"Optical Measurements"

Master Degree in Engineering
Automation-, Electronics-, Physics-, Telecommunication- Engineering

Optical Telemeters

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Some pictures are taken from the Book "Electro-Optical Instrumentation: Sensing and Measuring with Lasers" by Prof. Silvano Donati

Summary

- **Measurement principles and applications**
- **Triangulation**
  - passive
  - active
- **Time Of Flight (TOF)**
  - pulsed
  - Continuous Wave (CW) [with sine modulation]
  - *power budget* of a laser telemeter and system equations
  - *timing* and optimal filtering, noise and accuracy, ambiguity
  - optics (launch/receiving), instrumental development
- **LIDAR** (*LIght Detection And Ranging*)

Measurement principles in Telemetry (1/2)

- ***tele-metry*** = distance-measurement (or also measurement at distance)
  we measure the distance $\underline{L}$ between the <u>instrument</u> and a remote <u>target</u>
  (*range-finding* and *range-finder*)

1) by <u>triangulation</u> (trigonometric method)

the target is "triangulated" from **two points apart *D* on the same baseline** (see measurement of the distance of the stars); measuring the angle between the two lines of sight one gets the distance as $\boldsymbol{L \cong D/\underline{\alpha}}$ $(L = D/\mathrm{tg}\,\alpha)$

2) by <u>Time Of Flight</u> (measurement by counting a time interval)

pulsed laser or CW sine modulated (f_m) laser

$\underline{\boldsymbol{T}} = 2L/c \Rightarrow L = c/2 \cdot T \propto T$ ("2L" for a round-trip path)

$\underline{\boldsymbol{\Delta\varphi}} = 2\pi f_m \cdot T \Rightarrow \Delta\varphi/2\pi = f_m/(c/2L) \Rightarrow$

$L = c/2 \cdot \Delta\varphi/2\pi f_m = \lambda_m/2 \cdot \Delta\varphi/2\pi \propto \Delta\varphi$ ***" counting in terms of $\lambda_m/2$ "***

(measurement depends on res./acc. on T and $\Delta\varphi$, for CW also on f_m e λ_m)

Measuement principles in Telemetry (2/2)

- ***tele-metry*** = distance-measurement (or also measurement at distance)
  we measure the distance $\underline{L}$ between the instrument and a remote target

3) by interferometry (by means of counting optical wavelengths)

a laser beam (monochromatic) is sent to the target and the returning light is **coherently detected**, by beat note analysis on a photodetector; the detected signal goes as $\mathbf{\underline{\cos(2\mathit{kL})}}$**, with** $\boldsymbol{k=2\pi/\lambda}$, and from the phase of the cosine function we can **"count" the distance increment in terms of $\boldsymbol{\lambda/2}$ and its fractions**, from 0 to L or for small variations ΔL starting from a fixed L^*

$$\underline{\boldsymbol{\Delta\varphi}} = 2kL \Rightarrow \Delta\varphi = 2\cdot(2\pi/\lambda)\cdot L \Rightarrow \boldsymbol{L = \lambda/2\cdot\Delta\varphi/2\pi \propto \Delta\varphi}$$

" counting L in terms of λ/2 "

$$\underline{\boldsymbol{\cos[\,2\pi\cdot L/(\lambda/2)\,]}}$$

(resolution depends on laser's λ)

with λ typically $\approx 0.5\ \mu m$ (VIS)

we count L in terms of $\Delta L = \lambda/2 = 250$ nm (resolving "just" $\Delta\varphi = 2\pi$)

but with $\Delta\varphi = \pi$ o $\pi/2$ we obtain $\Delta L = 125$ nm or ~60 nm and less…

Measurement fields for Optical Telemeters

Distance and displacement measurements

BY TRIANGULATION
(meas. optical P and beam position)
short-range
[0.1-10 m]

BY INTENSITY MODULATION
(optical mod. incoherent meas.)

INTERFEROMETRY
(meas. optical φ coherent detection)
very high-resolution
[ΔL=100-10 nm]

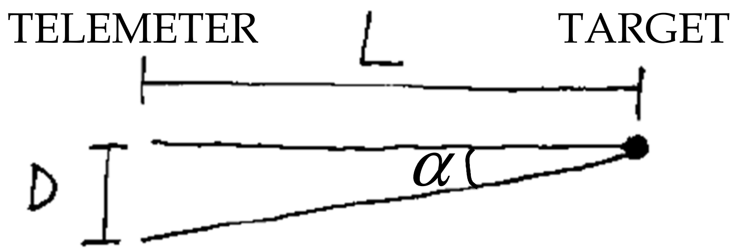
T E L E M E T E R S

PULSED
(meas. time of flight)
geodesy, radar, long-range
[0.1-10 km]

WITH SINE MODULATION
(electrical phase shift meas. on mod. light)
topography, short range
[1-1000 m]

Measurement method by triangulation

- Triangolazione
  - PASSIVI
  - ATTIVI

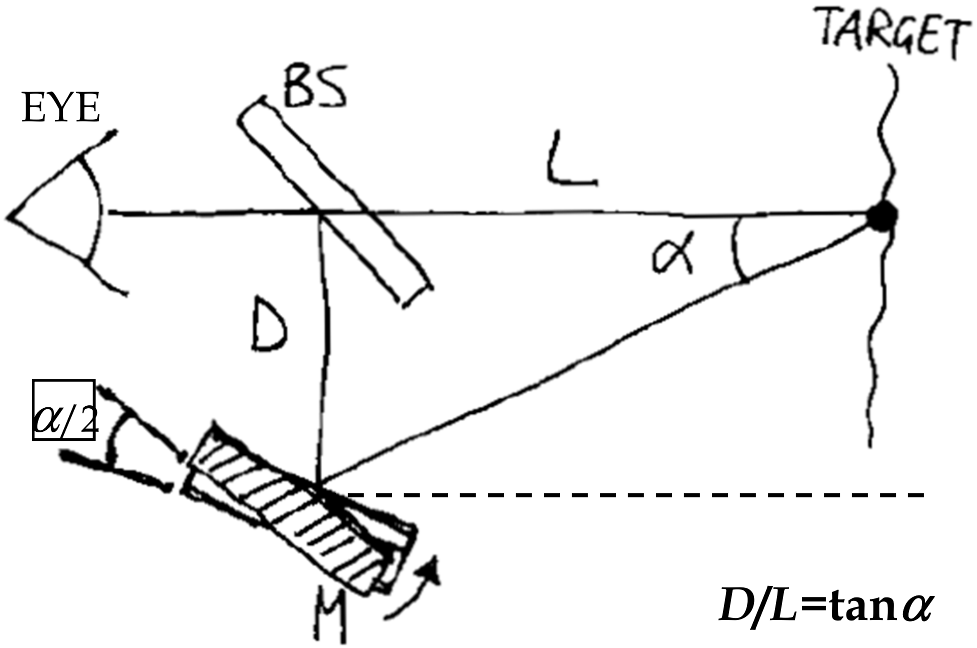


$$\frac{D}{L} = tg\,\alpha \approx \alpha \quad \text{se } \alpha \ll 1$$

$$L = \frac{D}{tg\,\alpha} \cong \frac{D}{\alpha}$$

- Measurement becomes **less accurate over long distance** (in practice for *L*>>*D*). In fact, if the detection angle gets small (α<10 mrad ≈ 0.5°) rel. uncertainty $\Delta\alpha / \alpha$ increases

Passive optical triangulator



$$L = \frac{D}{\tan\alpha} \cong \frac{D}{\alpha}$$

from angular meas. we get the distance L

L non-linearly depending from viewing angle $\boldsymbol{\alpha}$

D/L*=tan*α

sensitivity of L with α

$$\Delta L = -\frac{D}{\alpha^2}\Delta\alpha = -\frac{L^2}{D}\Delta\alpha = \kappa \cdot \Delta\alpha$$

error or unc. or res. (absolute) $\propto L^2$ or $\propto 1/\alpha^2$

note that ΔL increases as L^2

res. gets worse with increasing distance

$$\frac{\Delta L}{L} = -\frac{\Delta\alpha}{\alpha} \propto L$$

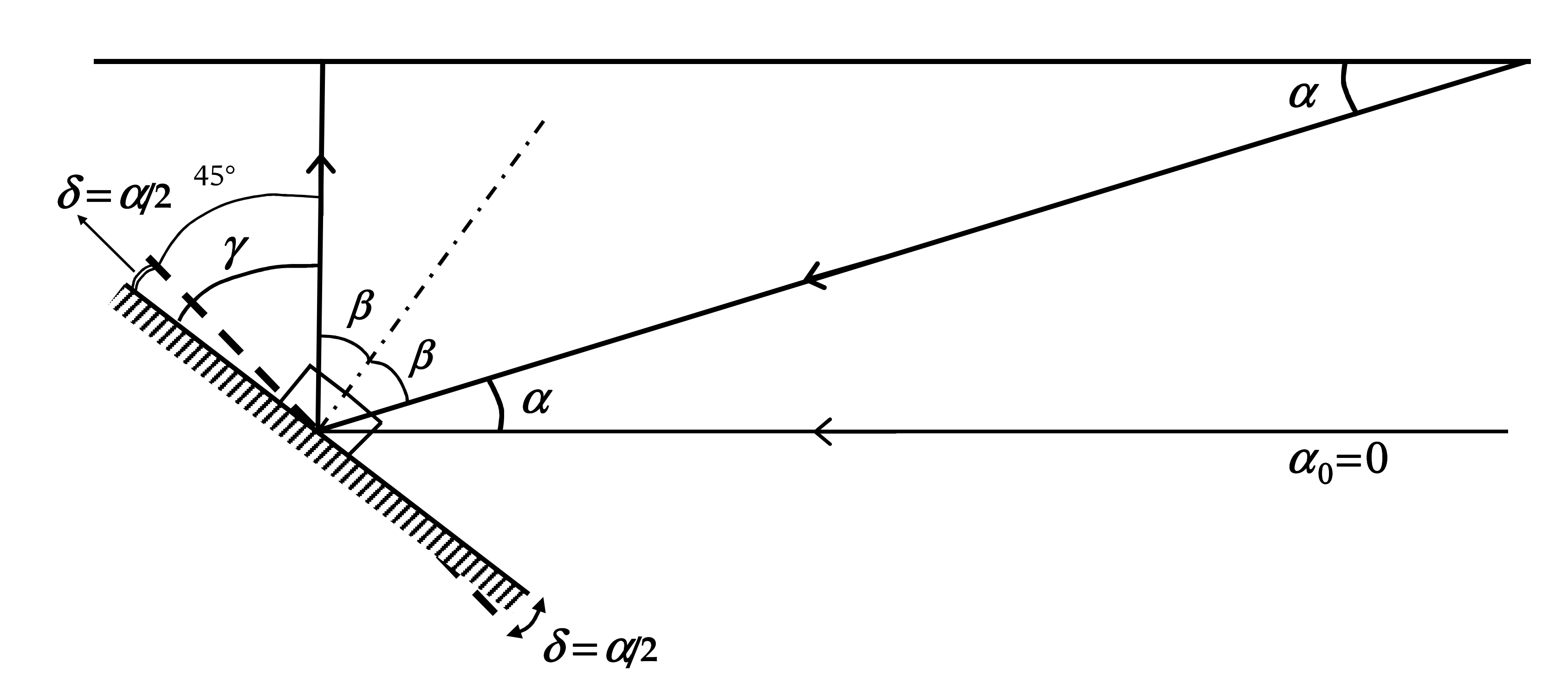
error or unc. or res. (relative)

UNCertainty

$$u(L) = \sigma(L) = \frac{\Delta L}{\sqrt{12}}$$

quantized meas. (limited res.)

“Optical lever” on the mirror



1. $\gamma + \beta = 90°$
2. $\alpha + 2\beta = 90°$
3. $\gamma - \delta = 45°$

from 1.–3. we get $\beta + \delta = 45°$
from 2./2 we get $\alpha/2 + \beta = 45°$
subtracting the two members,
do obtain $\delta = \alpha/2$

Resolution and accuracy in a passive optical triangulator (examples)

- Accuracy/resolution of distance (L) measurement depend on accuracy/resolution of the angle (α) measurement
- For example, with a micrometrical screw goniometer we can resolve $\Delta\alpha \approx 3$ mrad (0.17 °) while with an angular encoder we can achieve $\Delta\alpha \approx 0.1$ mrad (0.0057 °)

Ex.: for L=1 m we choose D=10 cm $\Rightarrow$ $\alpha \cong D/L$=0.1 rad

$$\frac{\Delta L}{L} = \begin{array}{lll} 3\% & \text{screw} & (3\text{ cm}) \\ 0.1\% & \text{encoder} & (1\text{ mm}) \end{array}$$

if for L=100 m we choose D=1 m $\Rightarrow$ $\alpha = D/L$=0.01 rad

$$\frac{\Delta L}{L} = \begin{array}{lll} 30\% & \text{screw} & (30\text{ m}) \\ 1\% & \text{encoder} & (1\text{ m}) \end{array}$$

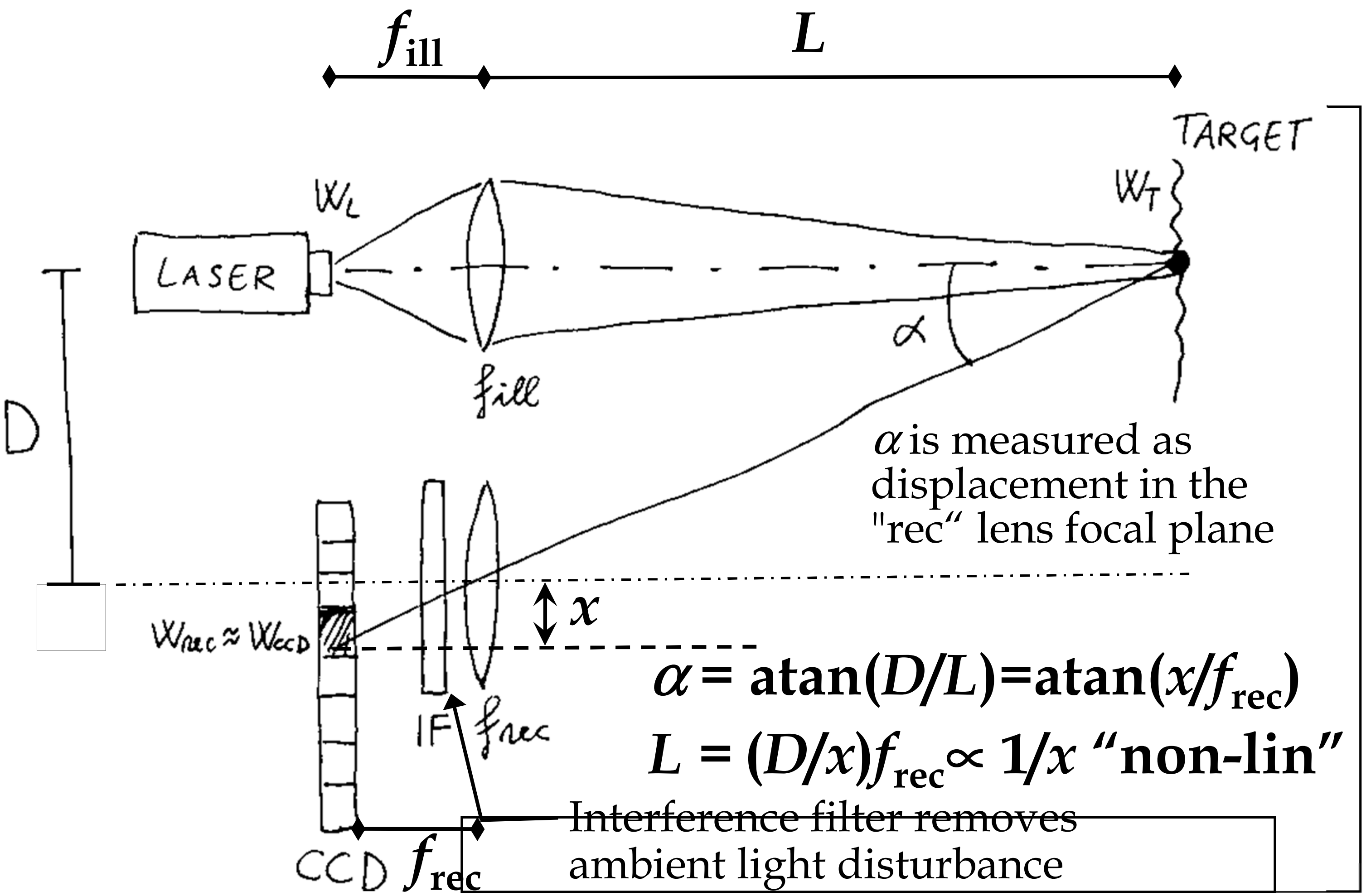
it would be insane to keep D=10 cm because it would be α= 1 mrad

- Performance is good until D/L is not too small and hence for medium-short ranges (L=0.1-10 m)

"Active" (laser) optical triangulator

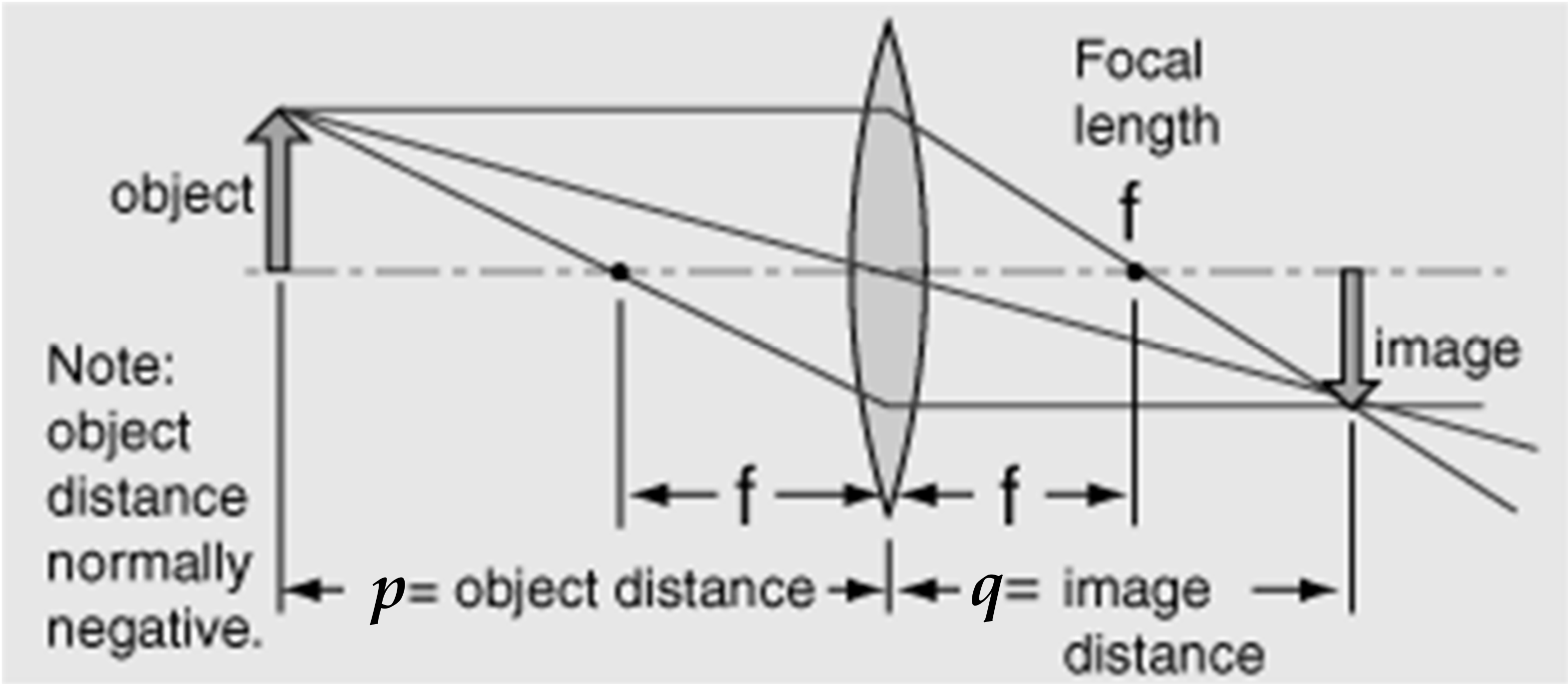
- We **remove moving parts** (no goniom. With respect to passive triangulator) and we achieve a very quick and accurate response, very repeatable
- We use **visible** λ for simplicity of "target viewing" (He-Ne at 633 nm or LD-VIS or Nd:YAG2×)
- The **laser beam** undergoes a **round trip** path from telemeter to target. **Misura 1D measurement** with optical **position sensor** (2Q/PSD/CCD) of the **angle** α between going and returning beam. Receiving optics is off-axis at a distance D from launching optics: we then retrieve $\boldsymbol{L=D/\tan\alpha}$

Laser triangulator or active triangulator



$$\frac{w_L}{f_{ill}} = \frac{w_T}{L} \qquad \frac{w_T}{L} = \frac{w_{rec}}{f_{rec}} \qquad \Rightarrow \quad w_{rec} = \frac{f_{rec}}{f_{ill}} w_L$$

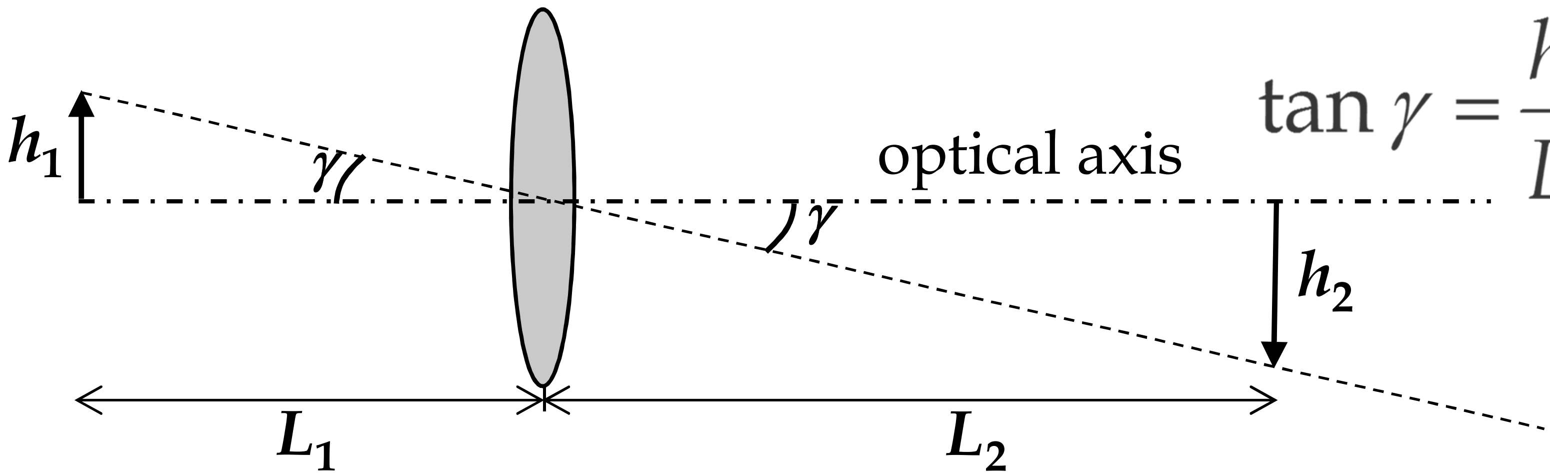
Image from a thin lens under "geometrical optics" [basics]



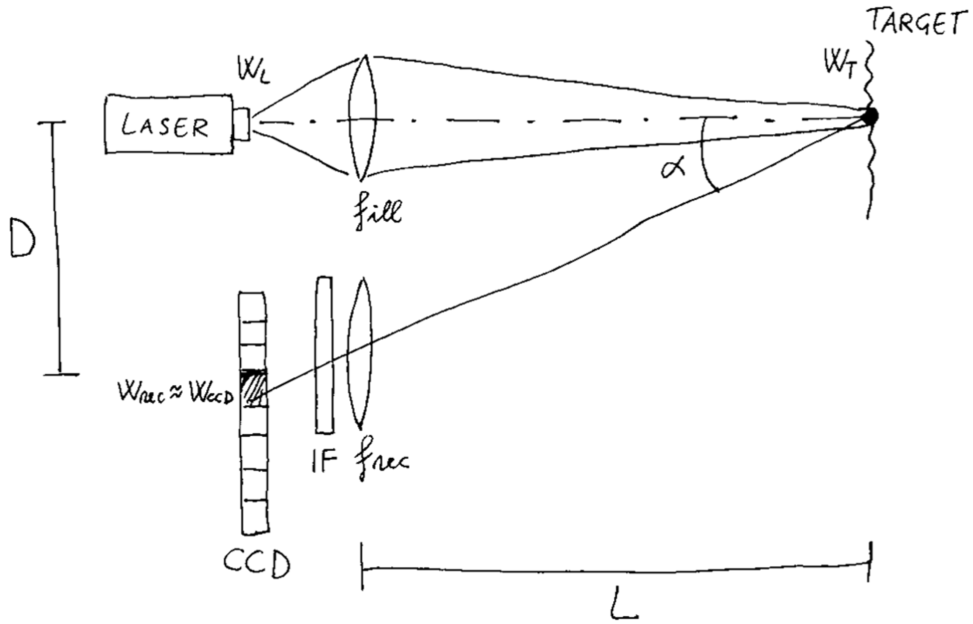
$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$$

For an object at distance $p=f$ (focal length) from the lens, image is formed to the "infinity" i.e. at a distance $q=\infty$

Dimensioning of the laser spot size in the active triangulator [calculus steps]



$$\tan\gamma = \frac{h_1}{L_1} = \frac{h_2}{L_2}$$



toward target

$h_1 = w_L$ e $L_1 = f_{ill}$

$h_2 = w_T$ e $L_2 = L$ $\Rightarrow w_T = \dfrac{L}{f_{ill}} w_L$

toward receiver

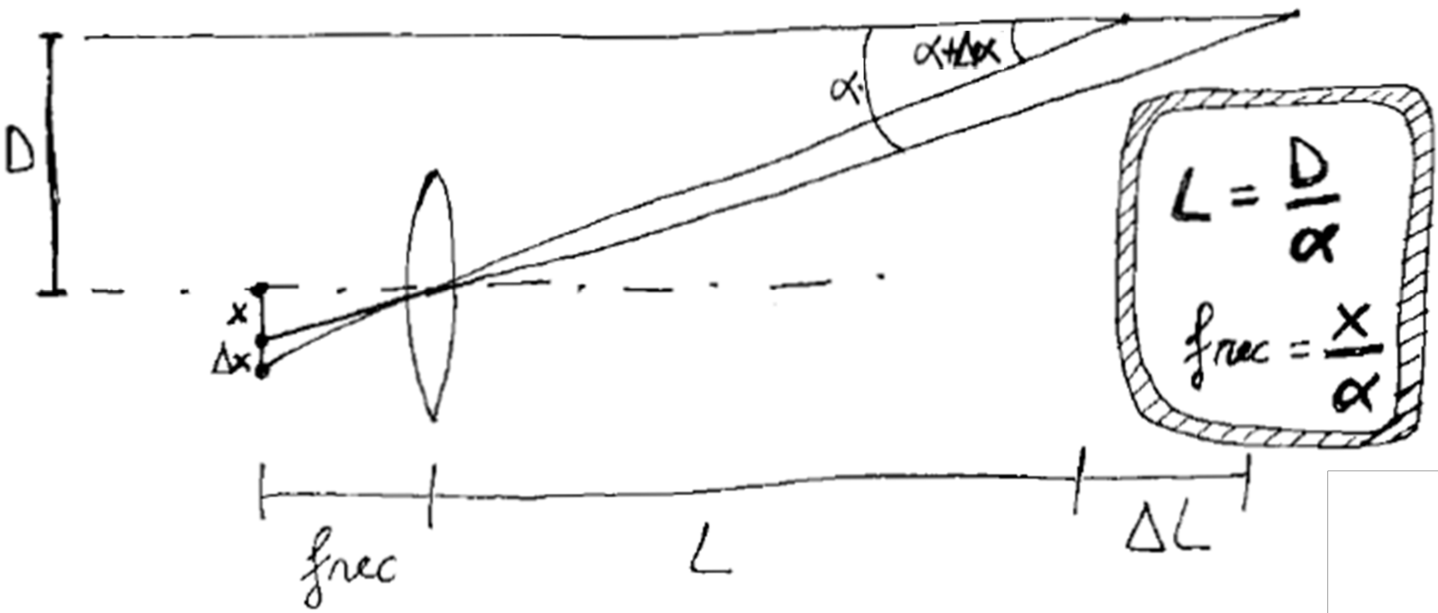
$h_1 = w_T$ $L_1 = L$

$h_2 = w_{rec}$ $L_2 = f_{rec}$ $\Rightarrow w_{rec} = \dfrac{f_{rec}}{L} w_T$

$$\boldsymbol{w_{rec} = (f_{rec}/f_{ill})\, w_L}$$

Measurement equations for the active optical triangulator (1/2)

Let x be the **distance** of the received spot on the CCD **from the receiving lens optical axis** (which is off-set at a fixed distance, D, from the launching optical axis)



⇐ distance measured as an agle

⇐ conversion from angle to displacement

$$x \propto \alpha \propto \frac{1}{L}$$

For a variation $L \pm \Delta L$ we get a corresponding var. $\alpha \mp \Delta\alpha$ and $x \mp \Delta x$ (the **lens converts** α **into** x **and** $\Delta\alpha$ **into** Δx)

Measurement equations for the active optical triangulator (2/2)

L measured from the position x on the CCD is

$$L = \frac{D}{x} f_{\text{rec}} \longrightarrow \frac{1}{\alpha}$$

hence, by differentiating in L and in x, we get

x is now electronically measured by a CCD
and we don't use a goniometer to measure α

$$\Delta L = -\frac{D}{x^2} f_{\text{rec}} \Delta x$$

As for the passive triangulator formula but with x e Δx instead of α and $\Delta\alpha$

$$\boxed{\Delta L = -\frac{L^2}{f_{\text{rec}} D} \cdot \Delta x \propto L^2} \qquad \boxed{\frac{\Delta L}{L} \propto L}$$

e infine

$$\frac{\Delta L}{L} = -\frac{\Delta x}{x} = -\frac{\Delta \alpha}{\alpha}$$

once again as for the passive optical triangulator

Exercise on the laser triangulator ("known" formulas: only calculations)

DATA: as in the case of passive optical triangulator, we work with D=10 cm (and f_{rec}=25 cm) for L=1 m and we now consider w_L=5 µm and w_{CCD}=10 µm:

$$x = -\frac{D}{L} f_{rec} = \frac{10\ \text{cm}}{1\ \text{m}} 250\ \text{mm} = 25\ \text{mm}$$

If we resolve Δx=10 µm ($\approx w_{CCD}$) on the photodetector,

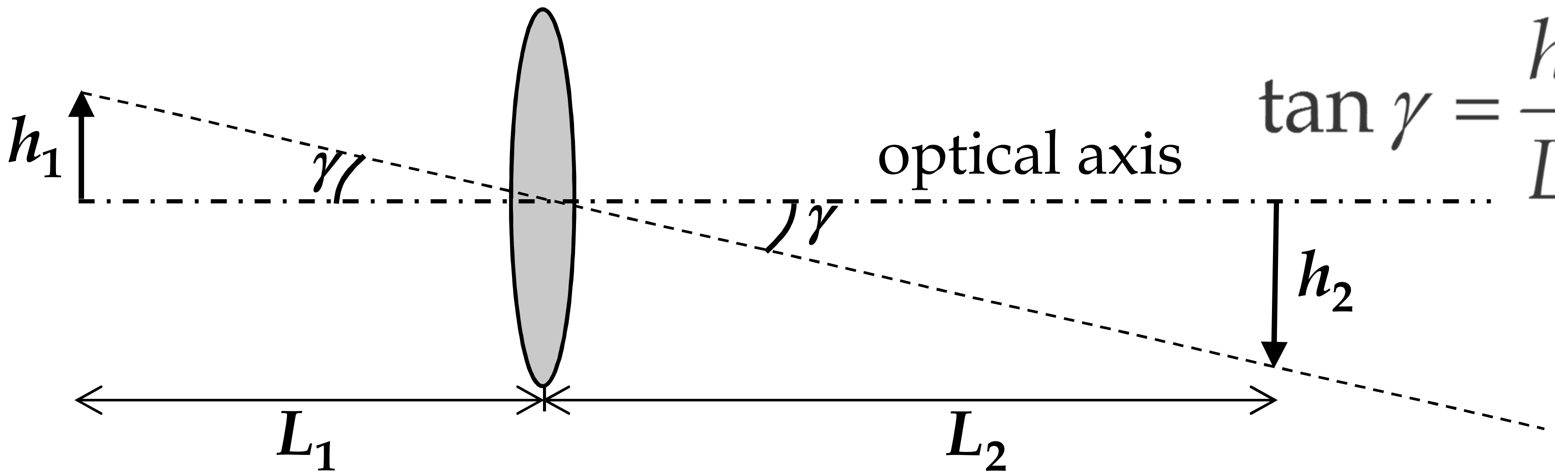
$$\frac{\Delta L}{L} = \left|\frac{\Delta x}{x}\right| = \frac{10 \cdot 10^{-3}}{25} = 4 \cdot 10^{-4}$$

(**ΔL=400 µm**)
(for L = 1 m)

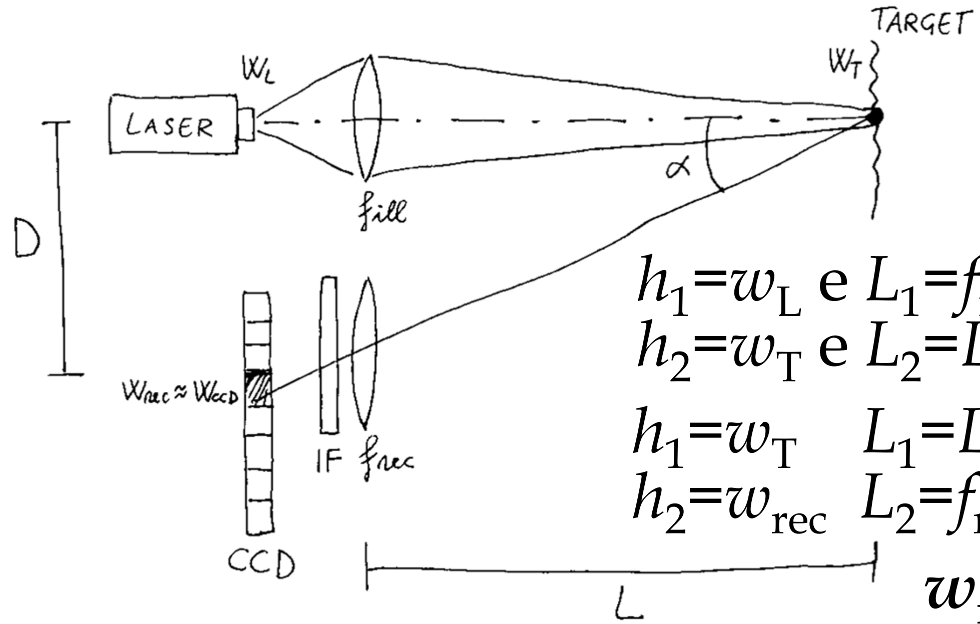
$$|\Delta\alpha| = 4 \cdot 10^{-4}\alpha \cong 4 \cdot 10^{-4}\frac{D}{L} = 40\ \mu\text{rad}$$

Remind how for the passive triangulator we had $\Delta\alpha \approx$ 3 mrad (micro-screw goniometer) and $\Delta\alpha \approx$ 0.1 mrad (angular encoder)

Exercise on the active triangulator
(with data, calculations and steps to get w_{rec})



$$\tan\gamma = \frac{h_1}{L_1} = \frac{h_2}{L_2}$$



DATA:

L=1 m w_L=5 μm

f_{ill}=125 mm f_{rec}=250 mm

h_1=w_L e L_1=f_{ill}
h_2=w_T e L_2=L

$$\Rightarrow w_T = \frac{L}{f_{\text{ill}}} w_L = \frac{1000}{125} \cdot 5\ \mu m = 40\ \mu m$$

h_1=w_T L_1=L
h_2=w_{rec} L_2=f_{rec}

$$\Rightarrow w_{\text{rec}} = \frac{f_{\text{rec}}}{L} w_T = \frac{250}{1000} \cdot 40\ \mu m = 10\ \mu m$$

w_{rec}= (f_{rec}/f_{ill})w_L=10 μm ≈ w_{CCD}

Interpolation in the laser triangulator

Spatial resolution Δx on the CCD limits angular resolution $\Delta\alpha$ and hence resolution ΔL in the measurement of distance L

Exploiting the spatial extension of the laser spot on the CCD, we can **interpolate over more bright pixels** and resolve even a sub-pixel laser spot position (e.g. Δx=0.2w_{CCD} or even less) with the resulting improvement in angular and distance resolution

EXERCISE (for home…):

Using a laser triangulator with Gaussian laser spot large w_{rec}=50 μm on a 1024 pixel CCD (with 12 bit amplitude resolution and w_{CCD}=10 μm),
we want to retrieve the position of the "spot center" obtained by interpolating over bright pixels

Exercise on the laser triangulator

QUESTIONS:

- if the telemeter laser is an He-Ne laser at 633 nm, what **kind** of **CCD** we should use? why?
- how many and which **pixels** on the CCD are "well **lit**" when the "background light" covers 1/100 (in amplitude) of the measurement dynamic range?
- how shall me measure the **position** of the **spot center** on the CCD? How much wide is the "visible" spot?
- which are the practical limits to the **accuracy**?
- Imagine we can achieve a resolution of 0.1 pixel: calculate the absolute **resolution** of the telemeter at the minimum measurable distance L_{min}=10 m

Exercise on the laser triangulator

ANSWERS:

- for a red He-Ne laser we can conveniently use a **Si CCD**, sensible in the visible range and inexpensive

- To calculate how many and which **pixels** on the CCD are "**well lit**", we must before define what we mean for "well lit": since the single pixel can resolve $N=2^n=2^{12}=4096$ levels of photocurrent and hence incident optical power, we can say that a pixel is well lit ($SNR=1$) if the current signal is equal to the minimum detectable current (due to quantization or electronic noise of the receiver + "background light")

[in general one pixel is lit if its signal level brings its output voltage to a greater than zero (just quantization limit) or here the pixel is well lit when its "signal" level is > 1/100 of the peak/dynamic-range of the whole optical signal (limits due to "noise" and background light)]

Exercise on the laser triangulator

The optical power on the single pixel is the optical intensity times the pixel area (precisely one should integrate the intensity over the pixel surface)

Optical intensity is decreasing as $\exp(-2r^2/w^2_{rec})$ getting farer from the peak (in $r=0$). So we obtain an **1/*M* part of the peak value** when $\mathbf{2r^2=w^2_{rec}\ln(M)}$ or equivalently $r/w_{rec}=(0.5\ln M)^{0.5}=[0.5\cdot 2.3\cdot \log_{10}M]^{0.5}$

We hence can calculate

$\boldsymbol{r\approx k\cdot w_{rec}}$ with $k\,\mathbf{=[0.5\cdot 2.3\cdot \log_{10}M]^{0.5}}$

clearly w_{rec} corresponds to a certain number of pixels and so r can be given by “counting pixels"

Exercise on the laser triangulator

with noise and background (at $1/M$=1/100 from peak):
we get 1/100 of the peak value for $2r^2=w^2_{rec}\ln(100)$
$r/w_{rec}=(0.5\cdot\ln 100)^{0.5}=[1.15\cdot\log_{10}100]^{0.5}=[2.3]^{0.5}\approx 1.5$
and $r\cong 75\mu m$ with a well-lit spot of about **150 μm** diameter or 15 pixel

for $1/M$=1/10 of peak, $r \cong 1.1\cdot w_{rec}$
[as we know exp(-2)=13 %≈1/10]

just quantization:
we get 1/4096 ≈ 1/4000 of the peak value for
$2r^2=w^2_{rec}\ln(4000)$ (≈ 4000 amplitude levels for n=12 bit)
$r/w_{rec}=(0.5\cdot\ln 4000)^{0.5}=[1.15\cdot\log_{10}4000]^{0.5}=[1.15\times 3.6]^{0.5}\approx 2$
we hence obtain $r\approx 2w_{rec}=2\cdot 50\mu m\cong 100\mu m$ corresponding to 10 pixels each of size10 μm (starting from the Gaussan center). The total number of well-lit pixels is finally 20 pixel (±10), for a "visible" spot size (diameter) of approximately **200 μm**

Exercise on the laser triangulator

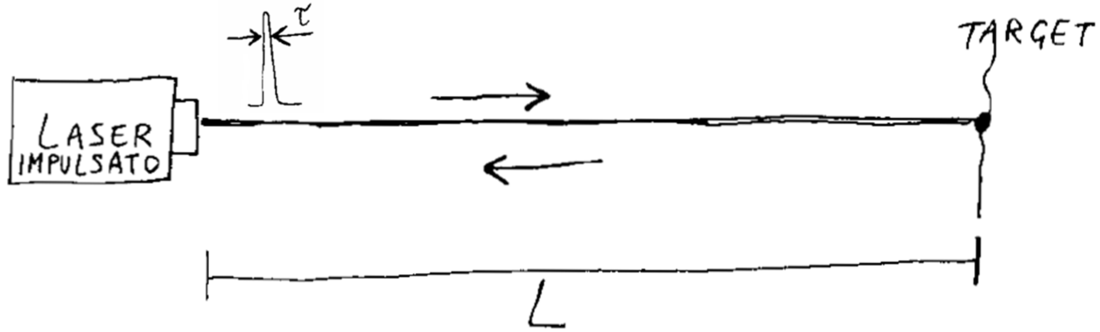
using as pixel weight the corresponding voltage

- The spot **position** on the CCD can be obtained by a weighted average of the points of the Gaussian profile, maybe after subtracting background level, and preferably by the average of only well-lit points; or by least squares fitting with function (Gaussian + offset): we can imagine achieving Δx=0.1pixel=1μm

- Limits to the **accuracy** are set by noise at the photodetector (ext. light, shot noise, CCD dark current, additional noise quantization/electronic) leading to a wrong estimate of the center position of ideal Gaussian

- The CCD is wide $x_{max}=1024\cdot 10\mu m \approx 1cm$ and, considering that $\Delta x/x_{max}=\Delta\alpha/\alpha_{max}=-\Delta L/L_{min}$, the **resolution** of the measure is $\Delta L=L_{min}(\Delta x/x_{max})=$ $=10m\cdot(1\mu m/1cm)=1mm$ [at 10 m distance it is 10^{-4}]

Time Of Flight ("TOF") telemeters (principles and working equations)



Laser radiation undergoes a round-trip path 2L (forth and back) in a time T, traveling at light speed $c \approx 3 \cdot 10^8$ m/s

$$T = \frac{2L}{c} \Rightarrow L = \frac{c}{2} T$$

ΔL is constant depending only on the time resolution ΔT

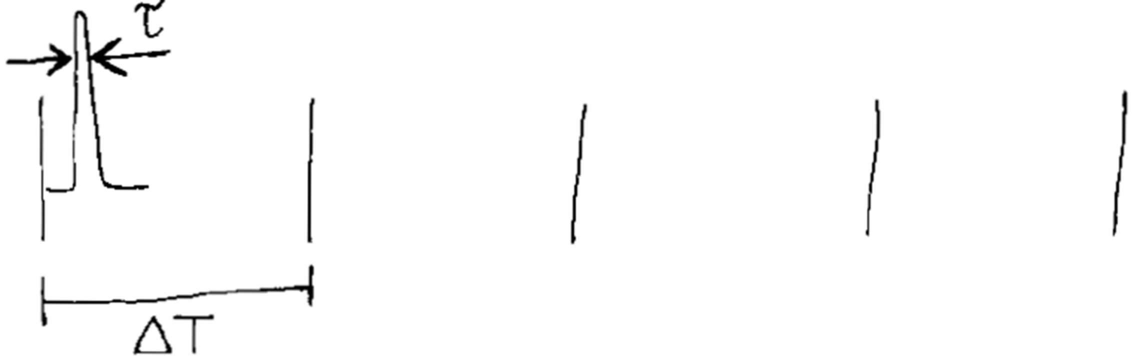
$$\Delta L = \frac{c}{2} \Delta T \qquad \frac{\Delta L}{L} = \frac{\Delta T}{T}$$

Example: to get **ΔL =1 m** we need **ΔT**=2·1m/3·10^8 m/s **≅7 ns** $\approx \tau$ (limit for a Q-switched laser; for shorter τ values we need a mode-locked laser; power...)

for **ΔL =1 mm → ΔT≅7 ps**… difficult resolving sub-mm with conventional TOF

Time Of Flight telemeters (length and time resolution)

- in impulsata $\tau \lesssim \Delta T$



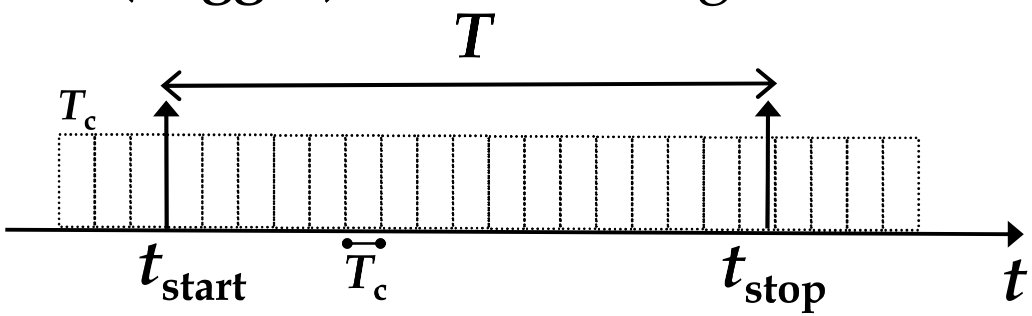
essendo $\Delta T = \dfrac{2\Delta L}{c}$

Length resolution ΔL in th measurement depends on the **time resolution** ΔT and so on pulse duration

To resolve a time ("distance") interval ΔT we must work with **pulse duration** $<\approx \Delta T$ and so with “fast” photodetector electronics with **bandwidth** $B \approx (1/\tau)$

Time Of Flight telemeters (time interval measurement)

The measurement of a time interval T is achieved with an **electronic counter**, "counting" the **distance between** $\boldsymbol{t_{start}}$ (transmitted pulse) and $\boldsymbol{t_{stop}}$ (pulse received), on the corresponding photodetected electronic signals.
Position of pulses on the time axis is determined by a **threshold discriminator (*trigger*)** on the voltage waveform/pulses



$$T = t_{stop} - t_{start} \cong N\,T_C$$

Not always an analogue t falls exactly on a clock transition.
So the measurement of t has the discrete resolution T_c of the electronic counter, with uncertainty $u_q(t)=\sigma(t)=T_c/\sqrt{12}$

Time Of Flight telemeters (measurement uncertainty of T=t_{stop}-t_{start})

Having uncertainty $u(t)=u_q(t)$ both on t_{start} and t_{stop} then the combined accuracy on the measured time of fligh T is

$$u(T) = [u^2(t_{stop})+u^2(t_{start})]^{1/2} = \sqrt{2}\, u_q(t) = T_C/\sqrt{6}$$

If we start the clock, with period T_C, exactly at t_{start} we then have $u(t_{start})=0$ [clock pulses starting with $t=t_{start}$]
and for the whole TOF we obtain $u(T) = u(t_{stop}) = T_C/\sqrt{12}$

In general, by choosing the clock period T_c short enough, the measurement uncertainty will depend by other factors (much more significant than the "small quantization"):
in particular from the **amplitude noise** at the trigger.

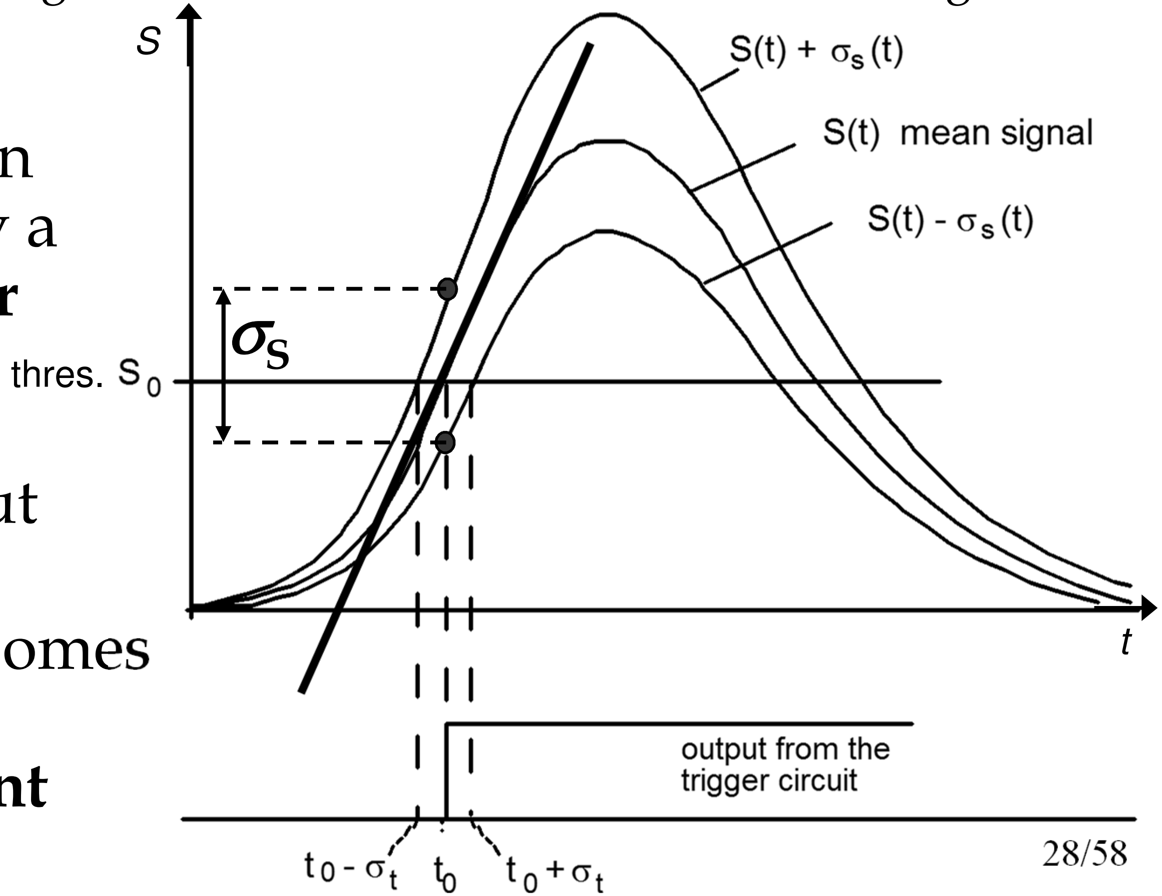
This noise depends on the circuit electronic noise and on the **amplitude noise of the detected signal**

Time Of Flight telemeters (threshold discrimination and noise: σ^2)

The time of flight is measured as $T = t_{stop} - t_{start}$ and its variance is $\boldsymbol{\sigma^2(T)=\sigma^2(t_{start})+\sigma^2(t_{stop})\cong\sigma^2(t_{stop})}$ **t_{stop} is more noisy**

Detection of light pulse providing t_{stop} is "more noisy" (with lower SNR) since the light signal returning from the target is much weaker than the transmitted signal $\boldsymbol{\sigma^2(t_{stop})>>\sigma^2(t_{start})}$

The location of pulses on time axis is obtained by a **threshold discriminator (trigger)** acting on the voltage signal $S(t)$ at the photodetector output



Amplitude noise σ_S becomes **time noise σ_t** with the **slope in the trigger point**

CW telemeters (principle)

Optical power sine modulated at frequency f_{mod}

$$P(t) = P_0\,[1+m\cdot\sin(2\pi f_{mod}t)]$$

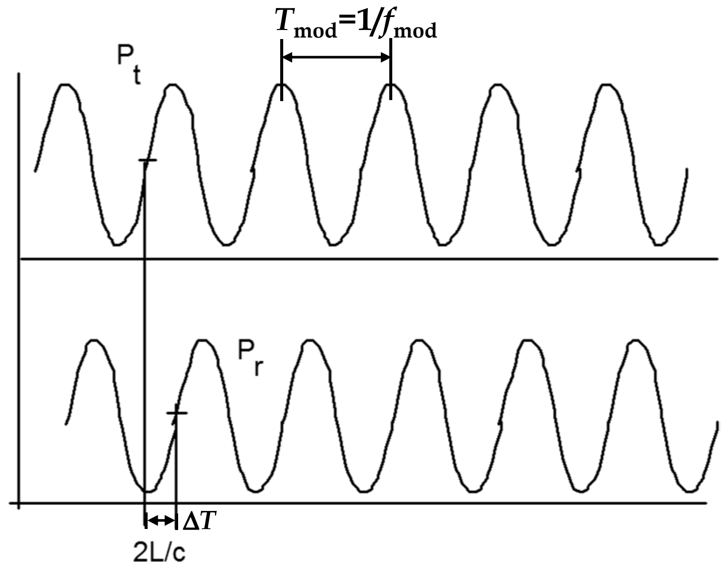
P_0 is the average power

• in CW con modulazione sinusoidale

Si misura lo sfasamento tra il segnale ricevuto e quello trasmesso

$$\frac{\Delta\varphi}{2\pi} = \frac{\Delta T}{T_{mod}}$$

con $\Delta T = \frac{2L}{c}$ tempo di volo



We detect the phase delay $\Delta\varphi$ between detected signal (P_r) and transmitted signal (P_t)

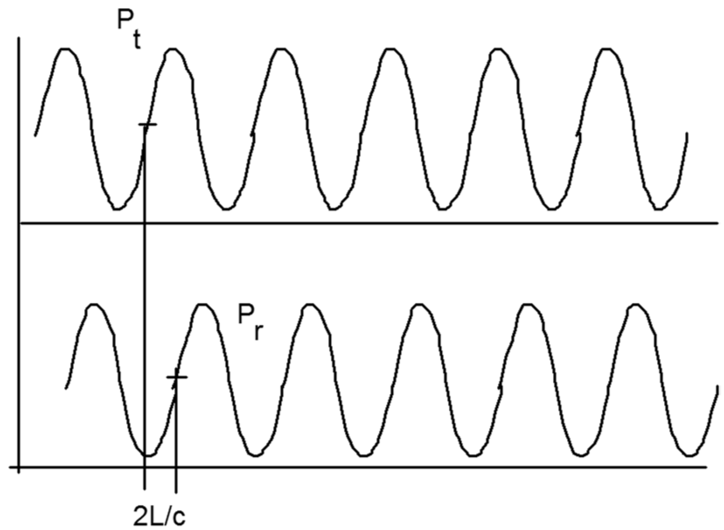
$$L = c/2 \cdot \Delta T = c/2 \cdot T_{mod} \cdot \Delta\varphi / 2\pi = c/2 \cdot \Delta\varphi / 2\pi f_{mod}$$

CW telemeters (working equations)

La misura di distanza si ottiene come

$$L = \frac{c}{2}\frac{1}{2\pi f_{mod}}\Delta\varphi = S^{-1}\Delta\varphi$$

la sensibilità S della misura cresce all'aumentare della f_{mod}



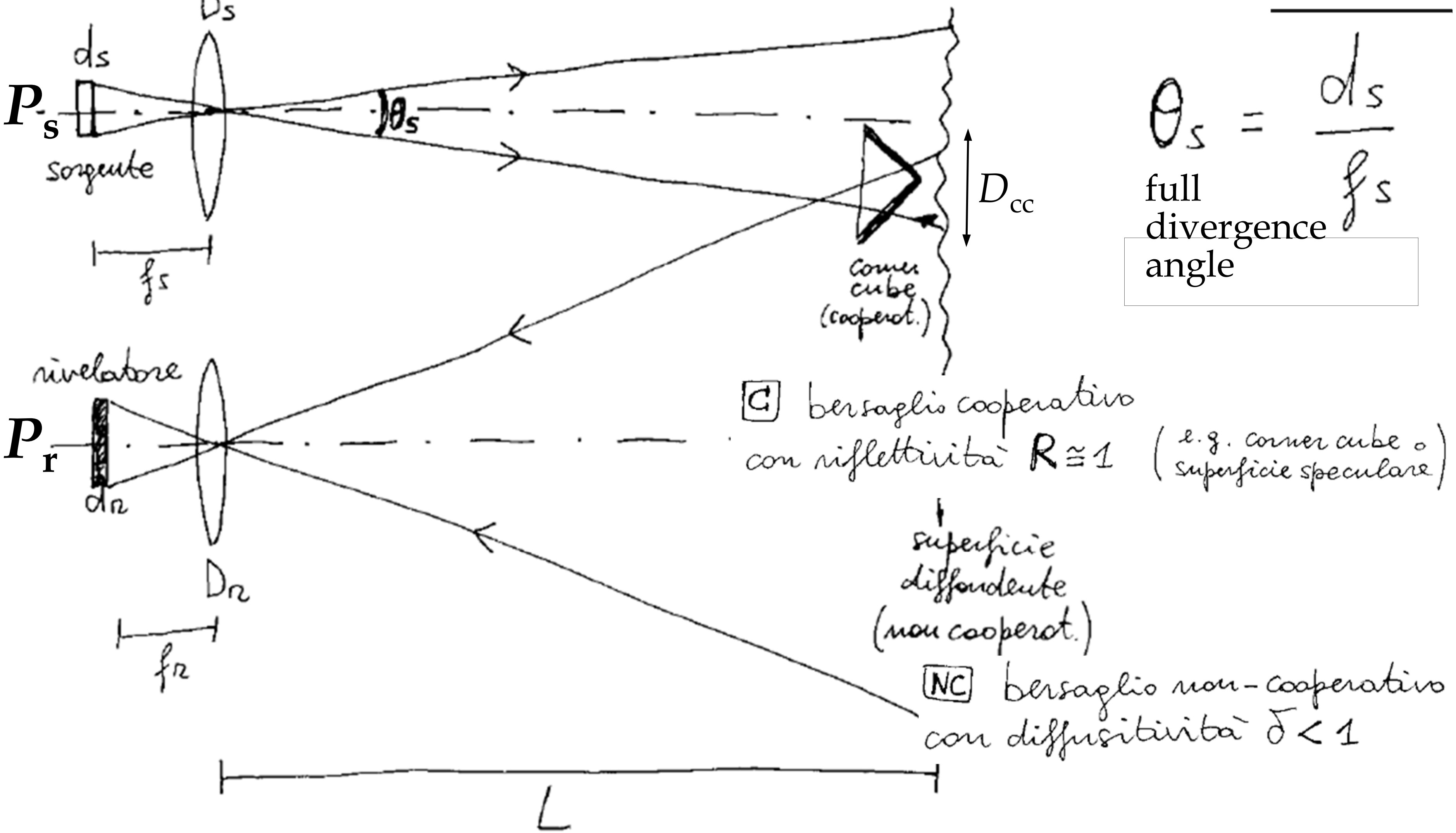
sensitivity tells how $\Delta\varphi$ (direct measure) varies with a variation of the distance L

$$S = \frac{\delta(\text{output})}{\delta(\text{input})} = \frac{\delta(\Delta\varphi)}{\delta(L)} = \frac{2\pi f_{mod}}{c/2} \propto f_{mod}$$

with f_{mod}=10 MHz
and $\delta(\Delta\varphi)$=2 mrad (1.2°)
we get $\delta(\Delta L)\cong$5 mm
Increasing f_{mod} we can measure smaller ΔL

Measurement sensitivity (on the phase $\Delta\varphi$) gets higher for increasing modulation frequency, BUT if f_{mod} is too high we can run into other problems (measurement ambiguity)

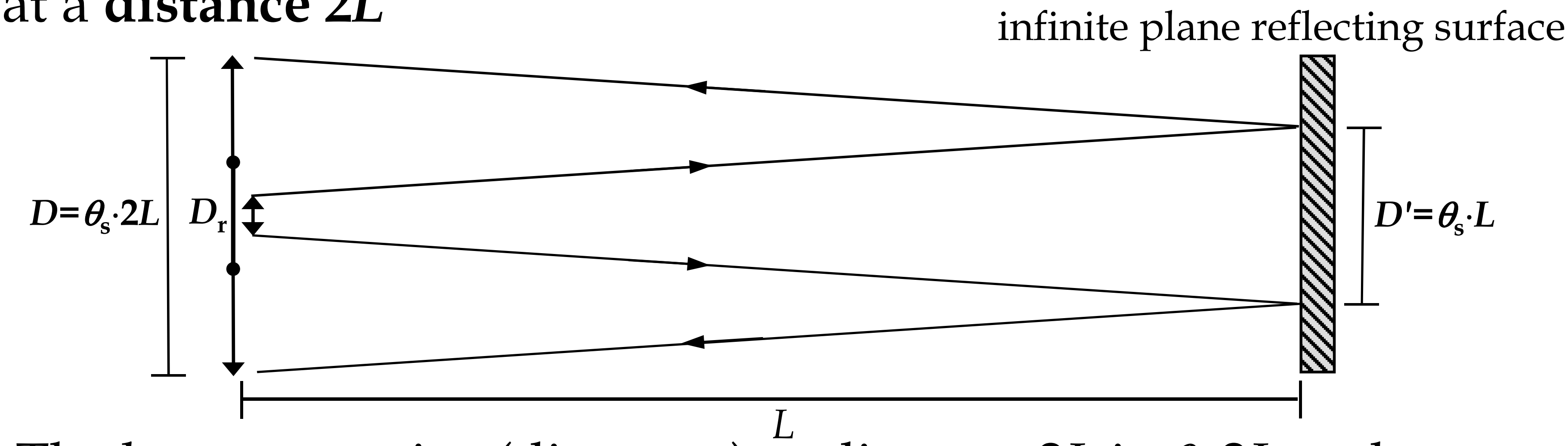
Power budget in optical telemeters



In practical applications $L >> f_s, f_r, D_s, D_r$

Cooperative target (reflecting) (1/2)

When the target is cooperative, e.g. *corner cube* reflector, it behaves **like a mirror** and hence the receiver sees the target at a **distance 2L**



The beam spot size (diameter) at distance 2L is $\theta_s \cdot 2L$ and hence the fractional **r**eceived power (respect to transmitted power from the **s**ource), on a circular area with diameter D_r (receiving lens) set at distance 2L, is equal to

$$\frac{P_r}{P_s} = \frac{\cancel{(\pi/4)}\, D_r^2}{\cancel{(\pi/4)}\, \theta_s^2\, 4L^2}$$

ratio of the receiver area to received beam spot size (area)

areas ratio at the receiver

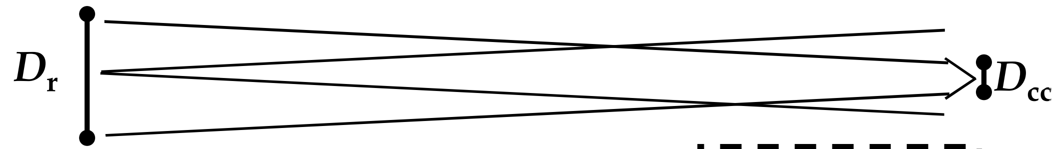
if all the receiver is lit ($D>D_r$)

Cooperative target (reflecting) (2/2)

If the corner cube has a diameter smaller then the laser spot size on it ($D_{cc}<\theta_s L$ i.e. "only corner cube cutting the beam") and the receiver collects all reflected beam, we have

$$\frac{P_r}{P_s}=\frac{D_{cc}^2}{\theta_s^2 L^2}$$

areas ratio at the CC



$D_r > D_{cc}+\theta_s L > D_{cc}+D_{cc} = 2D_{cc} \Rightarrow \boxed{D_{cc} < D_r/2}$

$$\frac{(2D_{cc})^2}{\theta_s^2 4L^2}$$

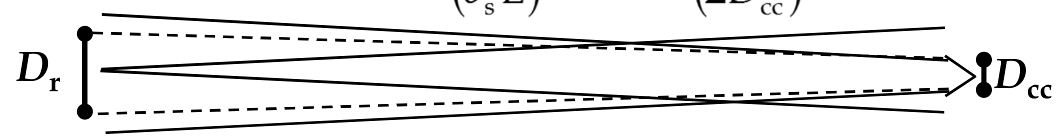
it is like receiver $D_r=2D_{cc}$

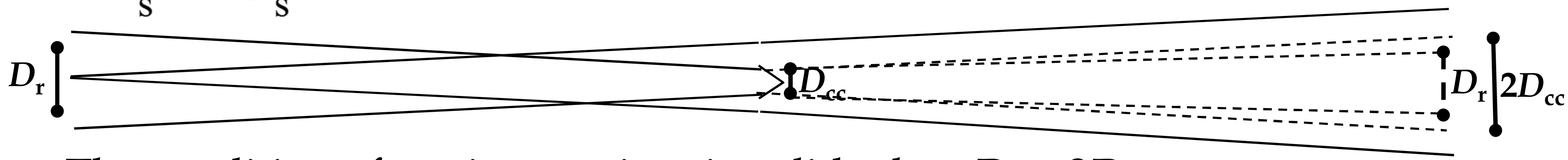
If in addition to the corner cube also the receiver is cutting the beam, we have again (like for the infinite plane reflector)

areas ratio at the receiver

$$\frac{P_r}{P_s}=\frac{D_r^2}{\theta_s^2 4L^2}$$

$$\alpha_1=\frac{D_{cc}^2}{(\theta_s L)^2} \quad \alpha_2=\frac{D_r^2}{(2D_{cc})^2} \quad \alpha=\alpha_1\cdot\alpha_2=\frac{D_r^2}{(\theta_s 2L)^2}$$



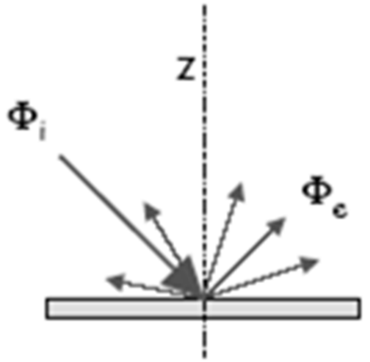


The condition of cutting receiver is valid when $D_r < 2D_{cc}$

ONLY when $D_{cc}< D_r/2$ we have only corner cube cutting the beam

Radiance of a Lambert diffusing surface

$$"I" = I_{\text{rad}} = \frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mathrm{d}\Phi}{\mathrm{d}\Omega} = \left[\frac{\mathrm{W}}{\mathrm{srad}}\right]$$ **Radiant Intensity**

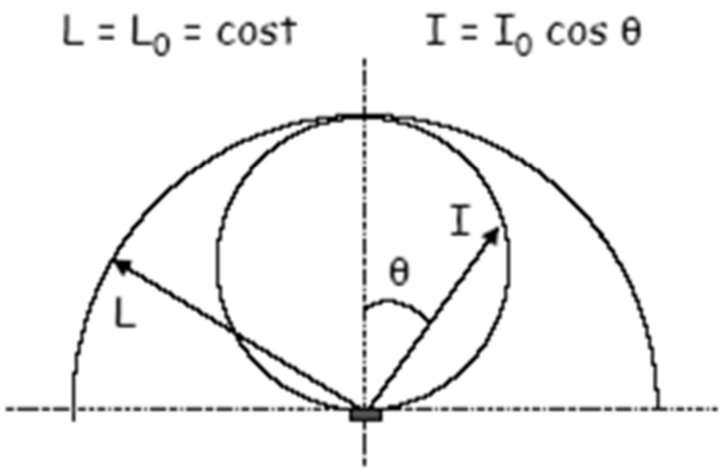


Radianza di un diffusore lambertiano

radiance or brightness

$$L = B = \frac{\mathrm{d}P}{A\,\mathrm{d}\Omega} = \frac{P}{A\pi} = \frac{I}{\pi}$$

Diffusore lambertiano



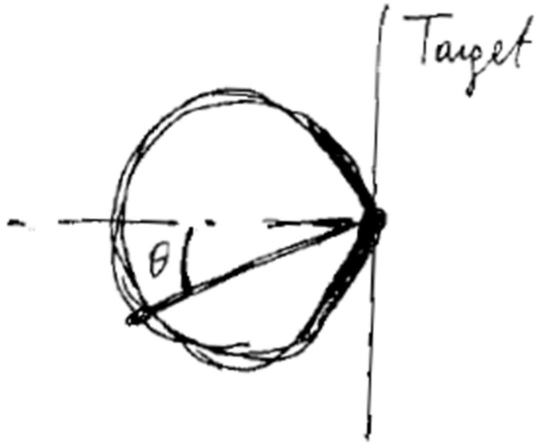
Flusso totale emesso: $\boxed{P} = \pi I_0$

Flusso totale emesso: $\boxed{P} = \pi A L_0$

Non-cooperative target (diffusing)

When the target is non-cooperative, the illuminated **surface**, with area A_T, is **diffusing** light with a diffusing coefficient $\boldsymbol{\delta < 1}$

la radianza del bersaglio è $1/\pi$ volte la intensità ottica (che senza attenuazione è $\delta P_s / A_T$)



Diffusore Lambertiano
"a $\cos\theta$"

$$B = \frac{1}{\pi} \delta \frac{P_s}{A_T} \qquad B = \frac{P}{A\Omega} = L$$

B is the target **brightness**

Non-cooperative target (diffusing)

Naming Ω_r the solid angle (θ_r the plane angle) by which light diffused from the target sees the receiver, we have

$$\Omega_r = \pi\theta_r^2 = \frac{\pi D_r^2}{4L^2} \quad \text{being } \theta_r = (D_r/2)/L$$

angle of sight of the receiver from the target

and hence the power collected at the receiver is

$$P_r = \Omega_r \cdot B \cdot A_T = \frac{\cancel{\pi} D_r^2}{4L^2} \cdot \frac{\delta P_s}{\cancel{\pi}\cancel{A_T}} \cdot \cancel{A_T} = \delta \frac{D_r^2}{4L^2} P_s$$

independent from the lit area (spot size) on the target

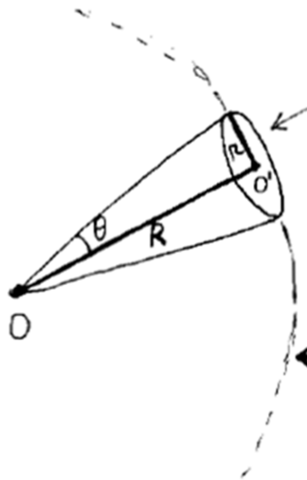
with a fractional received power

$$\frac{P_r}{P_s} = \delta \frac{D_r^2}{4L^2}$$

like for cooperative target
but with δ **instead of $1/\theta_s^2$**
and obviously $\delta \leq 1 << 1/\theta_s^2$

Solid angle $\Omega = \int \mathrm{d}\Omega = \int \frac{\mathrm{d}S}{R^2} = \int_0^r \frac{2\pi\rho \cdot \mathrm{d}\rho}{R^2} = \frac{2\pi}{R^2}\int_0^r \rho \cdot \mathrm{d}\rho = \frac{2\pi}{R^2}\frac{r^2}{2} = \pi\frac{r^2}{R^2} = \pi\theta^2$

ANGOLO SOLIDO $d\Omega = \frac{\vec{u}_n \cdot \vec{u}_R \, dS}{R^2}$



superficie "elementare" S, di raggio r, centrata attorno al punto O' della direzione di vista $\overline{OO'}$

superficie sferica, di raggio R, centrata nel punto di vista O

L'angolo piano è $\theta = \frac{r}{R}$ $(r \ll R)$

$S_{sfera} = 4\pi R^2$ e $\Omega_{giro} = 4\pi$

L'angolo solido è t.c.

$\Omega : \Omega_{giro} = S : S_{sfera}$

$\frac{S}{R^2}$

$\Omega = \frac{S}{S_{sfera}} \cdot 4\pi = \frac{\pi r^2}{4\pi R^2} 4\pi = \pi\frac{r^2}{R^2} = \pi\theta^2$

Def. # **Radian** $\boldsymbol{\theta}$ ________
is the **plane angle** subtended by a circular arc, as the length of the arc (r) divided by the radius of the arc (R):
$\boldsymbol{\theta = r/R}$ (1 rad is the plane angle where the arc is equal to the radius).

Def. # **Steradian** $\boldsymbol{\Omega}$
is the **solid angle** subtended at the center of a sphere, as the area (S) of the cap divided by the squared radius of the sphere (R^2): $\boldsymbol{\Omega = S/R^2}$

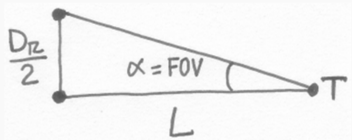
Power budget with diffraction and additional losses (optics and atmosphere)

Considering **power losses** due to Tx and Rx **optics** (***T*opt≤1**) and round-trip ($2L$) **propagation in the atmosphere** (***T*atm≤1**)

$$\left[\frac{P_r}{P_s}\right]_C = T_{opt} T_{atm} \frac{D_r^2}{\theta_s^2 4L^2}$$

$$\left[\frac{P_r}{P_s}\right]_{NC} = T_{opt} T_{atm} \delta \frac{D_r^2}{4L^2}$$

$$\frac{P_r}{P_s} = G \cdot FOV_{eq}^2$$



with $FOV_{eq} = \dfrac{D_r / 2}{L_{eq}}$

In the end we can write a **general expression**

$$\frac{P_r}{P_s} = G \frac{D_r^2}{4L_{eq}^2}$$

$G =$ T_{opt} / θ_s^2 cooperative, $T_{opt}\delta$ non-cooperative

equivalent GAIN

$$L_{eq} = L / \sqrt{T_{atm}}$$

equivalent length

Gain of the optical telemeter

The optical telemeter gain can be ***G*>>1**
in the case of **cooperative target** (since θ_s<<1)

$G = T_{opt} / \theta_s^2 \approx 10^6$ if θ_s=1 mrad and $T_{opt}\cong 1$

in this case the expression is analogous to the "gain of the antenna" in a radio transmission
(where it is extremely important to have low divergence)

Instead, the optical telemeter gain is always ***G*<1**
in the case of **non-cooperative target** ($\delta_{typ.}$=0.5-0.1)

Good **optics** (**antireflection** coated at λ laser) allow reflection losses <1%, at each air-glass interface, and absorption+scattering losses in the material (glass or quartz) $<10^{-3} \Rightarrow$ whole transmission $\boldsymbol{T_{opt}}$**>0.98-0.9≅1**

Attenuation coefficient

During its propagation (in air), the laser beam undergoes **absorption and diffusion losses** due to molecules or particulate always present **in the atmosphere**

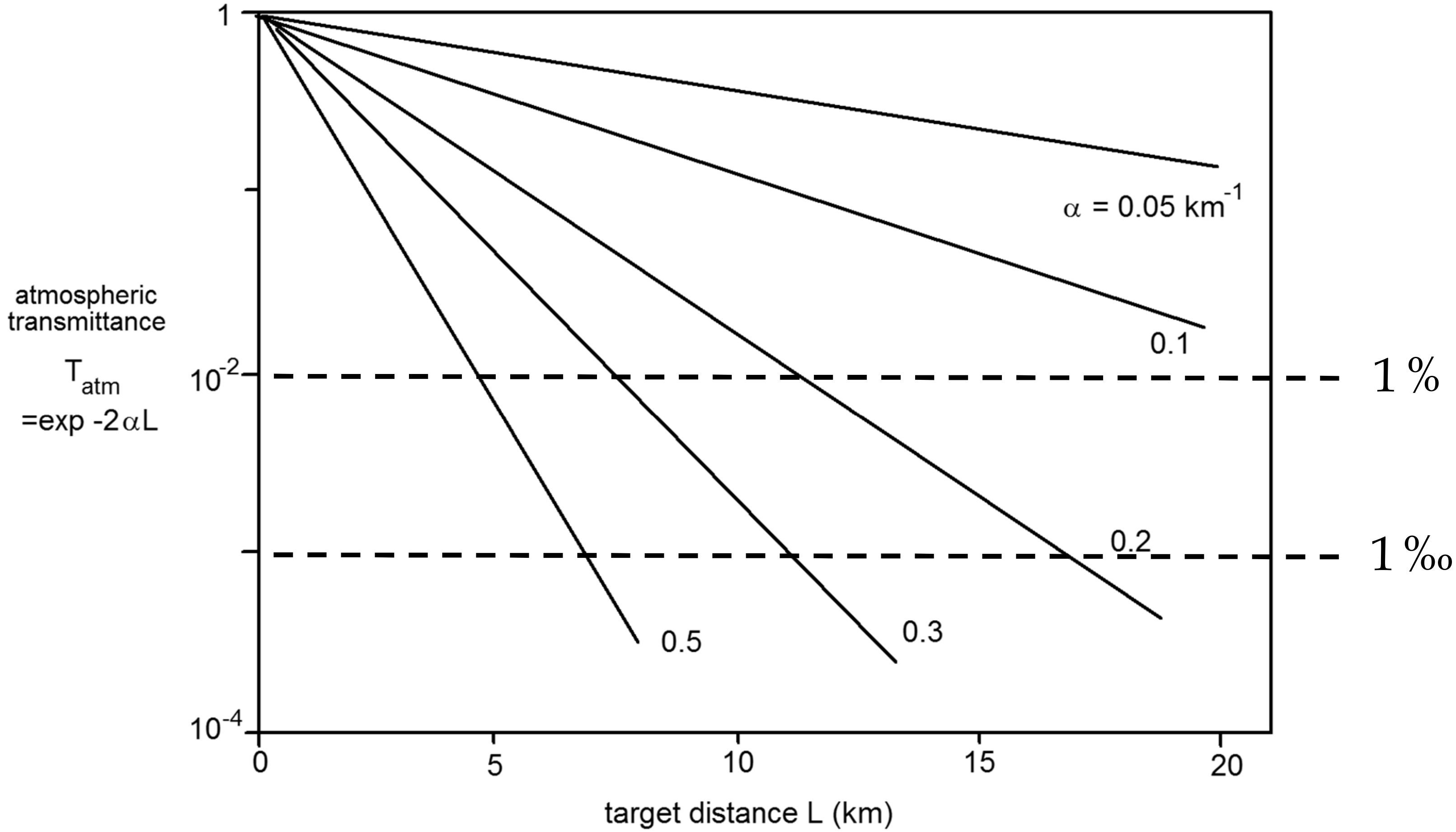
$$\mathbf{T_{atm} = \exp(-\alpha 2L)} = P(z=2L)/P(z=0)$$ from the law of Lambert-Beer

$$\alpha = \underbrace{a(\lambda)}_{\textbf{absorption}} + \underbrace{s(\lambda)}_{\textbf{scattering}\text{ (diffused light)}} = \alpha(\lambda)$$ attenuation coefficient

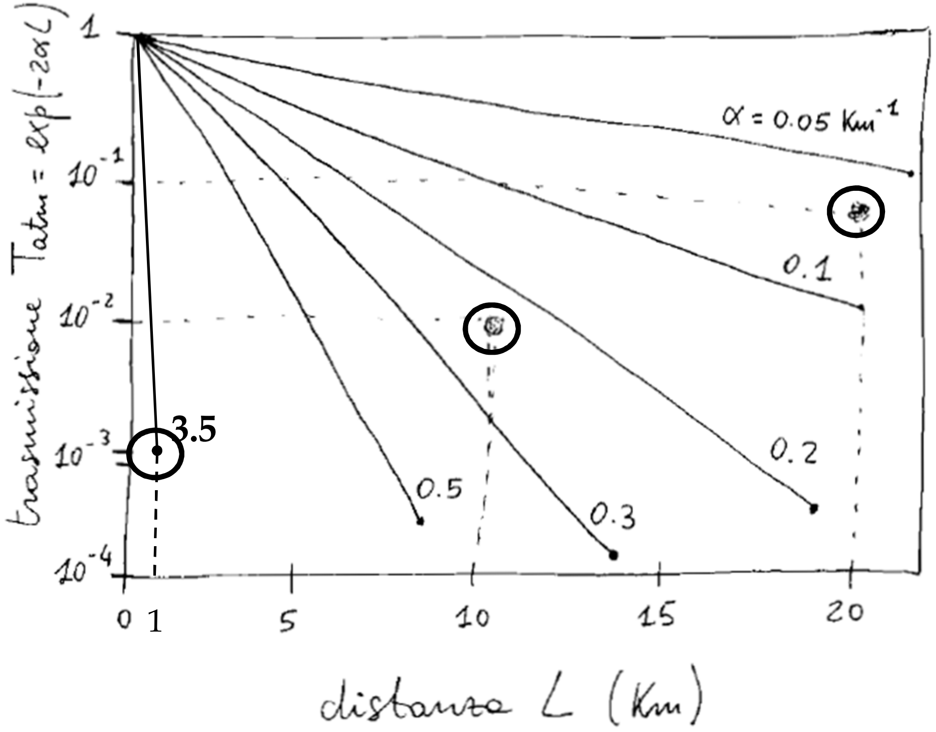
Carefully avoiding molecular absorption peaks, we can approximately adopt α=0.1km^{-1} for a very clear atmosphere, α=0.3km^{-1} for clear atmosphere, α=0.5km^{-1} for little haze, and α>>0.5km^{-1} if foggy. [in the case of no absorption $\alpha(\lambda) \cong s(\lambda)$]

Atmospheric attenuation

Clearly, at any fixed α, atmospheric transmission is exponentially decreasing with L (reducing received power P_r)



Atmospheric attenuation (examples)



In order to have the highest T_{atm}, we must avoid some peaks of atmospheric absorption (with $\alpha(\lambda)$ highest and T_{atm} lowest): *e.g.* 0.70, 076, 0.80, 0.855, 0.93, 1.13 µm: → lasers used are He-Ne (0.633 µm) or Nd:YAG (1.064 µm) or LD-GaAlAs (0.82-0.88 µm)

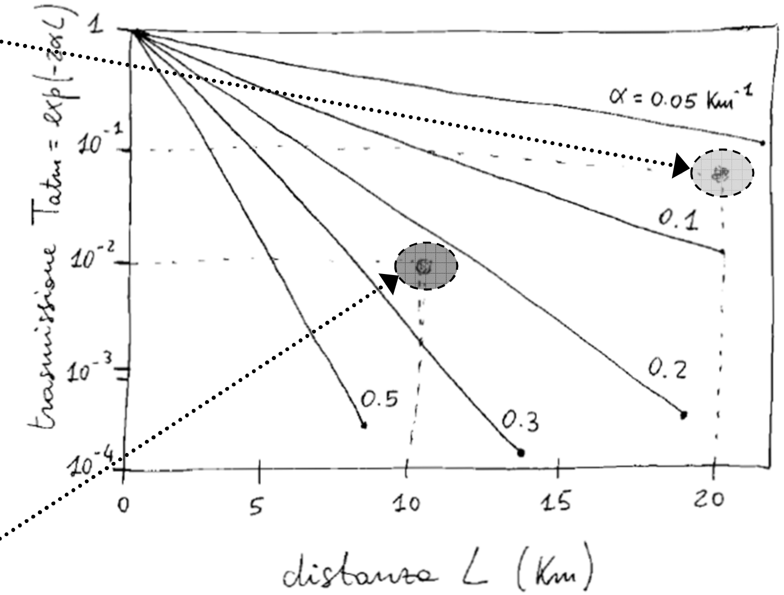
With **very clear atmosphere**, we can reach up to 20 km (and come back!) with an optical transmission **~10%**

With some **haze**, at a 10 km distance, transmission is **~1%**

With **fog**, just at 1 km distance, transmission is below **1‰**
(with α=3.5 km^{-1} we get T_{atm}=exp(-7)≅10^{-3} at 1 km)

Atmospheric attenuation (calculations)

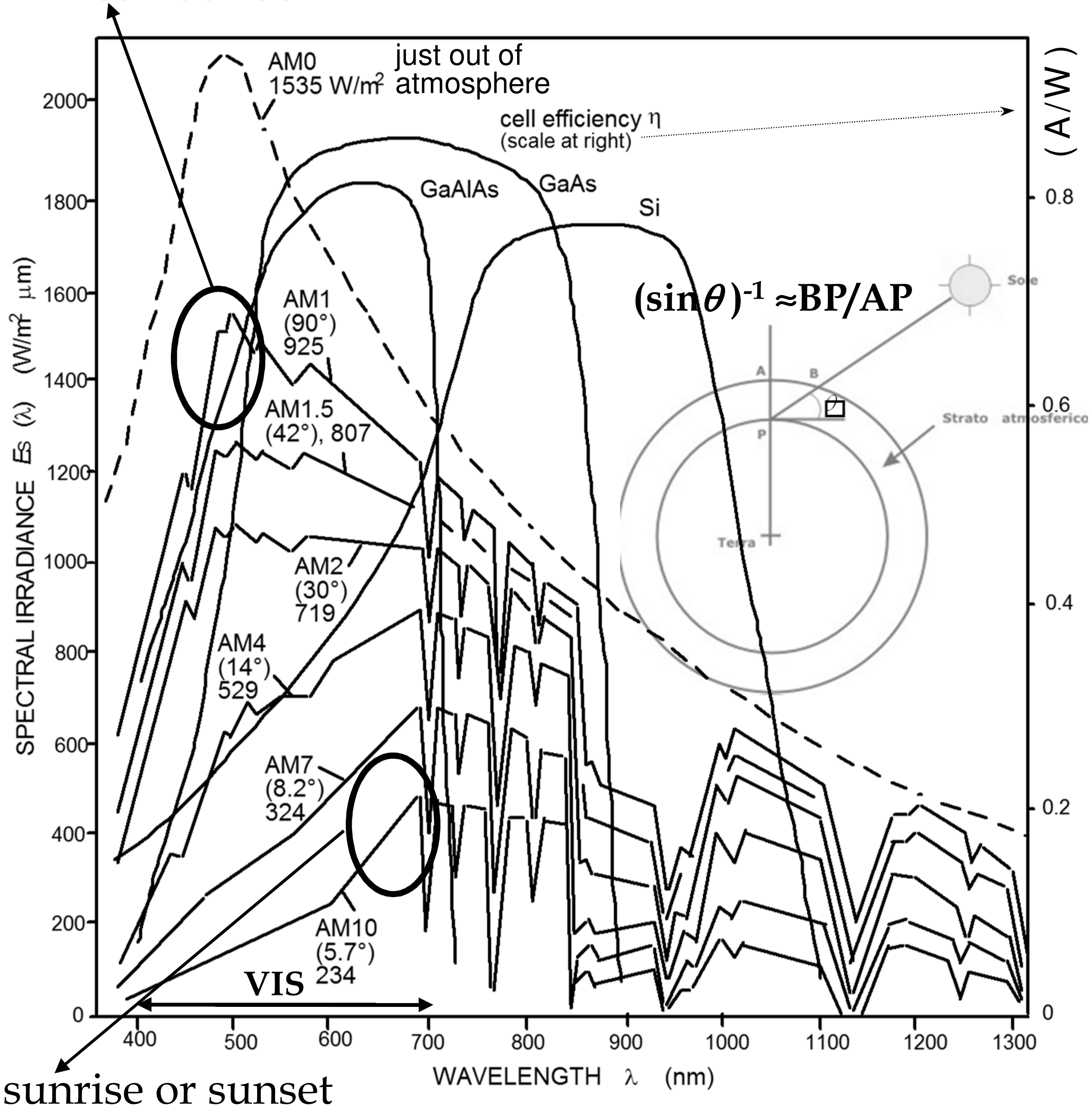
$$T_{\text{atm}} = \exp(-2\alpha L_{20\text{km}}) \quad = 10^{-1}$$
$$\log_{10}\left[\exp(-2\alpha L_{20\text{km}})\right] \quad = \log_{10}\left[10^{-1}\right]$$
$$-2\alpha L_{20\text{km}} \log_{10} e = -1$$
$$\alpha = \frac{1}{2L_{20\text{km}}}\frac{1}{\log_{10} e} = \frac{1}{40\text{ km}}\frac{\ln 10}{\ln e}$$
$$\alpha = \frac{2.3}{40}\text{km}^{-1} = 0.0525\text{ km}^{-1}$$



$$T_{\text{atm}} = \exp(-2\alpha L_{10\text{km}}) \quad = 10^{-2}$$
$$\log_{10}\left[\exp(-2\alpha L_{10\text{km}})\right] \quad = \log_{10}\left[10^{-2}\right]$$
$$-2\alpha L_{10\text{km}} \log_{10} e = -2$$
$$\alpha = \frac{1}{L_{10\text{km}}}\frac{1}{\log_{10} e} = \frac{1}{10\text{ km}}\frac{\ln 10}{\ln e} = \frac{2.3}{10}\text{km}^{-1} = 0.23\text{ km}^{-1}$$

Atmospheric attenuation (examples)

sun at noon



sunrise or sunset

Solar light spectrum reaching the Earth surface shows transparency windows and absorption peaks of the atmosphere

$AM = (\sin\theta)^{-1}$ **"Air Mass"** where θ is the arrival angle respect to the Earth surface ("to the horizon")
θ = *Sun elevation*

Note how for small elevation angles the solar light e.m. spectrum depletes of **blu** light (*scattering* $\propto \lambda^{-4}$) and enriches, relatively, of **red** light (less scatter)

System equations and telemeter SNR

The photoreceiver (photodetector+transimpedance amp.) detecting receiver optical power P_r has electronic noise to be added to optical and shot noise to obtain the **total noise power P_n** (as incident optical power on the photodiode)

The **received signal** optical power is $\boldsymbol{P_r}$ (depending on P_s)

In order to work with a given **ratio $(S/N)=P_r/P_n$** at the telemeter receiver, we must have:

$$GP_s = \frac{4L_{eq}^2}{D_r^2} P_r = \frac{4L_{eq}^2}{D_r^2}\left(\frac{S}{N}\right)P_n$$

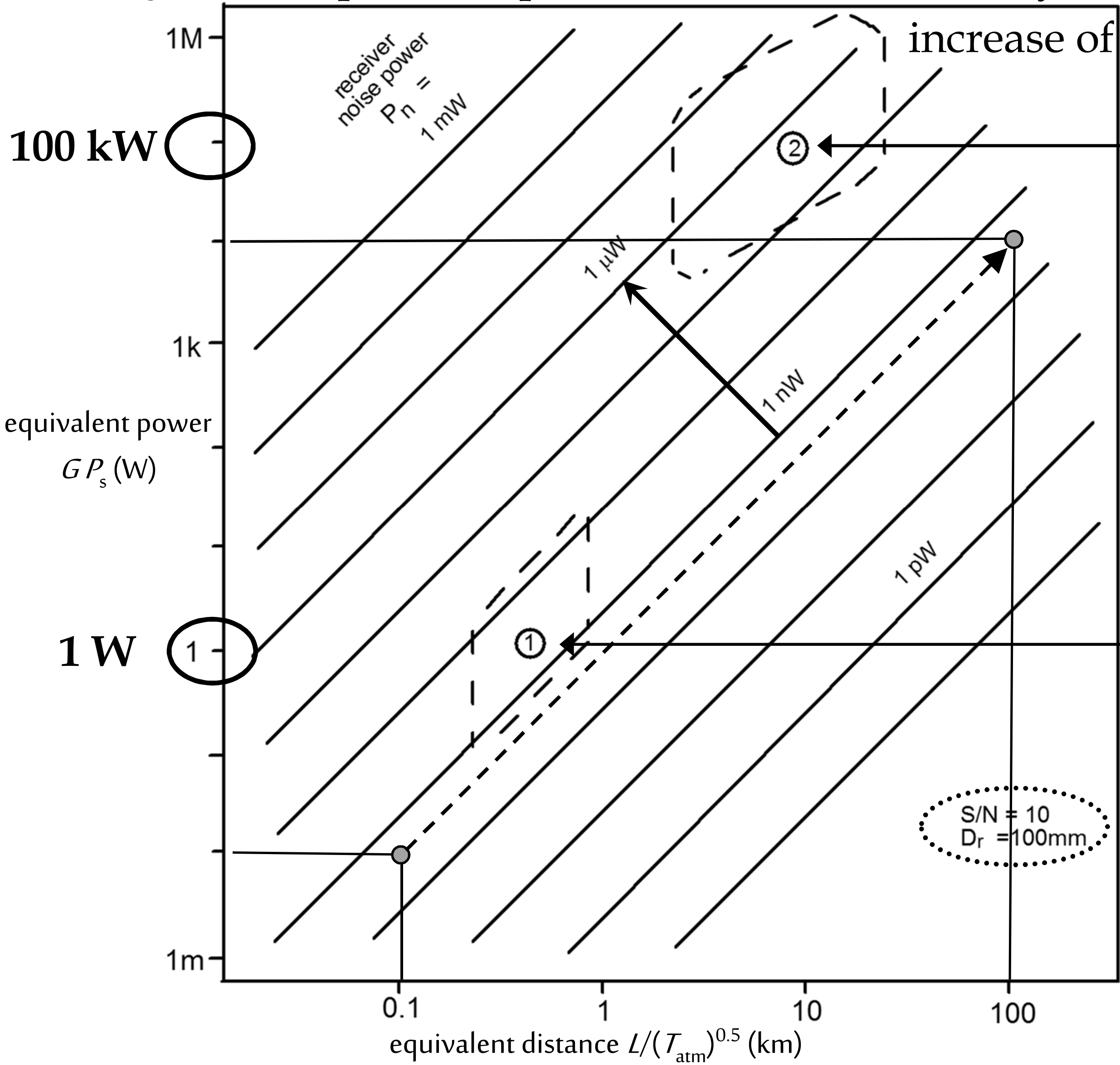
remind that we have $\dfrac{P_r}{P_s} = G\dfrac{D_r^2}{4L_{eq}^2}$

telemeter **equivalent power**

$$\Rightarrow \left(\frac{S}{N}\right) = \frac{GP_s}{P_n}\frac{D_r^2}{4L_{eq}^2} \propto \left\{\frac{D}{L}\right\}^2$$

Power vs distance (equivalent)

$GP_s \propto L^2$ → equivalent power must be increased by two orders of magnitude for an increase of L by one order of magnitude



Pulsed telemeter

$E_p \approx 2$mJ $\tau \approx 10$ns $P_{s,p} = 0.2$MW

$G \approx \delta \approx 0.5$ (non-cooperative)

$B \approx 1/\tau = 100$MHz

$\boldsymbol{P_n \approx}$ **"µW"**

$$i^2_{n,rec} = 2eI_{rec}B$$

eq. opt. power ← $P_n \propto i_{n,rec} \propto \sqrt{B}$

CW telemeter

$P_s = 0.1$mW $G = 10^4$ ($\theta_s = 10$mrad)

$T_{mis} = 10$ms-1s

$B = 1/2T_{mis} = 100$Hz-1Hz

$\boldsymbol{P_n \approx}$ **"nW"**

Noise at the receiver (of the telemeter)

P optical power; ***I*** DC current; ***i*** AC current

3 "optical" noise contributions to noise power P_n:

- noise $P_{n,r}$ associated to received signal (P_r)
- noise $P_{n,bg}$ associated to background light (P_{bg} on the detector)
- noise $P_{n,el}$ of photodetector and transimp. amp. (*front-end*)

$$\boldsymbol{P_n = P_{n,s} + P_{n,bg} + P_{n,el}}$$

$I_r=\rho P_r$ is the "useful" signal, $I_{bg}=\rho P_{bg}$ is the background and naturally $I_{rec}=I_r+I_{bg}$ (resp. $\rho=\eta e/h\nu$)

Let's evaluate the current noise i_{rec} on photodiode output I_{rec}:

- shot noise on $I_r \rightarrow$ $i_r^2 = 2eI_rB \rightarrow i_{n,s}$
- shot noise on $I_{bg} \rightarrow$ $i_{bg}^2 = 2eI_{bg}B \rightarrow i_{n,bg}$
- electronic noise $\rightarrow$ $i_{el}^2 = 2e"I_{el,0}"B \rightarrow i_{n,el}$

this noise is practically observed AFTER the photodiode but it is virtually "transferred" to its "input"

Noise at the receiver (of the telemeter)

Starting from the 3 noise contributions:

- shot noise on $I_r \rightarrow \; i_r^2 = 2eI_rB \; \rightarrow \; i_{n,s}$
- shot noise on $I_{bg} \rightarrow \; i_{bg}^2 = 2eI_{bg}B \; \rightarrow \; i_{n,bg}$
- electronic noise $\rightarrow \; i_{el}^2 = 2eI_{el,0}B \; \rightarrow \; i_{n,el}$

"equivalent" DC current producing a shot noise equal to the electronic noise transferred to the photodiode input

the global variance/power of current noise (sum of variances for uncorrelated quantities) is:

$$\boldsymbol{i_{rec}^2 = i_{n,s}^2 + i_{n,bg}^2 + i_{n,el}^2 = 2eB\,(\,I_r + I_{bg} + I_{el,0}\,)}$$

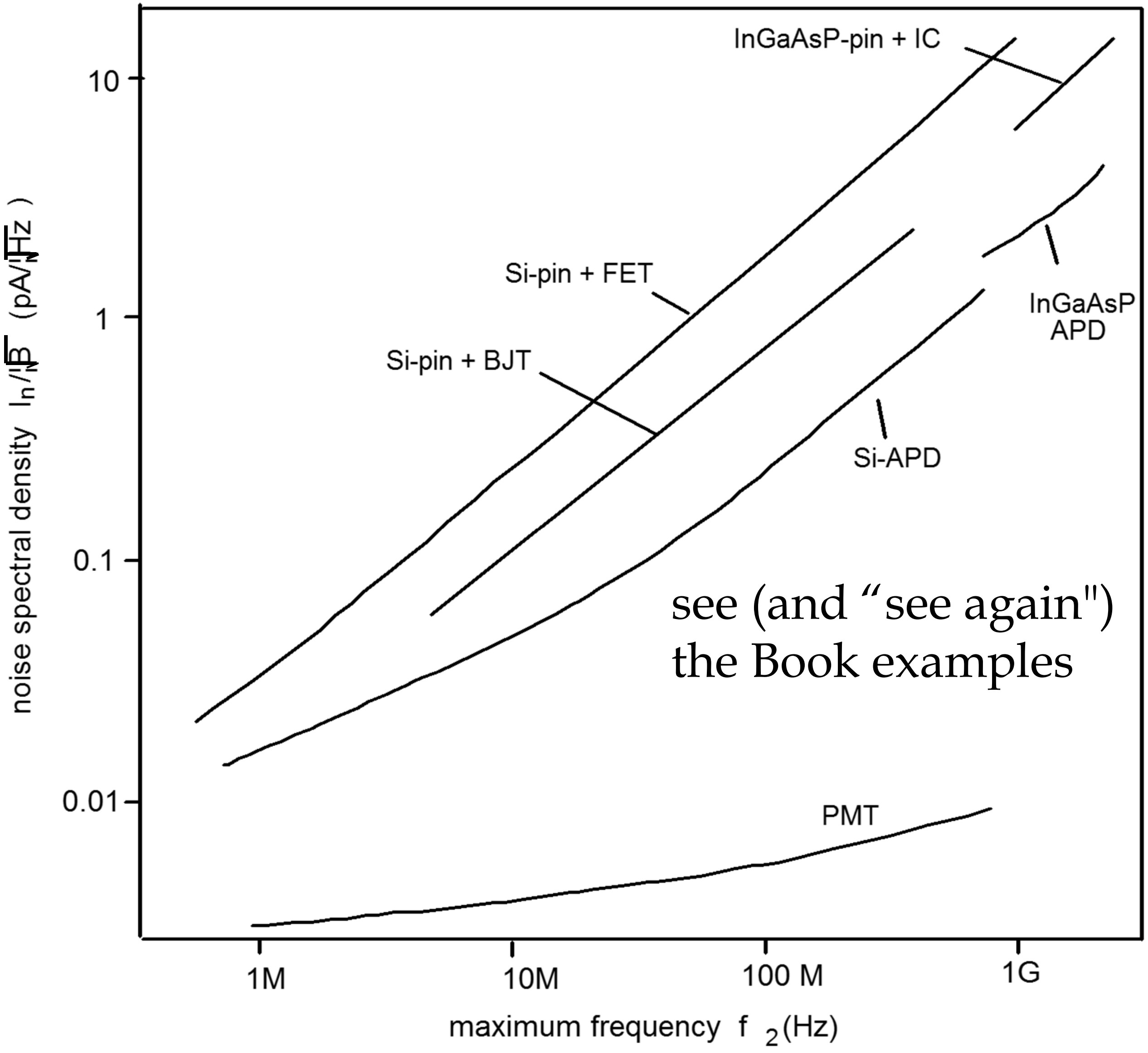
$\dots/\rho^2$

Dividing the two DC photocurrents and the noise equivalent current by the squared spectral responsivity (ρ^2) we get the optical power noise at the receiver:

$$\boldsymbol{P_n^2 = (\,2h\nu/\eta\,)\,B\,(\,P_r + P_{bg} + P_{el,0}\,)}$$

Let's see some typical behaviors of **electronic noise** $\boldsymbol{i_{n,el}}$ **(A/Hz$^{1/2}$) for photoreceivers** (photodiode+amp.), at different operating frequencies...

Electronic noise for different photoreceivers



see (and "see again") the Book examples

PD area A<0.5mm^2
Capacity C<0.5pF

Evaluation of the background light (optical background power P_{bg})

We start from the solar light spectrum (slide 44) and from working conditions (*AM*, clouds, *etc*.) we get the scene spectral radiance E_{scena} (W/m^{2}μm) that multiplied ("integrated") for the bandwidth $\Delta\lambda$ of the interference filter, gives the optical intensity of background light ("**sc**ene"):

$$\boldsymbol{I_{sc} = E_{sc} \cdot \Delta\lambda \quad (\mathrm{W/m^2})}$$ **scene "background" intensity**

Optical power collected on the receiver is 1/π times the background intensity (I_{sc}) times the scene diffusivity (δ_{sc}) times the viewing solid angle (Ω_{sc}) [received intensity I_{bg}] then multiplied by the receiver area ($A=\pi d_r^2/4$):

$$\boxed{\Omega_{sc}=\pi NA^2}$$

$$\boldsymbol{I_{bg} = (1/\pi)[\ \delta_{sc}\, I_{sc}\ ] \cdot \Omega_{sc} \qquad (\mathrm{W/m^2})}$$

$$\boldsymbol{P_{bg} = [\ \delta_s\, E_{sc}\, \Delta\lambda\, NA^2\ ] \cdot (\pi d_r^2/4) \qquad (\mathrm{W})}$$

being $\Omega=\pi\theta^2\cong\pi(NA)^2$ with $NA=\sin(D_r/2f)$ Numerical Aperture

(<u>in this slide "*I*" stays for optical intensity and not electric current</u>)

Accuracy of the pulsed telemeter

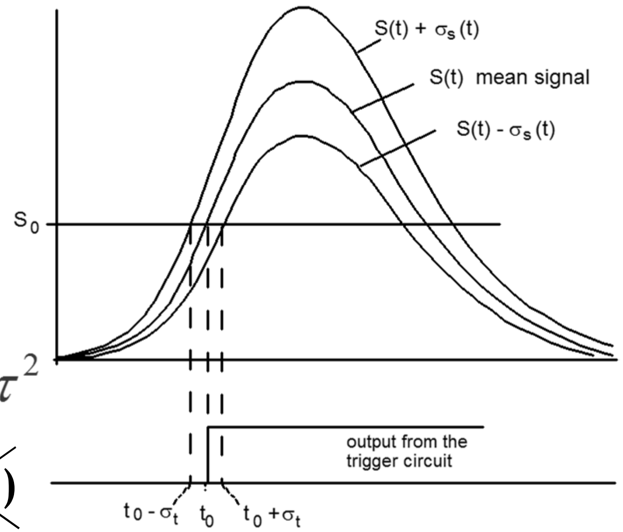
$$L = c\frac{T}{2} \qquad \Rightarrow \qquad \sigma_L = \frac{c}{2}\sigma_T$$

$$T = T_{\text{stop}} - T_{\text{start}}$$

$$\sigma_T^2 = \sigma_{\text{stop}}^2 + \sigma_{\text{start}}^2 \cong \sigma_{\text{stop}}^2$$

$$\sigma_T^2 = \frac{\sigma_S^2 \quad \approx P_\text{n}^2 \approx P_\text{r} \approx N_\text{r}}{\left|\mathrm{d}S/\mathrm{d}T\right|^2 \approx P_\text{r}^2/\tau^2 \approx N_\text{r}^2/\tau^2}$$

for SNL $P_\text{n}^2 = (2h\nu/\eta)\, B\, (P_\text{r} + \cancel{P_\text{bg}} + \cancel{P_\text{el,0}})$



In a well-designed receiver (***shot-noise limited***, *SNL*):

$$\sigma_T \propto \frac{\tau}{\sqrt{N_\text{r}}}$$

number of received photons (over a single pulse or as the sum/"average" over *N* pulses)

In general, if not SNL, the background light and electronic noise contributions do increase the whole noise worsening the situation!

Accuracy of the CW sine-modulated telemeter

$$\sigma_{\mathrm{T}} \propto \frac{1}{2\pi f_{\mathrm{m}}} \frac{1}{\sqrt{N_{\mathrm{r}}}}$$ and once again $$\sigma_L = \frac{c}{2}\sigma_T$$

In analogy to the pulsed telemeter, now **the term $\mathbf{1/2\pi} \boldsymbol{f}_{\mathbf{m}}$ is equivalent to the duration** τ of thepulse:

- **pulsed telemeter:** better working with short pulses (**small** τ)
- **CW sine-modulated telemeter:** better working with an high modulation frequency (**high** $\boldsymbol{f}_{\mathbf{m}}$)

Usually for QS $\tau \approx 10$ns << $(1/2\pi f_{\mathrm{m}}) \approx 1\mu$s for a typical $f_{\mathrm{m}} \approx 200$kHz
and hence $\sigma_{\mathrm{T,p}} << \sigma_{\mathrm{T,CW\text{-}mod.}}$ (in the SNL case) but due to electronic noise
$B_{\mathrm{p}} \approx 1/\tau \approx 100$MHz >> $B_{\mathrm{CW\text{-}mod.}} \approx 1/2T_{\mathrm{mis}} \approx 100$Hz-1Hz (SNL difficult when pulsed)

We would like to have an **high $\boldsymbol{f}_{\mathbf{m}}$** (or high pulse repetition rate, allowing for "pulse averaging" in the pulsed case) but this will onset other **measurement ambiguity problems**

Ambiguity in Time Of Flight telemeters

Working with a periodic signal (transmitted and hence also received), we have an **ambiguity problem** in distinguishing **targets at different distance** that may return an optical **signal with the same measurement information** (time of flight or phase delay in the round-trip):

In order to avoid measurement ambiguity, we must have:

- pulsed telemeter:

$\boldsymbol{T_{max}} = T(L_{max}) \leq \boldsymbol{T_{rep}}$ $\quad\Rightarrow \boldsymbol{T_{rep} \geq T_{max}}$

- CW sin-mod telemeter:

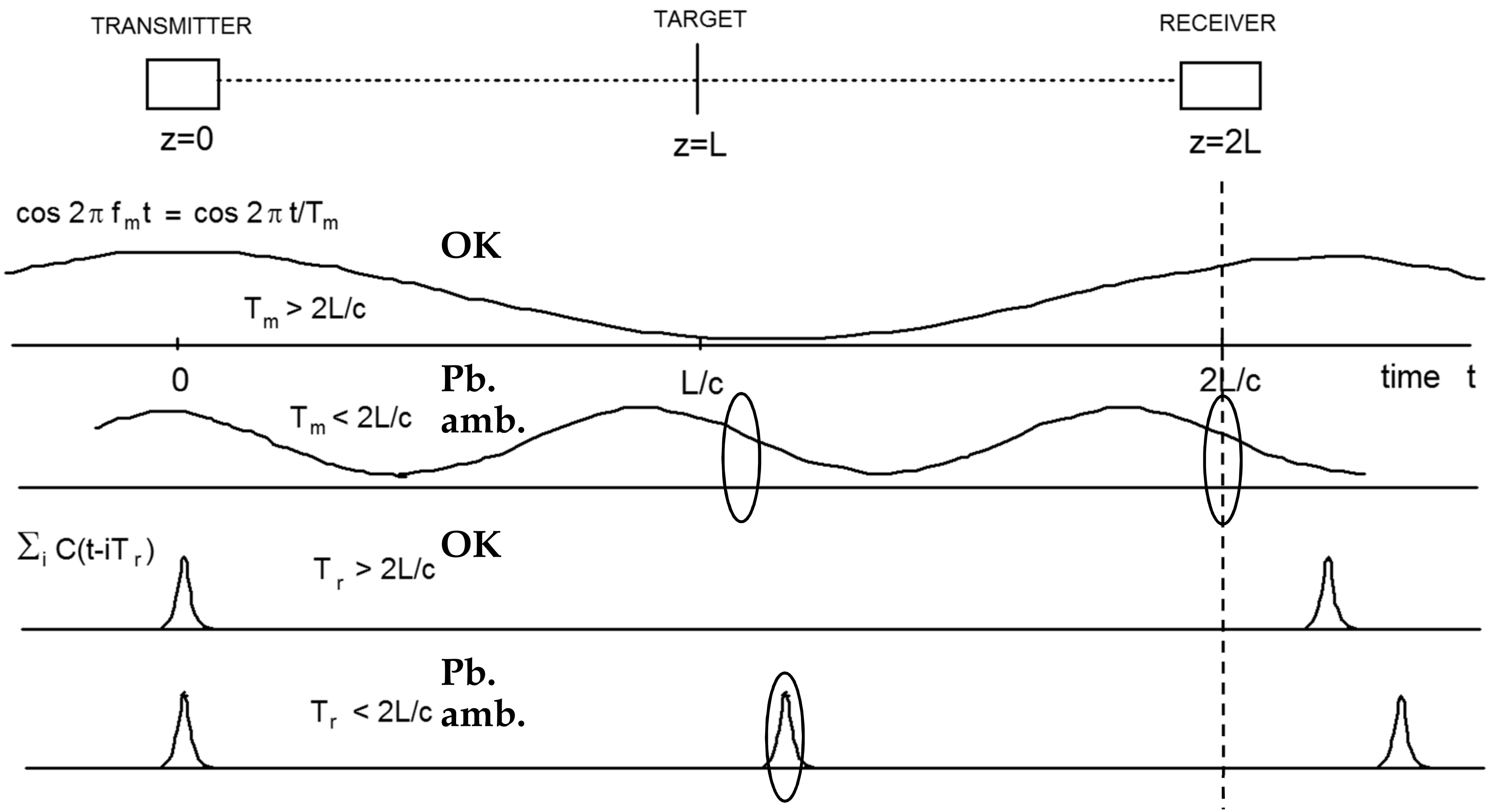
$\boldsymbol{\varphi_{max}} = \varphi(L_{max}) = 2\pi f_m T_{max} \leq \mathbf{2\pi}$ $\quad\Rightarrow \boldsymbol{f_m \leq 1/T_{max}}$

$$T_{max} = \frac{2L_{NA}}{c} \leq \frac{1}{f_{telem}}$$

$$f_{telem} = \left\langle \begin{array}{l} f_{rep} \\ f_m \end{array} \right.$$

where $\boldsymbol{T_{max}}$ **is the "maximum Time Of Flight"** corresponding to the maximum distance L_{max}, said $\boldsymbol{L_{NA}}$, correctly measurable

Ambiguity in Time Of Flight telemeters



$$T_{max} = \frac{2L_{NA}}{c}$$

When f_{mod} is "low" or T_{rep} is "high" (i.e. f_{rep} low) there is no ambiguity Pb. but litlle averaging and poor SNR

Ambiguity in Time Of Flight telemeters

- **pulsed** telemeter:

Q-switched laser $\tau \approx 10$ ns f_{rep}=10Hz÷10kHz (repetition rate)

from $T_{rep}=T_{max}$ we get $\mathbf{L_{NA}=(c/2)T_{rep}=c/2f_{rep}}$**=15000km÷15km**

the problem arises only at large distances and/or for high repetition frequency of the pulse [it is useful repeating measurements of single pulses in order to increase the accuracy ("averaging")]

- CW sine-**modulated** telemeter:

diode laser with f_m=10MHz÷10kHz (current modulated)

from $f_m=1/T_{max}=1/(2L_{NA}/c)$ we get

$\mathbf{L_{NA}=(c/2)T_m=(c/2)\cdot(1/f_m)}$**=15m÷15km**

the problem arises already at medium-short distances

To achieve high accuracy we want high f_m ("averaging") but to reach high distances we must keep f_m low... combined use of 2 frequencies f_{m1} e f_{m2}

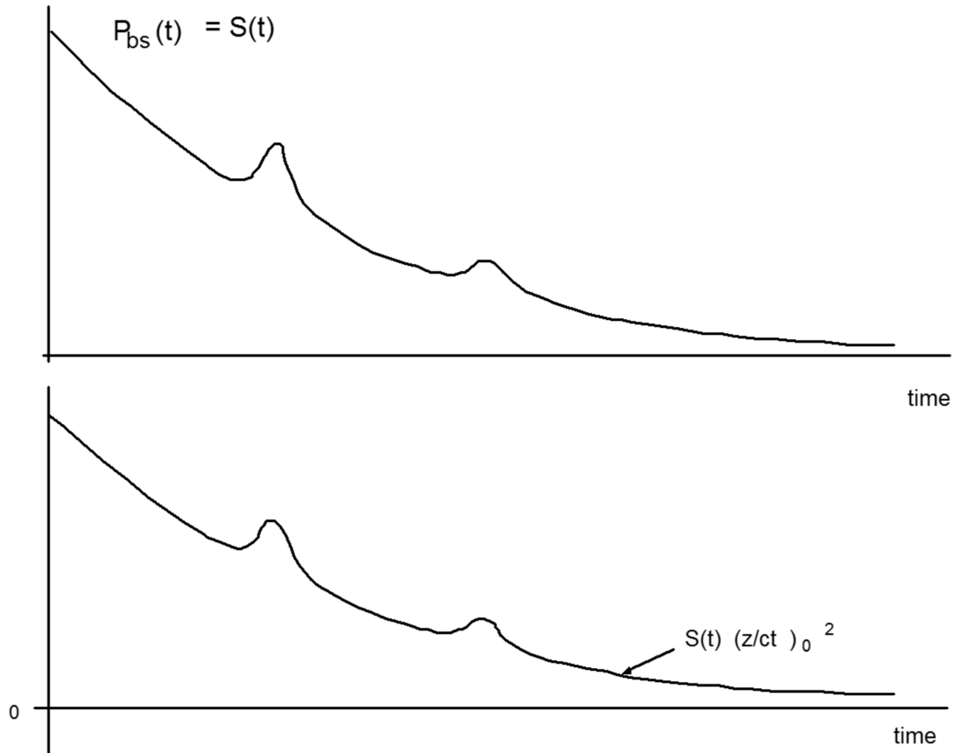
LIDAR

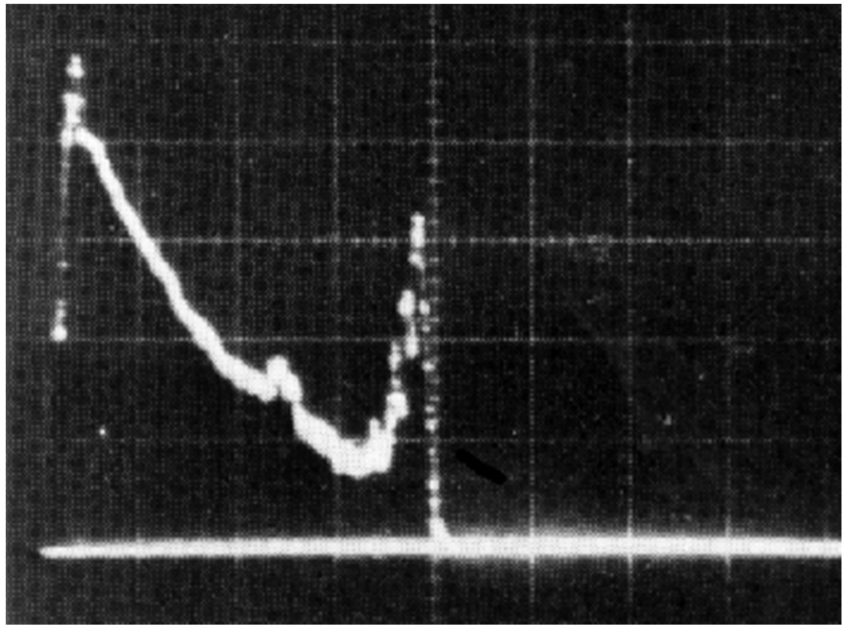
Light **I**dentification **D**etection **A**nd **R**anging

very similar to a telemeter, it is an instrument for measuring "at distance" the **properties of a media** trough which the **optical pulse** undergoes transmission and **backscattering**

LIDAR

Laser source with high peak power (**Q-switched**) at one or **more suitable wavelengths** to detect **absorption** and **scattering** from the investigated component within the medium (**gas** or **particulate in the atmosphere**, or **pollutants** or plankton/chlorophyll/seaweed **in water**, *etc*.).





backscattering signal
(technique OTDR)

LIDAR

From measured Time Of Flight $t = 2L/c$ we get the distance of the investigated target ($\tau \rightarrow$ size of investigated volume);

from measured backscattered signal we get the composition (chemical/physical) of the investigated volume;

we obtain **maps, even fake-colored, as a function of the telemeter <u>elevation angle</u> and <u>distance</u>**