mm/s} \Rightarrow f_D = 2 \text{ kHz}$.

Agevolmente misurabile dopo conversione A/D del segnale di luce diffusa in funzione del tempo.

LDV: properties

PRESTAZIONI DI MISURA

La sensibilità del velocimetro Doppler è:

$$\Delta v \approx \frac{2v \sin \theta}{\lambda} \approx \frac{2v}{\lambda} \text{ per } \theta \ll 1$$

per cui, in termini di incertezza relativa:

$$\frac{u(S)}{S} = \sqrt{\frac{u^2(\theta)}{v^2} + \frac{u^2(\lambda)}{\lambda^2}}$$

e potendo considerare un solo contributo d'incertezza alla volta si ha:

$$\frac{u(S)}{S} \approx \frac{u(\theta)}{v}$$

oppure

$$\frac{u(S)}{S} \approx \frac{u(\lambda)}{\lambda}$$

Solitamente l'incertezza nella lunghezza d'onda non è un problema in quanto si può controllare

$$v = \frac{\lambda}{2 \sin \theta} f_D \approx \frac{\lambda}{2\theta} f_D$$

scere λ con incertezza relativa $\frac{u(\lambda)}{\lambda} < 10^{-4}$. Invece essendo

$$\theta = \arctg \frac{r}{f} \approx \frac{r}{f}$$

$\begin{matrix} \nearrow & \text{distanza} \\ & \text{del fascio nella} \\ & \text{lente} \end{matrix}$
 $\searrow & \text{focale della} \\ & \text{lente}$

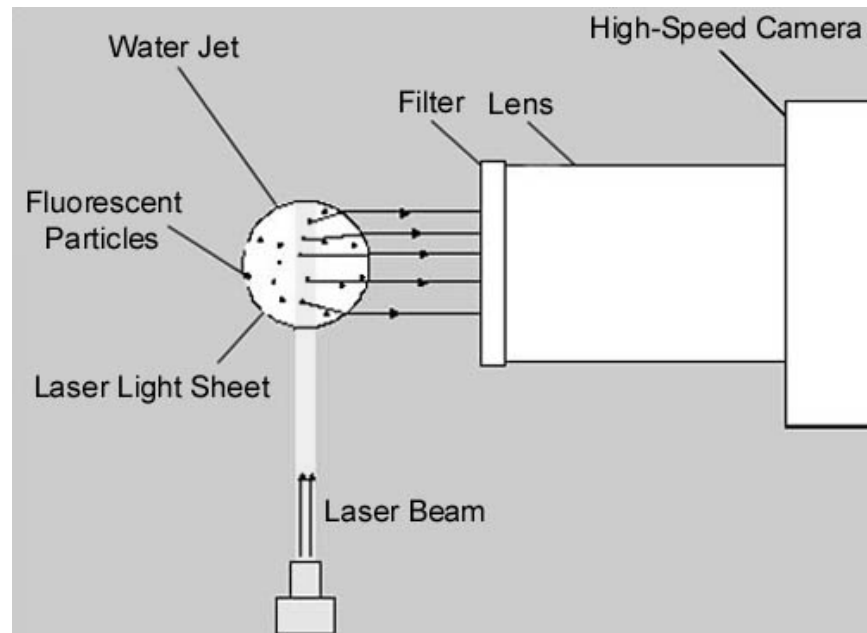
mentre f può essere creata con incertezza di 10^{-4} , il valore di r può essere mantenuto con incertezza relativa di qualche parte per mille (a causa della instabilità a lungo termine dei componenti meccanici).

Pertanto un limite di accuratezza nel coefficiente di sensibilità, e naturalmente sulla conseguente misura di velocità, può essere nell'ordine di 10^{-3} .

La misura della frequenza f_D naturalmente si può fare con incertezza anche minore di 10^{-6}

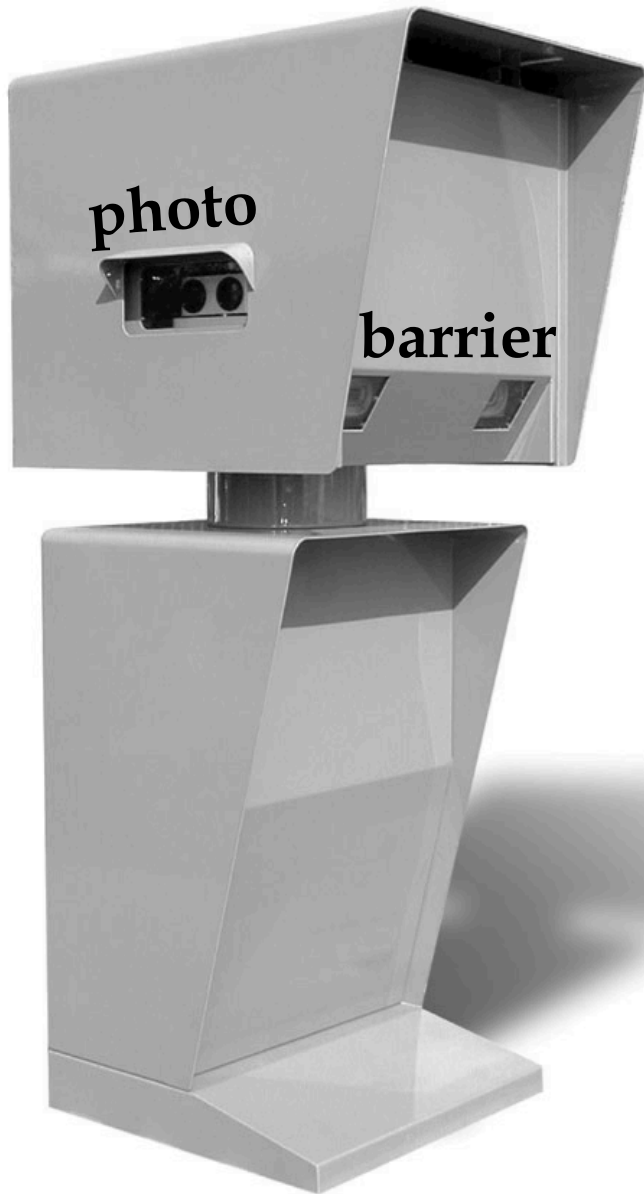
PIV (only measurement "principle")

The velocity measurement of the particles in the fluid is done "lighting" the investigation zone, with laser light (or with fast light flashes and using a multi-exposure slow camera)



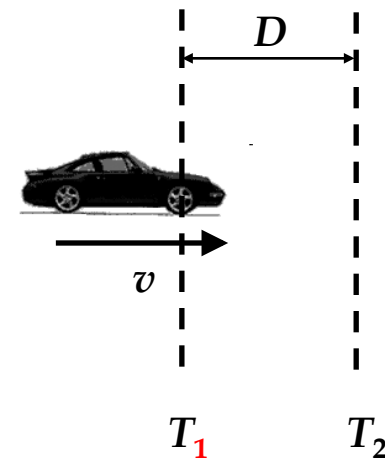
Images of the particles as a function of time are detected with a fast CCD camera, from which one can reconstruct the entire spatial profile (field) of velocities: $v_i = \Delta l_i / \Delta t$

Autovelox with optical barrier (e.g. 104 C2)



optical barrier

$$D = 0.5 \text{ m}$$



$$v = D / (T_2 - T_1) = D / \tau_{21}$$

E.g. for $v = 150 \text{ km/h}$ $\tau_{21} = 12 \text{ ms}$

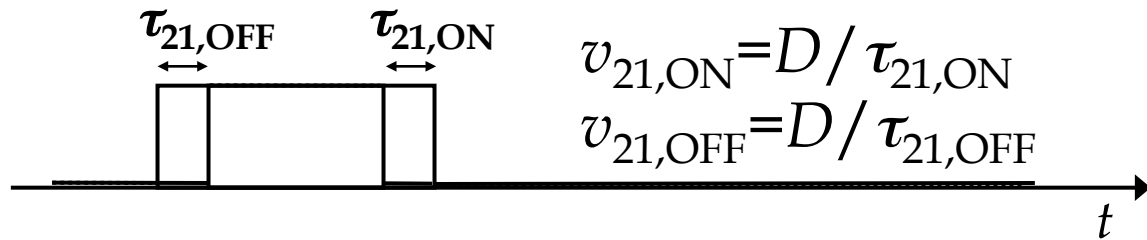
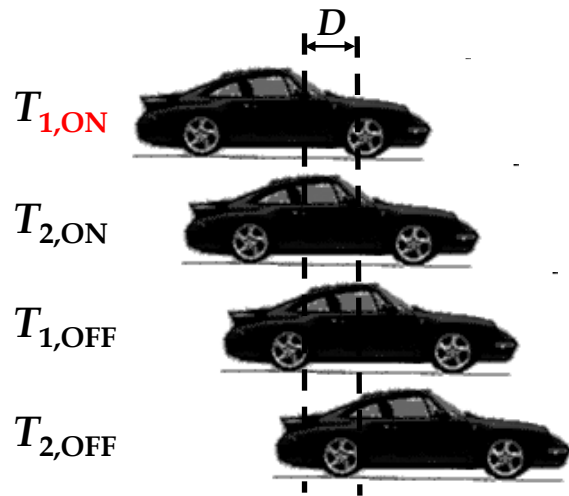
$\Delta \tau_{21} \approx 10 \mu\text{s}$ $\Delta v = -D / (\tau_{21})^2 \cdot \Delta \tau_{21} \approx 0.1 \text{ km/h}$

Detection with NIR laser (Class 1)

No interception/countermeasures

PLATE imaging by photo/video

Autovelox with optical barrier (e.g. 104 C2)

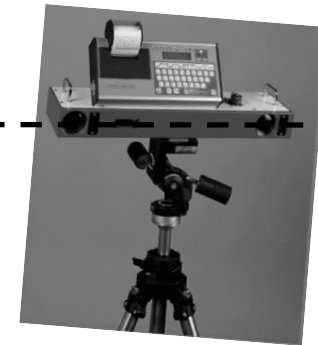


$$v_{21,ON} = D / \tau_{21,ON}$$

$$v_{21,OFF} = D / \tau_{21,OFF}$$

Measurement independent from vehicle profile and double-detection ($v_{21,IN} \cong v_{21,OFF}$ within 1 km/h) allows discarding fake readings and meas. errors

The target is hit by the laser and retro-diffused light is detected

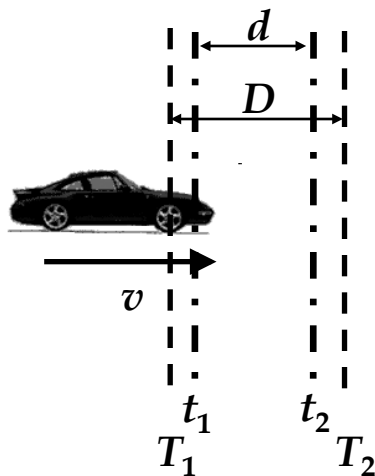


Autovelox with optical barrier (e.g. 104 C2)

Co-planarity ($\theta_{\text{Orizz.}} = \theta = 0$) of the optical barrier with the road is important and instrument **calibration** is mandatory by law every year

horizontal vs. inclined

$$d = D \cdot \cos\theta < D$$



The horizontal instrument measures

$$v = v_{21} = D / (T_2 - T_1) = D / \tau_{21}$$

but if inclined by θ the measure is

$$\tau_{21}^* = (t_2 - t_1) = d / v < \tau_{21} \text{ and hence}$$

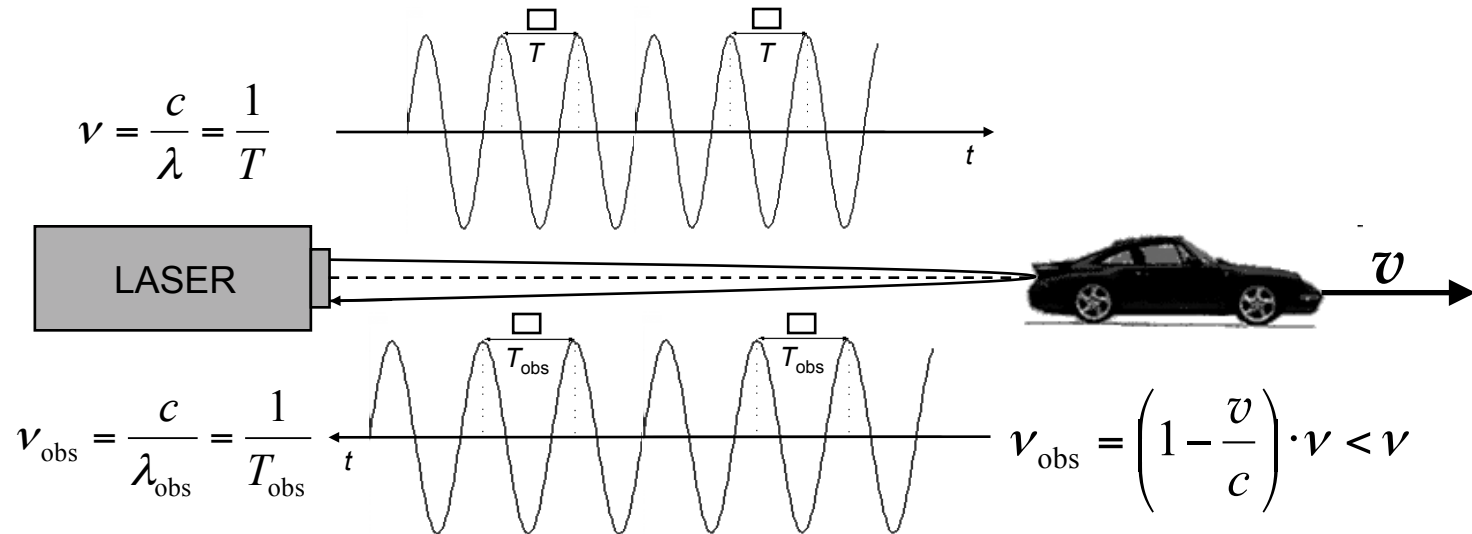
$$v^* = D / \tau_{21}^* = v \cdot D / d = v / \cos\theta > v$$

the velocity can be “overestimated”

A misalignment barrier-to-road of 10° gives a measurement error of 1.5% (in excess!), corresponding to +2.3 km/h at 150 km/h

If well aligned and calibrated, the instrument can reach accuracy levels well below 1% that – “poor us!” – are more than adequate for the measurement, given with a “tolerance” of 5%

Tele-laser (Doppler velocimeter) “???”



By beat-note analysis we “could” detect a the **Doppler shift**

$$\Delta \nu_{\text{Doppler}} = (\nu - \nu_{\text{obs}}) = (v/c) \cdot \nu = (1/\lambda) \cdot v$$

[for $\lambda=1 \mu\text{m}$ ($\nu=300 \text{ THz}$), $v = 108 \text{ km/h} = 3 \text{ m/s} \rightarrow \Delta \nu = 3 \text{ MHz}$]

[for $\lambda=1 \mu\text{m}$ ($\nu=300 \text{ THz}$), $v = 36 \text{ km/h} = 1 \text{ m/s} \rightarrow \Delta \nu = 1 \text{ MHz}$]

[for $\lambda=1 \mu\text{m}$ ($\nu=300 \text{ THz}$), $v = 180 \text{ km/h} = 5 \text{ m/s} \rightarrow \Delta \nu = 5 \text{ MHz}$]

Given the high value of $\Delta \nu_{\text{Doppler}}$, $T_{\text{Doppler}} < 1 \mu\text{s}$, repeated v_j measurements “could” be averaged in a short time, reducing measurement uncertainty

— Tele-laser “pistol” (TOF telemeter)

Technical specs.:

$$T_{\text{meas}}=0.4 \text{ s}$$

$$L\text{-Range}=610 \text{ m}$$

$$v\text{-Range}=\pm 320 \text{ km/h}$$

$$\lambda=904 \text{ nm}$$

$$\theta_{\text{div}}=3 \text{ mrad}$$

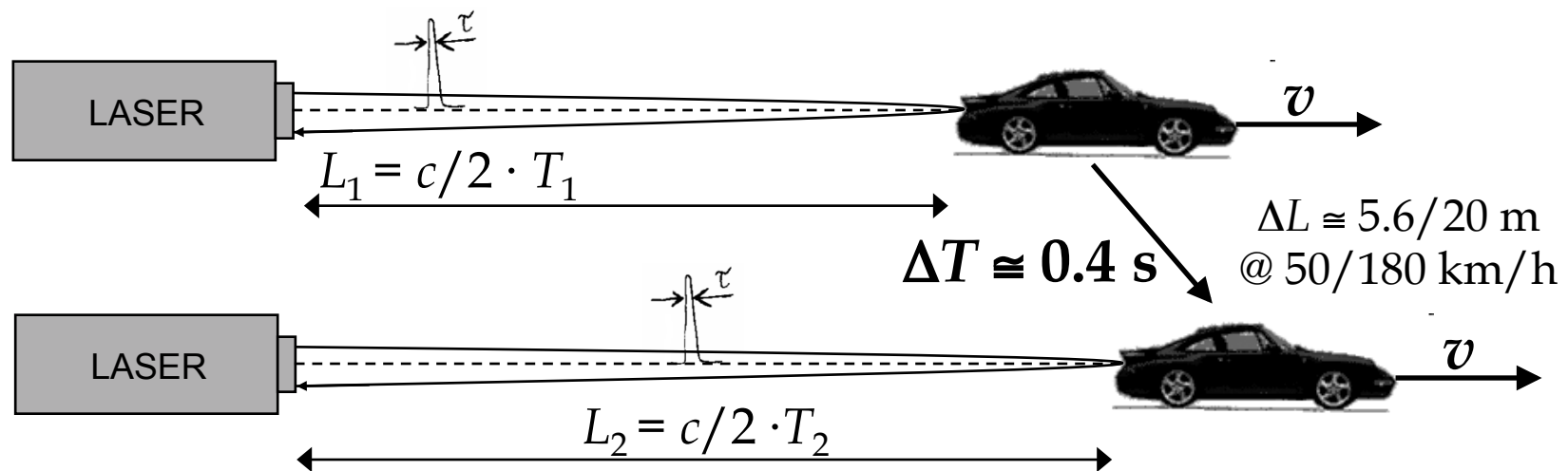
laser class 1

$$u(L)=15 \text{ cm}$$

$$u(v)=2 \text{ km/h}$$



Tele-laser (TOF telemeter)



$$v = (L_2 - L_1) / T_{\text{rep}} = c/2 \cdot (T_2 - T_1) / T_{\text{rep}}$$

$$T_{\text{rep}} = T_{\text{NA}} = 2L_{\text{NA}} / c \approx 4 \mu\text{s} \quad (f_{\text{rep}} = 250 \text{ kHz}) \quad \text{with } L_{\text{max}} \approx L_{\text{NA}} = 600 \text{ m}$$

Given the high value of f_{rep} , repeated TOF measurements can be averaged in a short time ($T_{\text{ave}} = 10 \text{ ms}$ and $T_{\text{meas}} = 0.4 \text{ s}$: average of 40 readings of TOF for L_i meas. and 39 readings of v_j for $v_{\text{ave@400ms}}$) reducing measurement uncertainty