“Optical Measurements”

Master Degree in Engineering
Automation-, Electronics-, Physics-, Telecommunication- Engineering

Alignment/Pointing and Dimensional Measurements

Prof. Cesare Svelto
Politecnico di Milano

Some pictures are taken from the Book “Electro-Optical Instrumentation: Sensing and Measuring with Lasers” by Prof. Silvano Donati

Summary

- **Propagation and transformation** (focusing, widening-and-collimation) of gaussian beams
- **Position sensitive detectors** of laser beam
  - 4-quadrants photodiodes
  - PSD (*Position Sensitive Detector*)
  - reticle detectors
- **Laser level**
- **Wire diameter** measurement
- Optical measurement of **particulate dimensions**

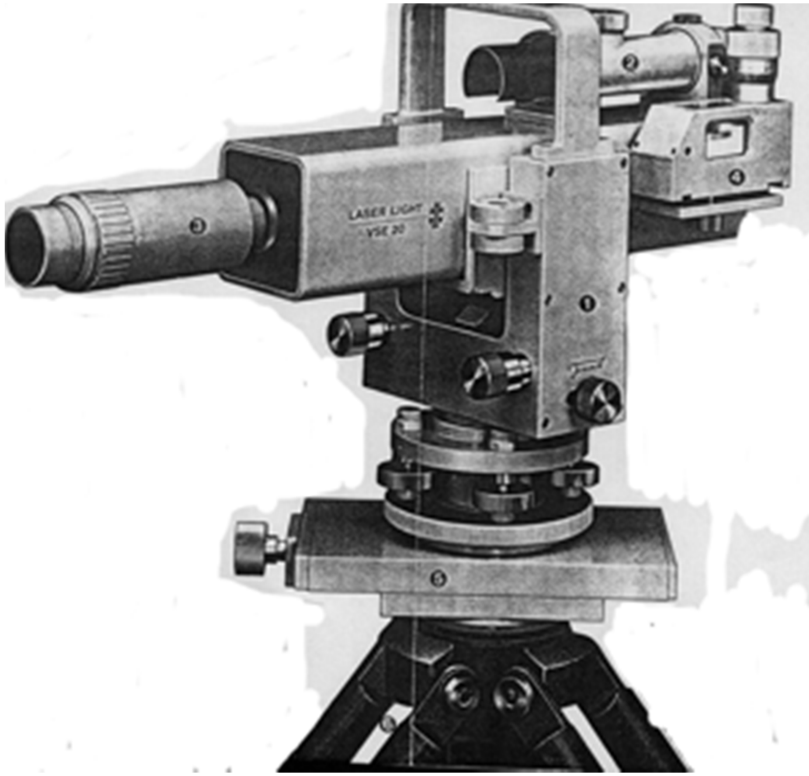
Laser alignment

One property of **laser sources** is the possibility of keeping the **optical bem well collimated** (slightly divergent and hence with **“constant spot size"** during propagation)

The **divergence limit** posed by **diffraction theory (TEM_{00})** is “easy to meet”: *e.g.* for an **He-Ne LASER (633 nm)**. Visible light is useful for **alignment** in a specific **direction** ("filo a piombo" not only vertical)

We must **minimize the laser spot dimension** on the whole working region (range $\pm z^*$) and to this aim we must design an **optimal value of the *beam waist*** (w_0) in the center of the range: for this purpose we use a **telescope** to “**widen the spot size** to the desired dimension"

Laser alignment in constructions



Typical instrument for laser alignment and its use in the construction of gas pipeline (LaserLight AG, Munich)

Propagation of a laser Gaussian beam

$$w(z) = w_0 \cdot \sqrt{1+\left(\frac{z}{z_R}\right)^2}$$

$$w(z) = w_0\sqrt{1+\left(\frac{\lambda z}{\pi w_0^2}\right)^2} \cong \frac{\lambda z}{\pi w_0} = \theta z \quad \text{for } z >> z_R$$

Divergence of the **laser spot** (free space)

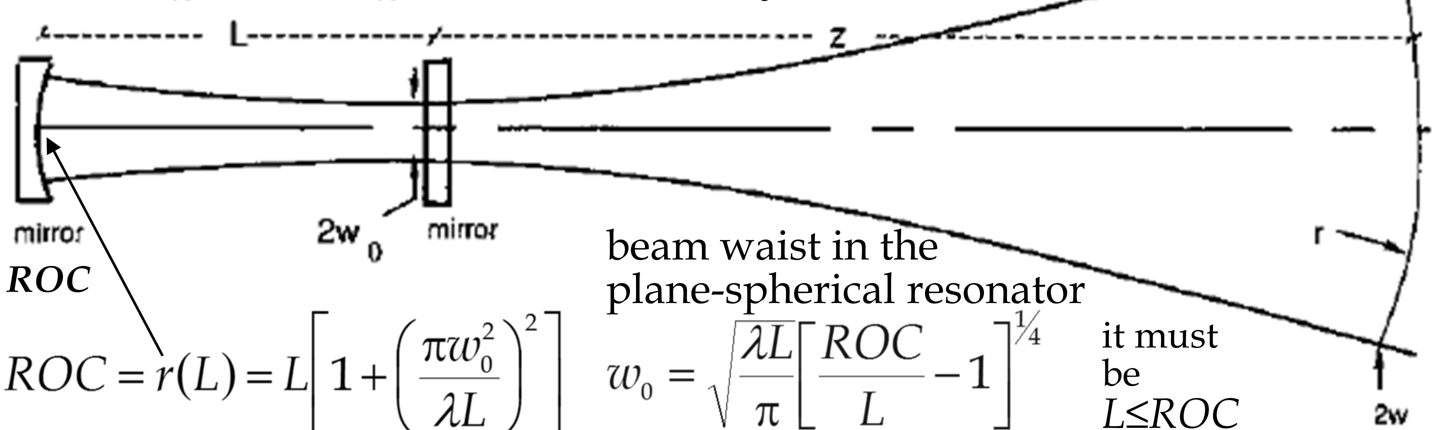
$$r(z) = z \cdot \left[1+\left(\frac{z_R}{z}\right)^2\right]$$

$$r(z) = z\left[1+\left(\frac{\pi w_0^2}{\lambda z}\right)^2\right] \cong z \quad \text{for } z >> z_R$$

Curvature Radius of the wave front

$r=\infty$ ot z=0 and at $z=\infty$ (plane wave) minimum $r_{MIN}=2z_R$ **at** $z=z_R$

Output beam from a laser (*e.g*. He-Ne) with plane-spherical cavity



ROC

$$ROC = r(L) = L\left[1+\left(\frac{\pi w_0^2}{\lambda L}\right)^2\right]$$

beam waist in the plane-spherical resonator

$$w_0 = \sqrt{\frac{\lambda L}{\pi}\left[\frac{ROC}{L}-1\right]^{1/4}}$$

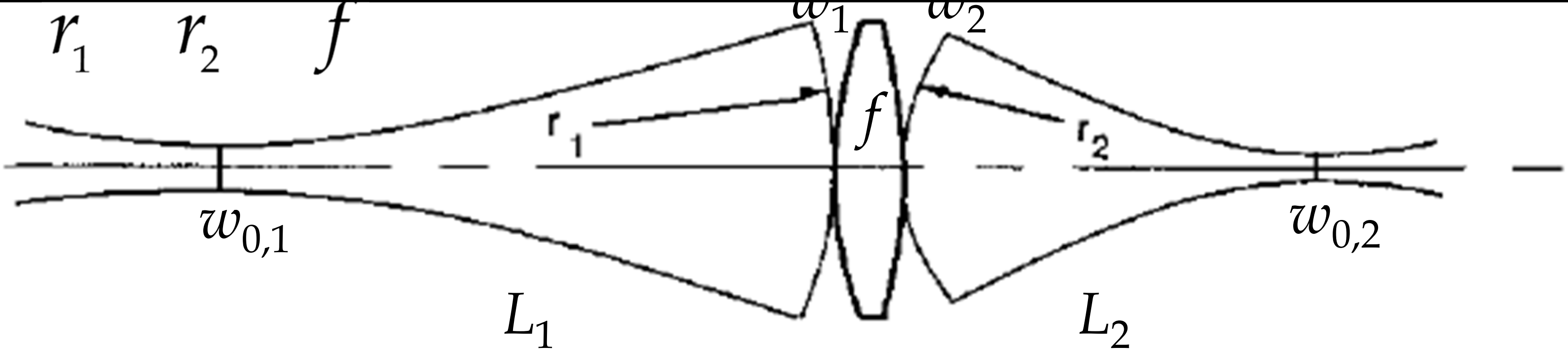
it must be $L \le ROC$

Propagation trough a lens

analogous of $1/p+1/q=1/f$ for geometrical optics

$$\frac{1}{r_1}+\frac{1}{r_2}=\frac{1}{f}$$

Transformation of the curvature radii trough a lens (for a "thin lens" $w_2=w_1$)



1. know/measure the incident beam ($w_{0,1}$ or w_1,r_1)
2. derive r_2 [as $(1/f - 1/r_1)^{-1}$]
3. use $w_2=w_1$ ["thin lens"]
4. derive $w_{0,2}$ [propagation of a Gaussian beam over L_2]
   (knowing both r_2 and w_2, from 2. and 3.)

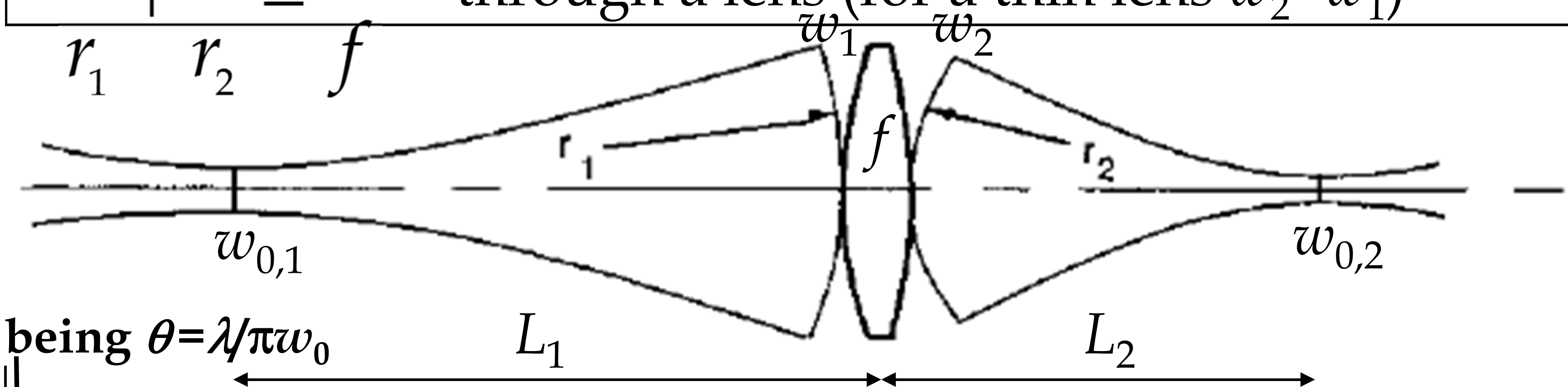
Before ("object") and after ("image") the lens we have <u>w_0/r =const.</u>
and also <u>w_0/L =const.</u> we see in the next side how …

Propagation through a lens

$r_{1,2}$ are not easy to measure while measuring $L_{1,2}$ is simple

$$\frac{1}{r_1}+\frac{1}{r_2}=\frac{1}{f}$$

Transformation of the radii of curvature through a lens (for a thin lens w_2=w_1)



being $\theta=\lambda/\pi w_0$

If $z>z_R$ ($z>>z_R$) $\Rightarrow$ $\mathbf{r_{1,2}\cong L_{1,2}}$ and hence $\theta_1 r_1 \cong w_1 = w_2 \cong \theta_2 r_2 \Rightarrow$

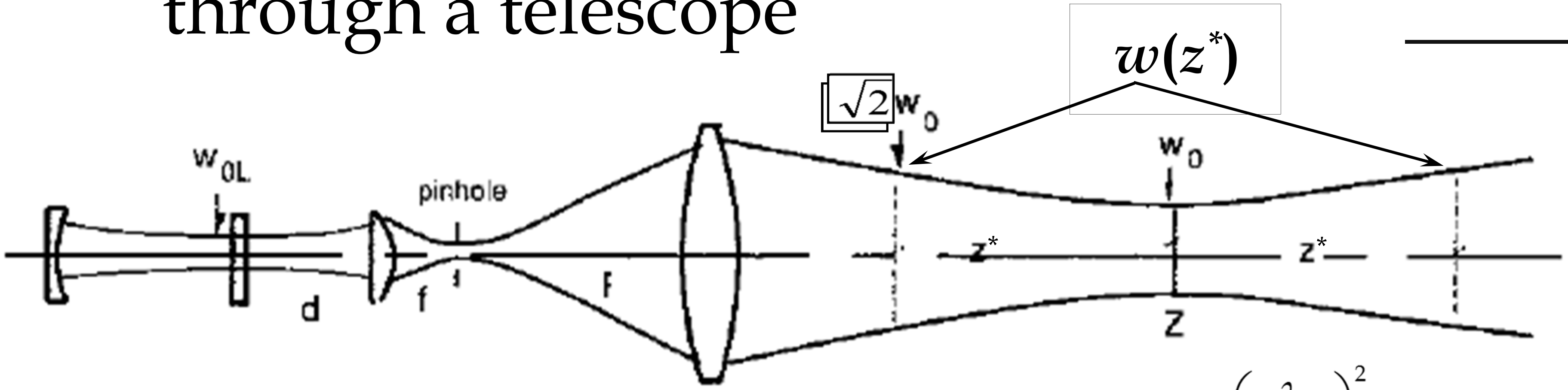
$r_1/w_{0,1} \cong r_2/w_{0,2}$ $\quad \mathbf{w_{0,1}/w_{0,2} \cong r_1/r_2 \cong L_1/L_2}$ e $\underline{\mathbf{w_{0,1}/L_1 \cong w_{0,2}/L_2}}$

After propagating through a lens the beam undergoes a **magnification** $\mathbf{m=w_{0,2}/w_{0,1}=r_2/r_1=L_2/L_1}$

$$L_2=\left(\frac{1}{f}-\frac{1}{L_1}\right)^{-1}$$

To “enlarge" $w_{0,1}$ with respect to $w_{0,2}$, one must work with $r_1>r_2$ and hence with the lens more distant from $w_{0,1}$ ($L_1>L_2$) then the distance from $w_{0,2}$ (or for $L_1<L_2$ one has $w_{0,1}< w_{0,2}$)

Collimation over a range $\pm z^*$ through a telescope



$$w^2(z) = w_0^2 + \left(\frac{\lambda z}{\pi w_0}\right)^2$$

condition for collimation over a range $\pm z^*$

Varying w_0 we search the **minimum** $\boldsymbol{w(z^*)}$, at fixed $\pm z^*$ distance from the beam waist... We differentiate the expression of the spot size w respect to w_0, or $y=[w]^2$ respect to $W=[w_0]^2$:

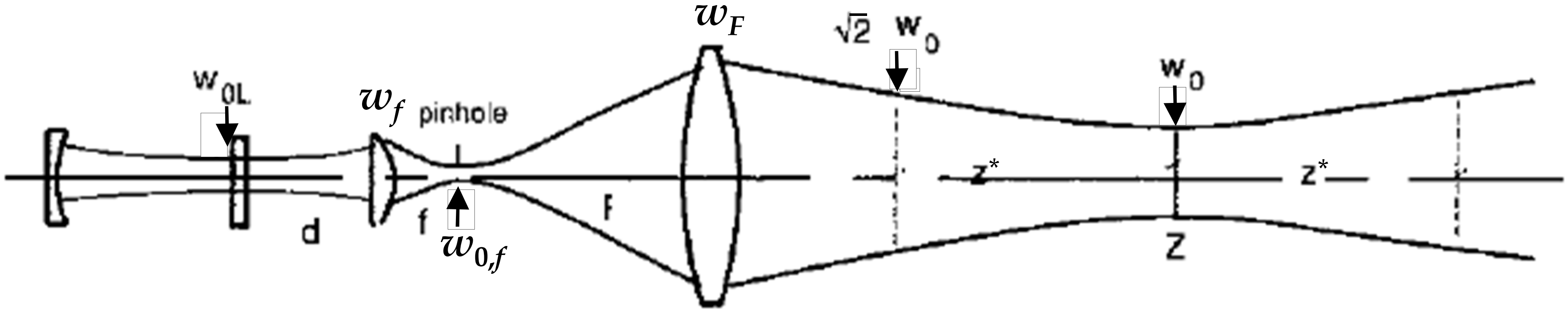
$$y = W\left[1+\left(\frac{\lambda z^*}{\pi W}\right)^2\right] = W + kW^{-1} \qquad \frac{\partial y}{\partial W} \cdots$$

$$w_0 = \sqrt{\frac{\lambda z^*}{\pi}} \qquad \frac{\partial y}{\partial W} = \frac{\partial w^2}{\partial w_0^2} = 1 - \left(\frac{z^*}{z_R}\right)^2 = 0 \Rightarrow z^* = \frac{\pi w_0^2}{\lambda} = z_R$$

the half-width of the collimation range is

with $w(z^*) \cong 1.4 w_0$

Beam-sizing of the laser spot after a telescope



$$w_{0,f}/w_{0L} \cong f/d \;\text{ e }\; w_0/w_{0,f} \cong Z/F \;\Rightarrow\; w_0 \cong (Z/F)\cdot(f/d)\, w_{0L}$$

Spot **magnification**: $m = w_0/w_{0L} = (Z/d)\cdot(1/M)$
with $M = F/f = w_F/w_f$ telescope magnification

Typically one has $f \ll F$, and it is relatively easy to "adjust" the dimension w_0 ($\propto f$) and the distance Z by slightly moving the ocular (lens with focal length f)
(in fact, in terms of relative variations: $\Delta w_0/w_0 = \Delta f/f$)

Example of collimation of an He-Ne LASER for alignment

DATA:

He-Ne LASER with plano-concave cavity (***L*=20 cm**, ***ROC*=1 m**).

We want to cover $\pm z^*$**=±20 m** with minimum spot dimensions:

<u>calculate the magnification ***m*** of the laser spot and the one ***M*** of the telescope.</u>

Imagine we use a telescope with ocular distance d=10 cm and we want to work with $Z \cong z^*$=20 m.

From $w_{0L} = \sqrt{\frac{\lambda L}{\pi}} \left[\frac{ROC}{L} - 1 \right]^{1/4}$ we obtain $\mathbf{w_{0L}}$**=282 μm ≈ 0.3 mm**

From $z^* = \frac{\pi w_0^2}{\lambda}$ =20 m we obtain $\mathbf{w_0}$**=2 mm** (diam. $2w_0$=4 mm)

From $w_0 \cong (Z/F) \cdot (f/d)\, w_{0L}$ we get $m = w_0/w_{0L} = 7.1 = (Z/d)/M$ as magnification of the laser spot, whereas the magnification of the telescope is $M = F/f = (Z/d)/m = (20/0.1)/7.1 = 28$

At ±20 m from w_0, beam size is $\boldsymbol{D} \cong 2 \cdot 1.41 w_0 \cong 2.8 \cdot 2\,\text{mm} = \mathbf{5.6\,mm}$

Diapositiva 10

CS7 qui termina la 6a lezione dell'AA 2005/2006

Cesare Svelto; 31/03/2006

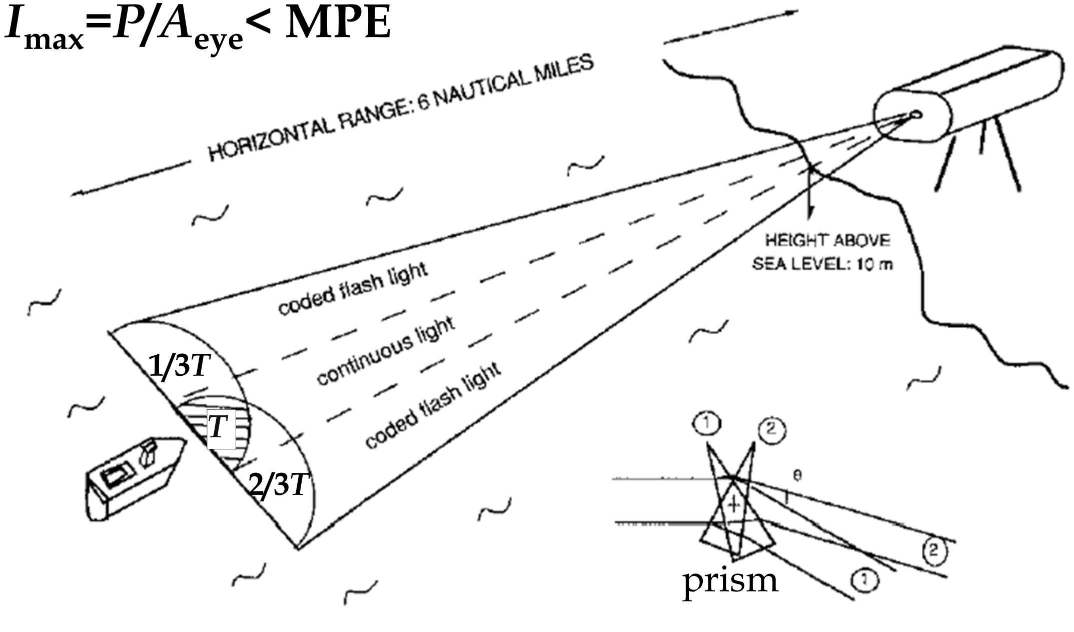
Telescope for alignment and marine channel directions

Typically an alignment system uses an He-Ne (0.5-2 mW) laser and a 50-mm diameter telescope with magnification M=20-50. In practice the laser beam can remain collimated in a range from a few tens to a few hundreds of meters.

Using a telescope with D=100 mm and a 10 mW He-Ne laser, the beam can be seen at a few miles distance:

$$I_{max} = P/A_{eye} < \text{MPE}$$

Plot 3 square waves with different d.c. val. at left, center, right

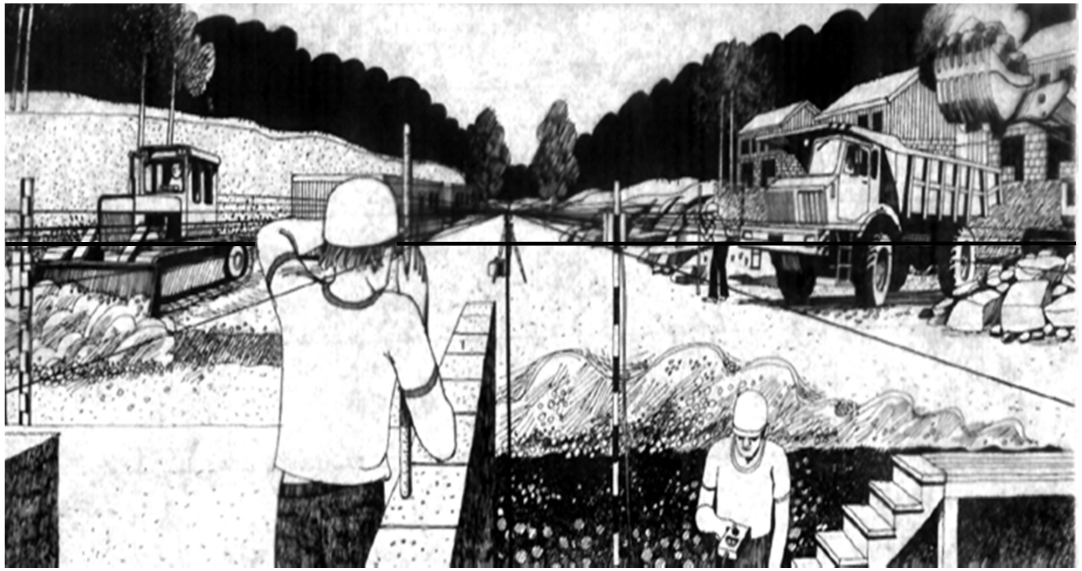


Diapositiva 11

CS8 qui inizia la 7a lezione dell'AA 2005/2006

Cesare Svelto; 05/04/2006

Alignment with the laser level



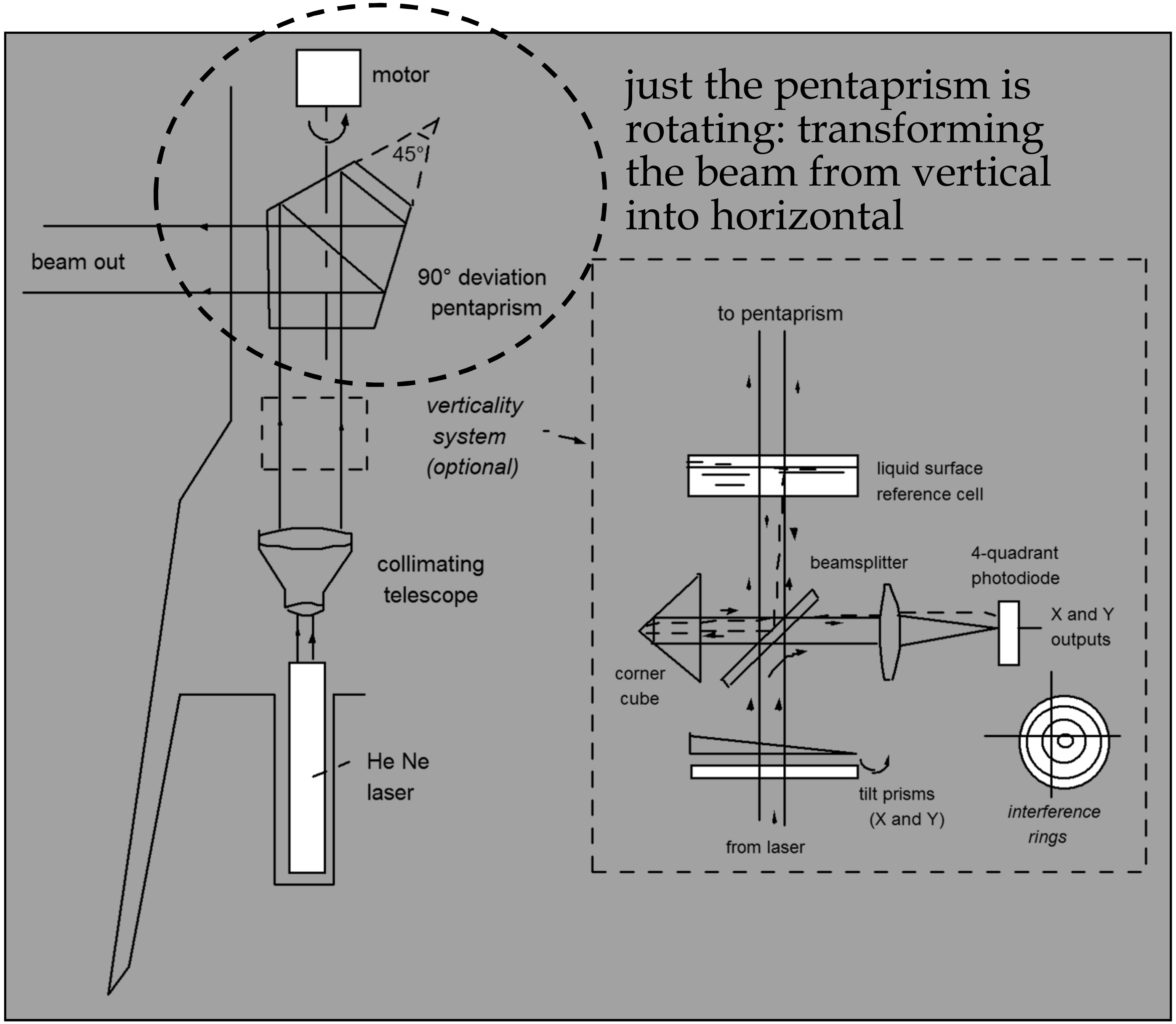
When we need to **measure the height** ***h*** **or angle** φ over a **working surface** (construction area, pool, rice field, ...)

A laser level distributes, over an area of radius 20-50 m, a beam "horizontal fan", at constant height, by changing the rotation angle

We need to **"level" the laser beam**: laser+telescope shine vertically (from bottom) a 45° mirror, or to a pentaprism, reflecting light at 90° and hence in the horizontal direction

Typical laser level instrumentation tripod mounted

Horizontal leveling of the laser level



The verticality reference is the normal to the surface of a fluid (water) in the bowl Reflected beam at the air-water interface is recombined with the launched beam and perfect alignment is observed trough interference at the detector (screen or 4-quadrant photodetector). With 2 prisms launch X and Y directions can be regulated

Beam centering on the target and position-sensitive photodetectors

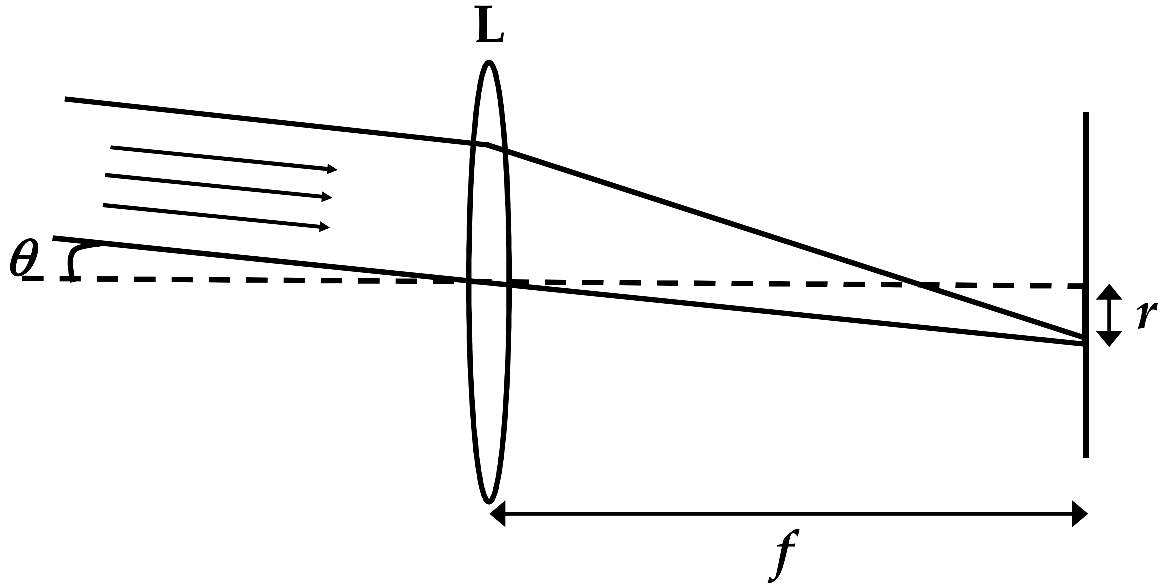
For less stringent applications, like in constructions, it is sufficient an **eye alignment** ($\Delta x \approx \Delta y \approx 1$ mm)

For more accurate measurements, we use a **photodetector** to provide for an **electric signal proportional to the alignment error**. A feedback system allows the **alignment control** by **minimizing the error signal**.

The position-sensitive photodetector can be a special **"photodiode"** (**4-quadrant photodiode**, **PSD** sensor or even a **CCD**) or a normal photodiode coupled to a spatial reticule/mask (**rotating reticule**) **transmitting light as a function of the impinging beam position**

Transformation from angular into spatial (position) coordinate

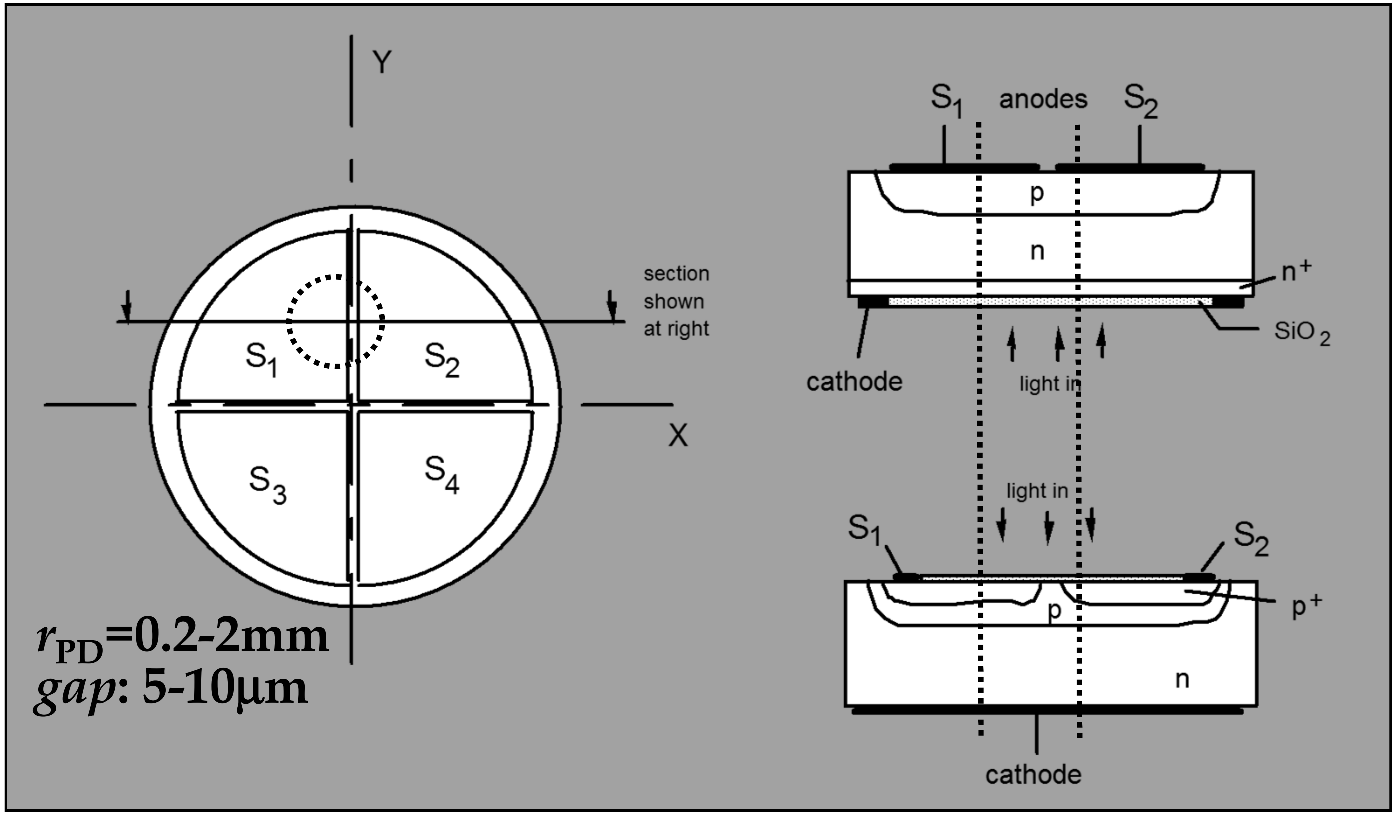
When we need to measure the **arrival direction (angle)** of the optical beam, we use a collecting lens with focal distance f and we observe the **displacement (position)** off-axis of the **laser spot in the lens focal plane**:



Transformation law between angular and spatial coordinate is:

$$r = f \cdot \tan(\theta) \approx f \cdot \theta \quad \text{for } \theta << 1$$

4-Quadrant photodiode (position sensor)



r_{PD}**=0.2-2mm**
***gap*: 5-10μm**

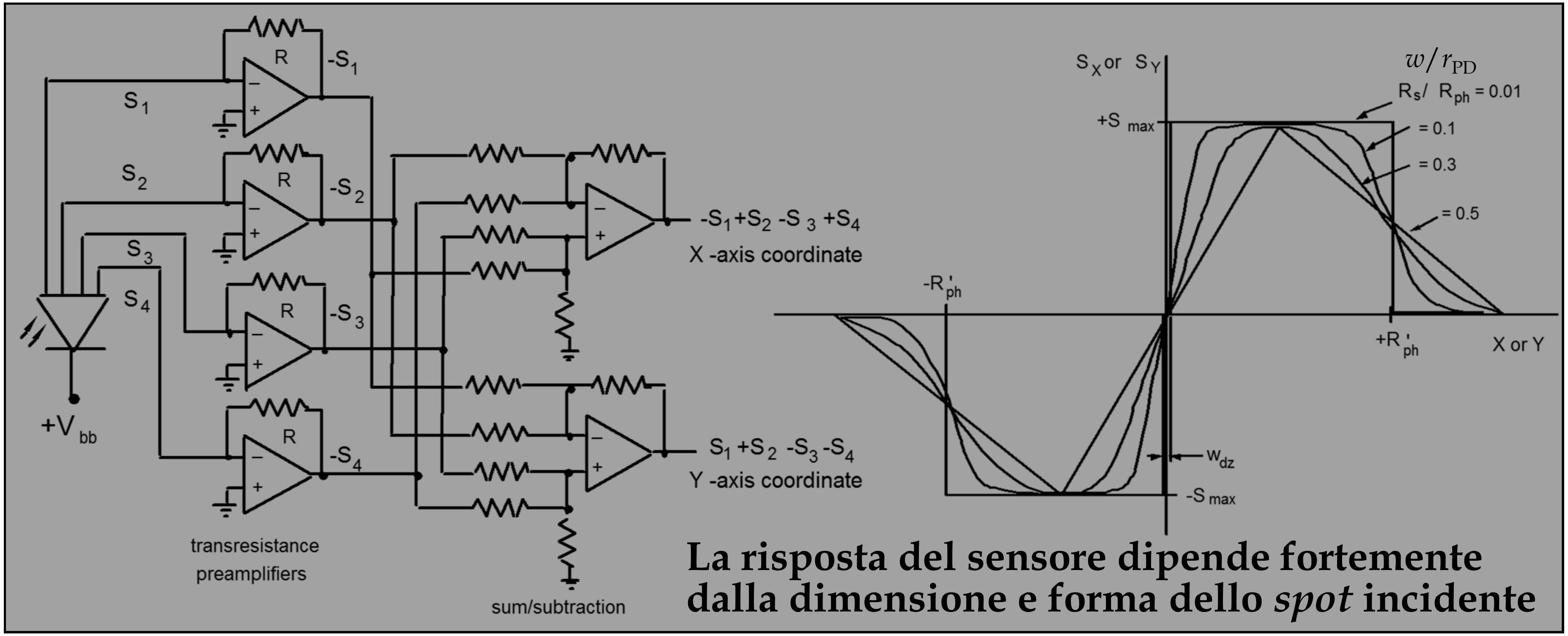
In the depletion region of the p-n junction, incident photons produce a current that can flow toward 4 distinct electrodes (one for each circular sector S_1, S_2, S_3, S_4)

The 4 photocurrents can be combined to obtain two signals proportional to X and Y coordinates of the beam respect to the photodiode center:

$$S_X=(S_2+S_4)-(S_1+S_3) \qquad S_Y=(S_1+S_2)-(S_3+S_4)$$

We can also normalize respect to $P_0 \propto S_0=(S_1+S_2+S_3+S_4)$

Extraction of X and Y coordinates from 4-Quadrant photodetector

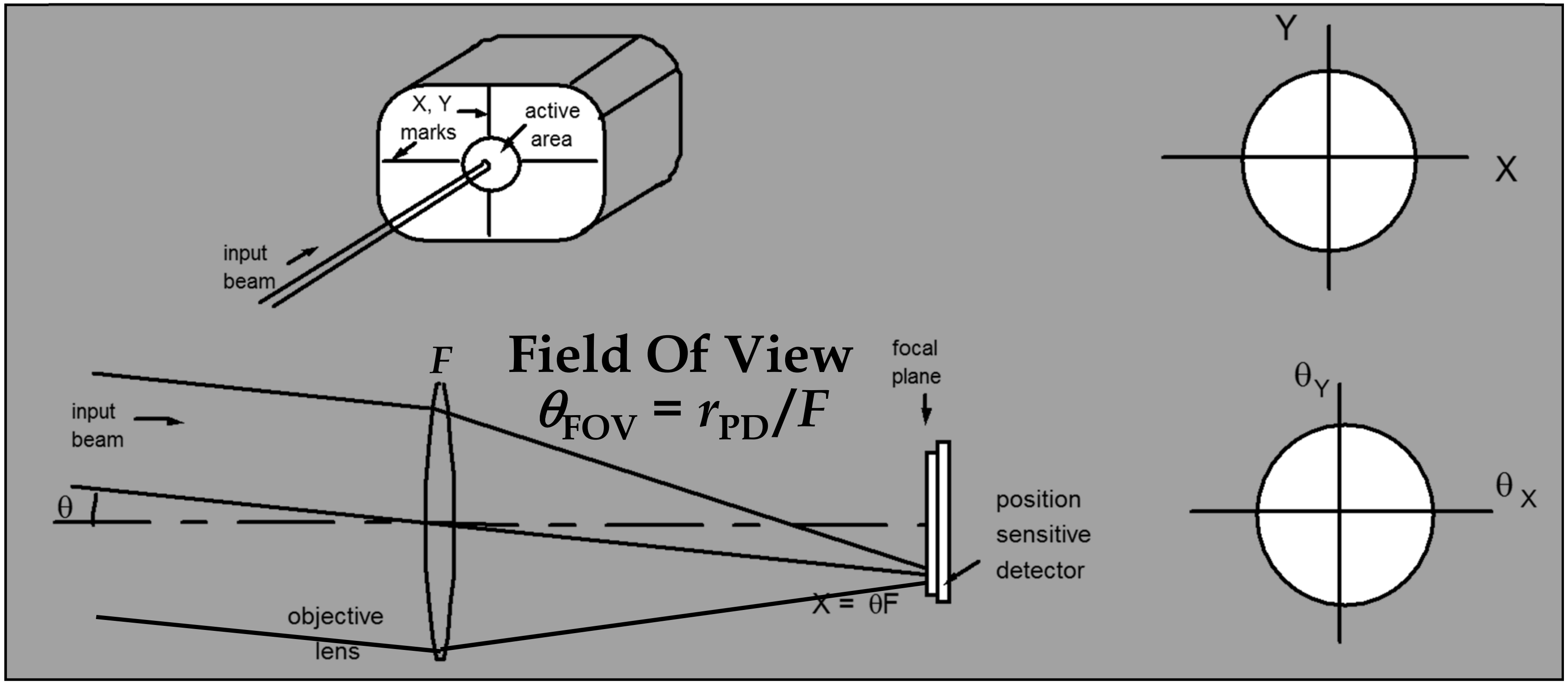


OP-AMP circuit (transimpedance conversion of photocurrents and voltage sum/subtraction) to achieve the coordinate signals S_X e S_Y

Spatial localization accuracy on the 4-Q depends on P_{spot} and w and spot shape, and *gap* and r_{PD}: $\sigma_{X,Y}$=10%-3%·r_{PD}

Dependence of the coordinate signal S_X (S_Y) from the coordinate value X (Y) of the laser (or light) spot.
For small spot ($w_0 << r_{PD}$) response is squared with a small dead-zone at X=0 (or Y=0). For larger spot the signal is "linearized" (in the central region)

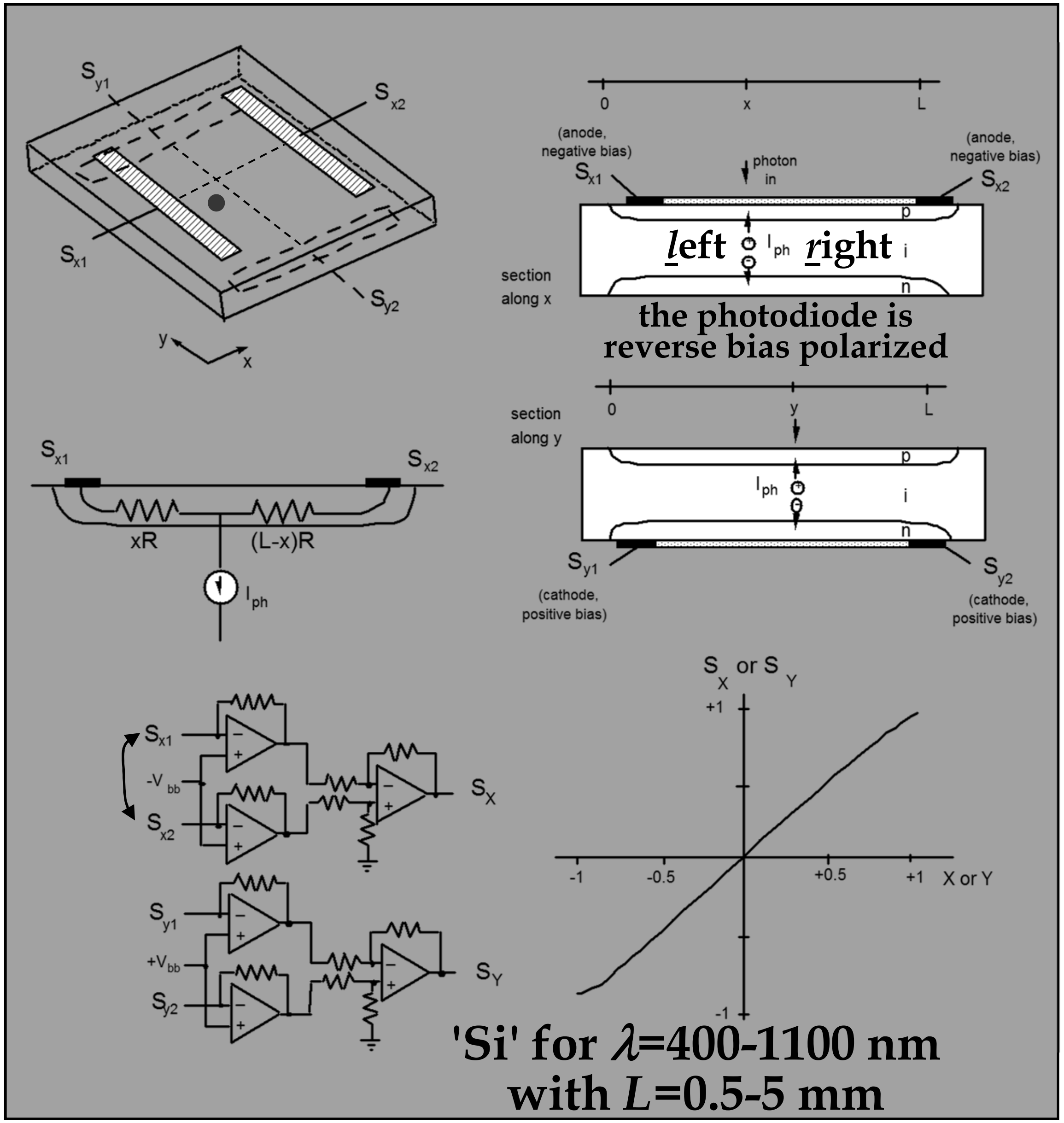
Angular position sensor with 4-Quadrant photodiode



A position sensitive detector (4-Q, PSD, reticule) other than indicator of X and Y coordinates can be used to detect angular (θ_X and θ_Y) coordinates of the arriving beam

If the sensor is placed in the focal plane of a lens, **angular coordinate** is transformed into a corresponding deflection coordinate : $X = F\boldsymbol{\theta}_{\mathbf{X}}$ and $Y = F\boldsymbol{\theta}_{\mathbf{Y}}$

PSD photodiode (scheme and principle)



High **linearity** over the whole measurement range

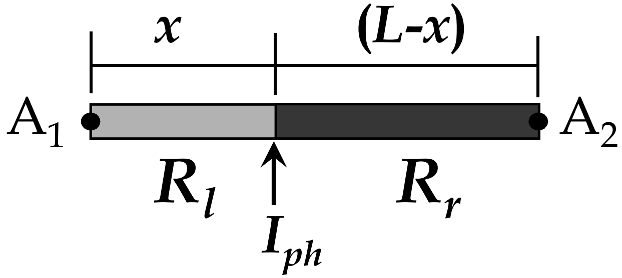
Position **S**ensitive **D**etector is a normal PIN photodiode with thin p and n regions lightly doped (to enhance the series resistance of p and n volumes at the border of the depletion region)

Incident spot of (*X*,*Y*) coordinate produces a photocurrent flowing from electrodes Y (cathode) to electrodes X (anode)

The current, passing trough regions p and n of high resistivity is divided with the partitions rule between two resistors.

The difference in the detected currents on the same electrode pairs (X or Y) gives the coordinate (*X* o *Y*)

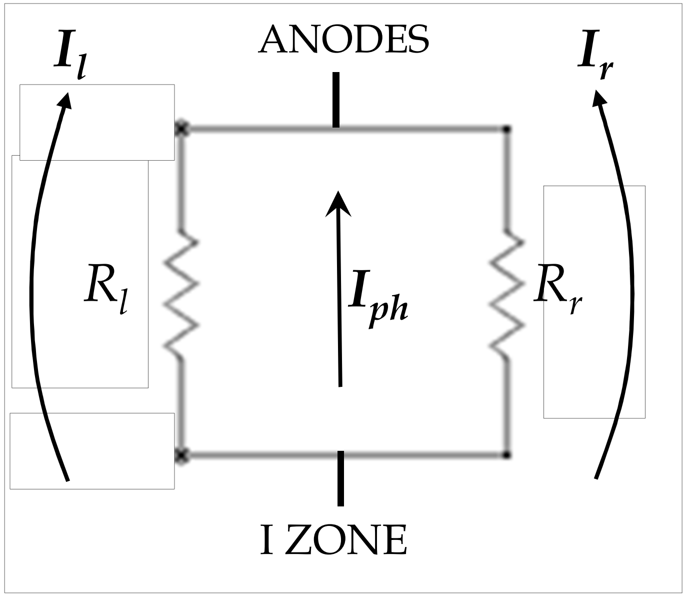
PSD photodiode (electrical model)



$R_l = x\rho^*$

$R_r = (L-x)\rho^*$

$R_l + R_r = L\rho^*$



ρ^* is the resistivity per unit length (in the slightly doped p region)

we have **current partitioning** toward the two anodes A_1 , A_2 **such as $I_{ph}=I_l+I_r$ with voltage**

$$\frac{R_l R_r}{R_l + R_r} I_{ph} = R_l I_l = R_r I_r$$

$$I_l = \frac{R_r}{R_l + R_r} I_{ph} = \frac{L-x}{L} I_{ph} \qquad I_r = \frac{R_l}{R_l + R_r} I_{ph} = \frac{x}{L} I_{ph}$$

PSD photodiode (working equations)

$$I_{x1} = I_l = \left(1 - \frac{x}{L}\right) I_{ph} \qquad I_{x2} = I_r = \frac{x}{L} I_{ph}$$

Similarly

$$I_{y1} = \left(1 - \frac{y}{L}\right) I_{ph} \qquad I_{y2} = -\frac{y}{L} I_{ph}$$

From the OP-AMP circuit we obtain

$$S_X = -R\left(I_{x1} - I_{x2}\right) = \left(\frac{2x}{L} - 1\right) R I_{ph} \propto X$$

$$S_Y = -R\left(I_{y2} - I_{y1}\right) = \left(\frac{2y}{L} - 1\right) R I_{ph} \propto Y$$

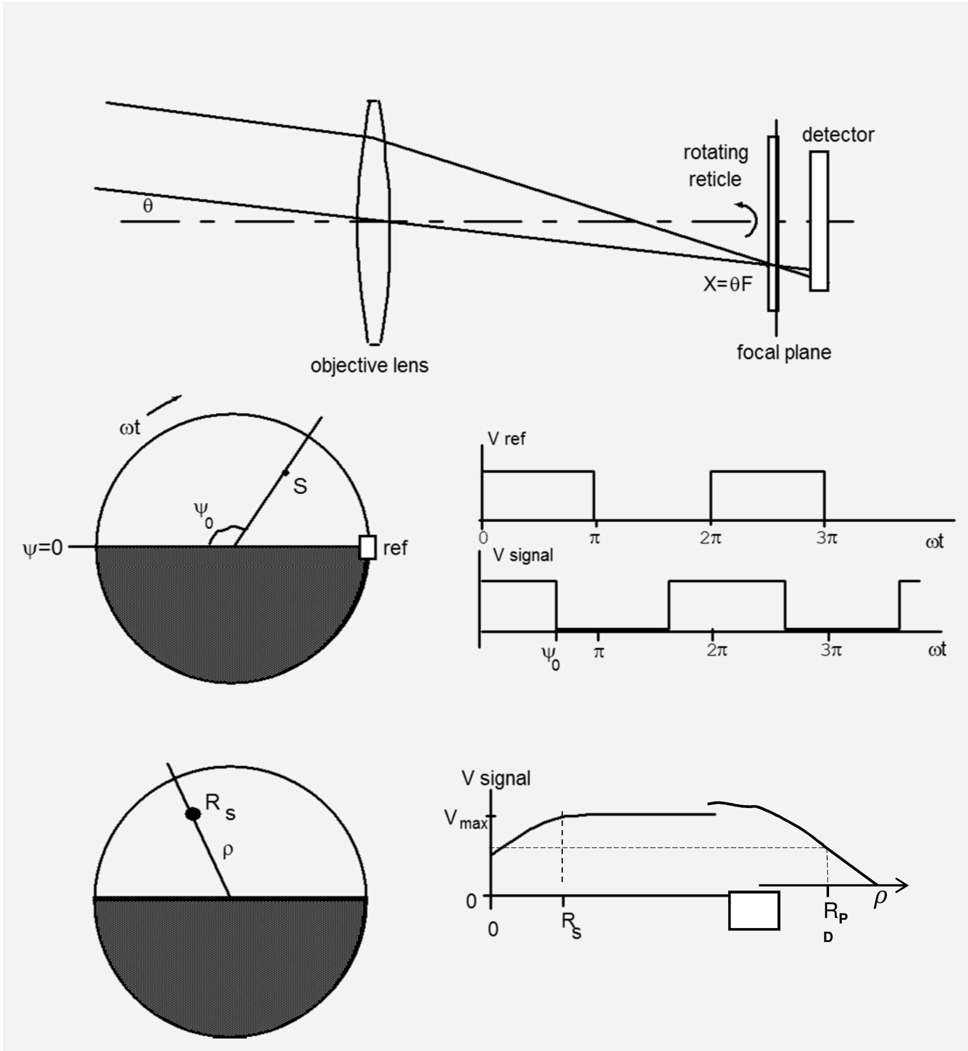
$I_{ph}=\rho P$
varying with P
(and also with ρ!)

The output becomes independent from photocurrent I_{ph} (and P) **dividing for the sum signal** $\Sigma_{X/Y}=R(I_{x/y1}+I_{x/y2})=RI_{ph}$
⇒ measurement independent from P and ≈*responsivity* (ρ)

Position Sensing with Reticules

Transformation from angular into spatial coordinate (lens focal plane) and measurement of spatial coordinate (x,y) from polar coordinates (ρ ,$\theta = \Psi_0$)

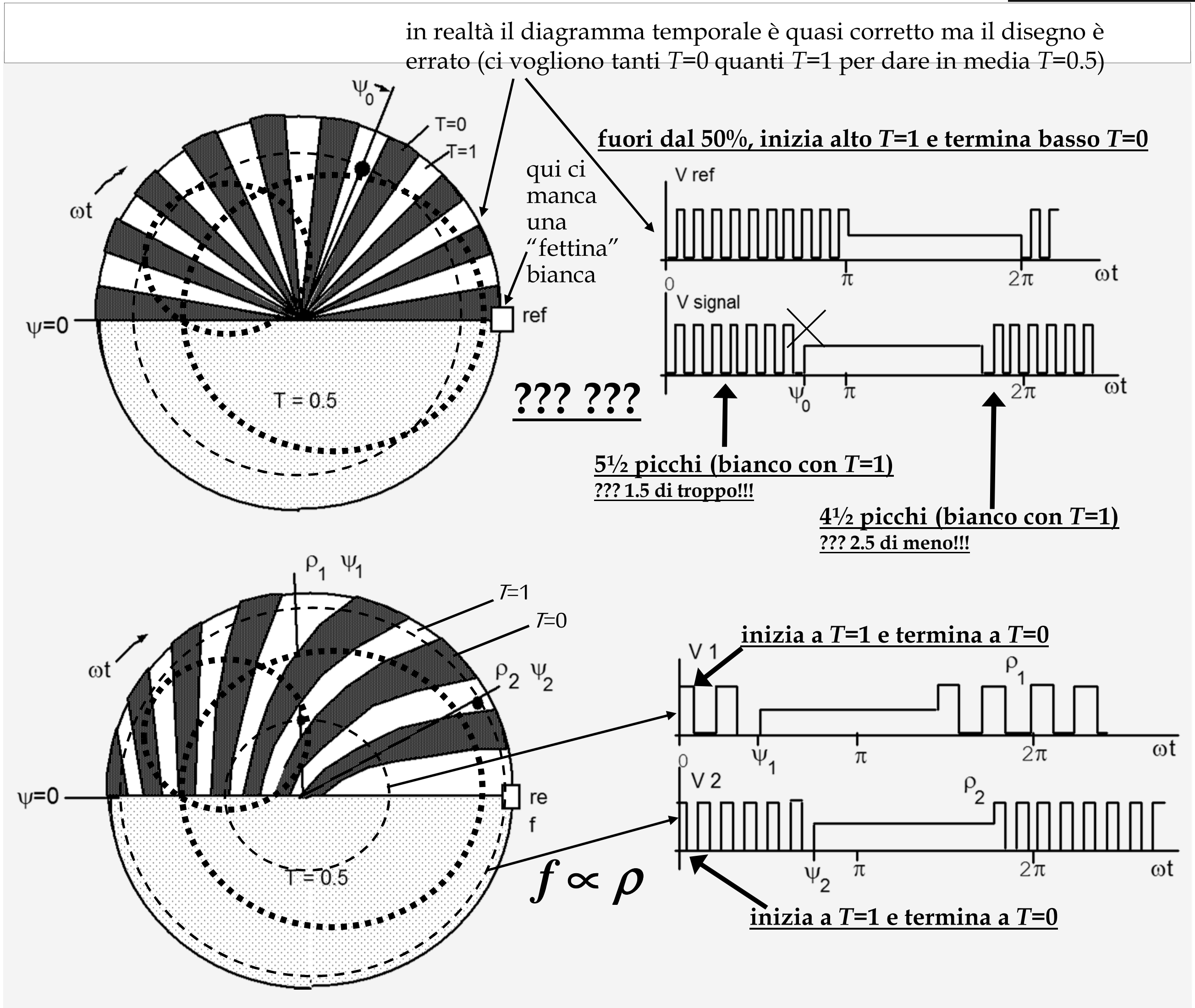
Position sensing by a rotating reticule: light from a bright spot at the angle θ is imaged by the objective lens on the focal plane, where it is chopped by the reticule placed in front of the photodetector. By comparing the **phase-shift of the square waveform from the photodetector and of a reference**, the **angular position ψ_0** of the source is determined.
The **amplitude** of the signal, V_{signal} , carries information on the **polar coordinate ρ**, similar to that of the quadrant PD.



Position Sensing with Reticules

The rising-sun (top) reticule provides a better suppression to extended sources of **disturbance** and digital counting of the angle ψ_0.

The digital readout reticule (bottom) supplies both ρ and ψ coordinates.

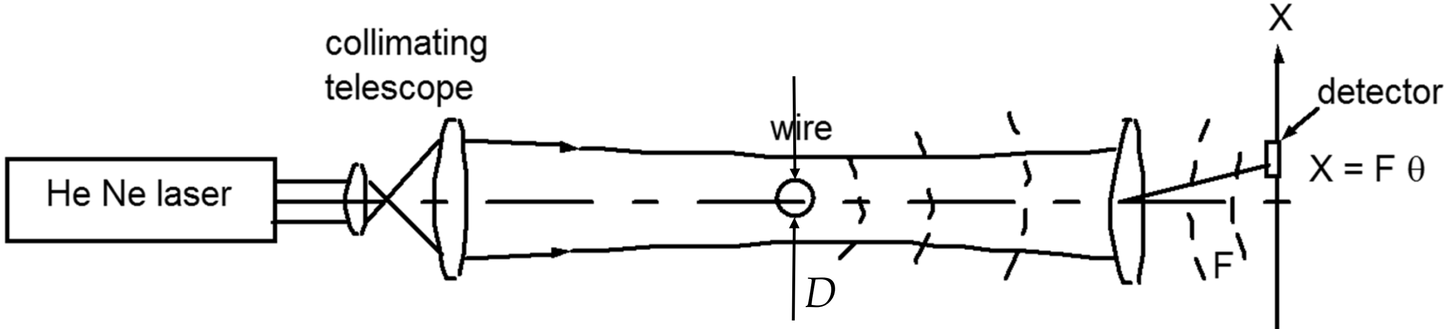


Diapositiva 23

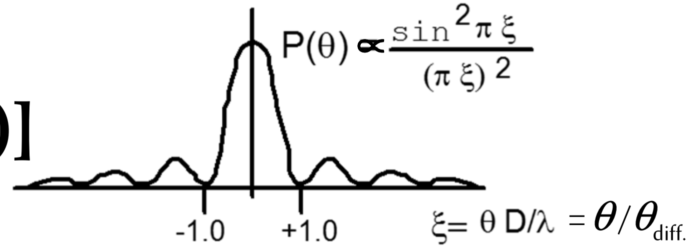
CS10 qui termina la 7a lezione dell'AA 2005/2006

Cesare Svelto; 05/04/2006

Measurements wires diameters from diffracted light analysis



electric field on the detector is the Fourier transform of the aperture:
transf. of rectangle is $\mathbf{sinc}[\pi\theta/\underbrace{(\lambda/D)}_{\theta_D}]$



angular distribution is $\boldsymbol{I(\theta) = E_0^2 / \eta_0 \cdot \mathrm{sinc}^2 \, \pi\theta/\theta_D}$

first zeroes of sinc are at $\theta_D = \theta_{diff.} = \pm\lambda/D$ e $X_{zero} = \pm F\lambda/D$

hence we can obtain $\boldsymbol{D = F\lambda/X_{zero}}$

For small wires (small D) we have X_{zero} large and vice versa

(it is easier – higher sensitivity – to measure wires with small diameter)

The distance between zeroes (or peaks) on the detector is proportional to $1/D$

CS11 qui inizia la 8a lezione dell'AA 2005/2006
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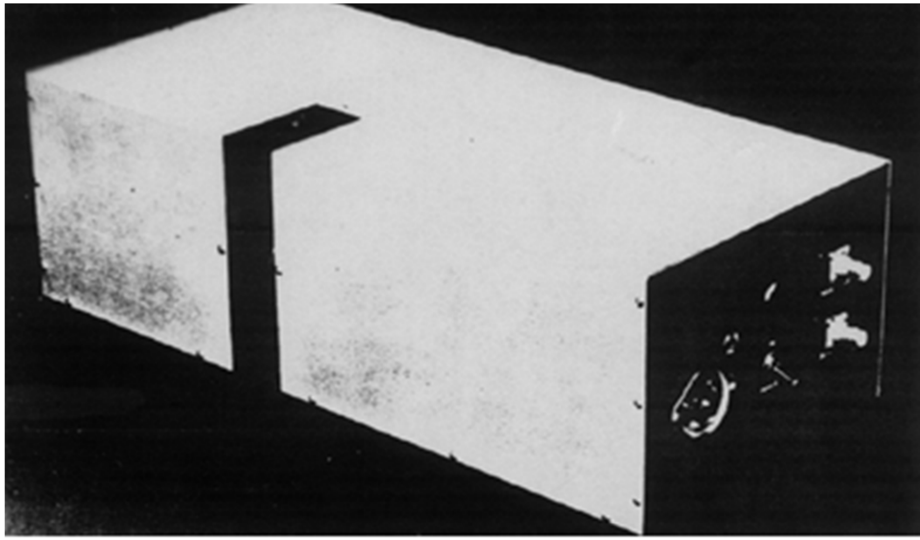
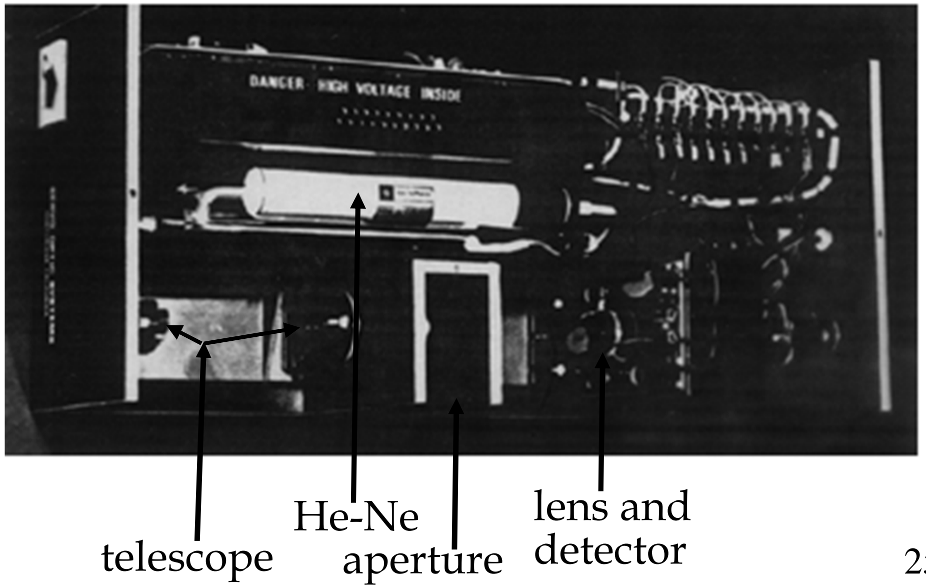
Instrument for wire diameter measurement

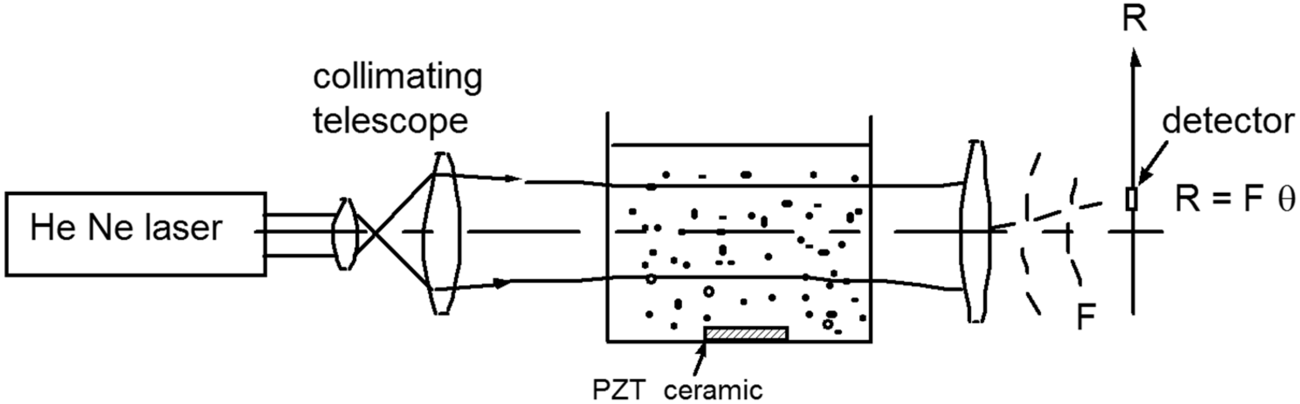
Commercial instruments for wire diameter measurement, measurable diameters can range from 10 µm (±1% acc.) up to more than 2 mm (±5% acc.)



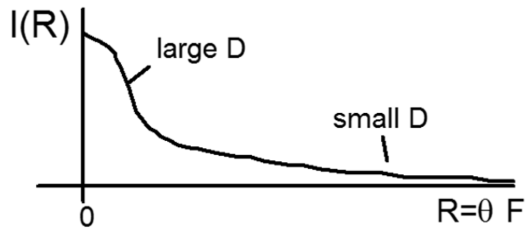
The wire is passed trough an "U“ aperture: direct monitoring during production, with online correction during the spinning process



Particle diameter measurement



LAELS: Low-Angle ELastic Scattering
electric field on the detector is the Fourier transform of the aperture:
transf. of circle is **somb[$(R/F)/(\lambda/D)$]**
[$(\theta/\theta_{diff.})$]



The diameter analyzer measures diffracted light from suspended particles within a fluid. At the cell exit a lens converts angular diffraction profile into a corresponding spatial profile in the focal plane ($\theta \rightarrow R$). A photodetector (scanned PD or CCD) **measures $I(R)$:** the **distribution of particle diameters**, $p(D)$, **is calculated inverting**

$$I(R) = I_0 \int_{0-\infty} \mathrm{somb}^2[(D/\lambda)(R/F)] \cdot p(D)\, dD \quad \text{with} (R/F)=\tan\theta \cong \theta$$

$$\mathrm{somb}(x)=2[J_1(\pi x)]/\pi x$$

PDF

Particle Size Measurement 2

Methods to solve for p(D) from measured data $I(\theta)$

- *Analytical Inversion*

$$p(D) = -[(4\pi/D)^2/\lambda] \int_{\theta=0-\infty} K(\pi D\sin\theta/\lambda)\, d[\theta^3 I(\pi D\sin\theta/\lambda)] / I_0$$

a theoretically nice result but impractical to be used.

- *Least Square Method*

Using a discrete approximation for $p(D)=p_k$ and $I(\theta)=I_n$ and letting $S_{nk}= \mathrm{somb}^2[(D_k/\lambda)\sin\theta_n]$, we get a set of equations:

$$I_n = \Sigma_{k=1..K} S_{nk}\, p_k \qquad (n=1..N)$$

N is the number of angular measurements performed on the intensity, K is the number of unknown diameters.We start with K<N and close the set adding N-K equations from the LSM condition, sought from:

$$\varepsilon^2 = \Sigma_{n=1..N} [\, I_n - \Sigma_{k=1..K} S_{nk}\, p_k]^2 = \min$$

Particle Size Measurement 3

Taking the derivative respect p_k's and equating to zero gives:

$$0 = \partial(\varepsilon^2)/\partial p_k = \Sigma_{n=1..N}\, 2[\, I_n - \Sigma_{k=1..K}\, S_{nk}\, p_k](-S_{nk})$$

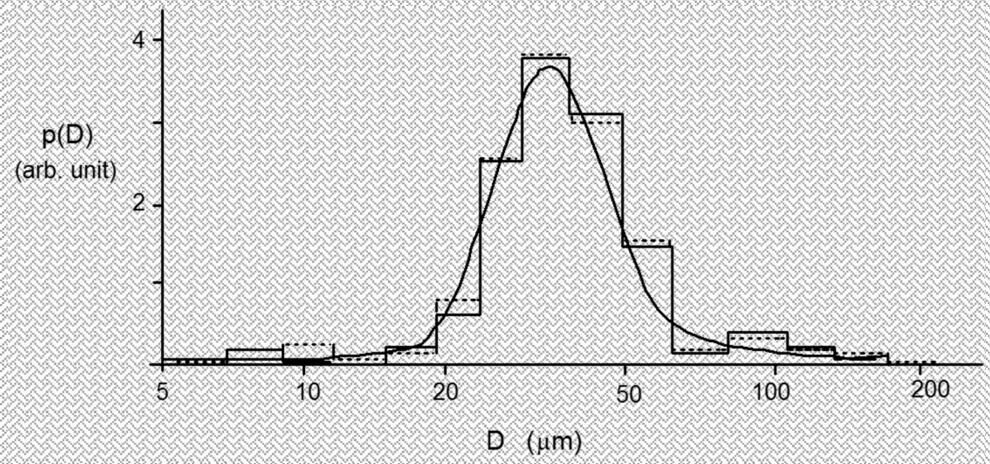
and rearranging we get

$$J_h \;=\; \Sigma_{k=1..K}\, Z_{hk}\, p_k \qquad\qquad (h=1..K)$$

where we have let $J_h = \Sigma_{n=1..N}\, I_n S_{nh}$ and $Z_{nk} = \Sigma_{n=1..N}\, S_{nk}^{\;2}$

Now, the number of equations is equal to the number of unknown and we can solve for p_k with standard algebra.

Usually, the range of diameters of interest may be large (for example, two decades from 2 to 200μm) but the number of affordable diameter is modest (e.g. K=6-9) at ±10% accuracy.



Particle Size Measurement 4 *

-*Iterative Methods.* They are based on the following approach: if the set of diameter p_k is correct, it should give the measured distribution $I_{n.calc} = \Sigma_k C_{nk}\, p_k$. If these values $I_{n.calc}$ differ from experimental values $I_{k.meas}$, then we may expect to approach the solution if we multiply p_k by $I_{k.meas}/I_{k.calc}$.

Using $p_{k+1} = I_{k.meas}/I_{k.calc}\; p_k$ and repeating an adequate number of times, p_k should converge to the correct solution (there is no clear sign of convergence, however)

A refinement of Chahine's method consists in weighting the iteration by the normalized kernel, $S_{nk}/\, \Sigma_{n=1..N} S_{nk}$, using

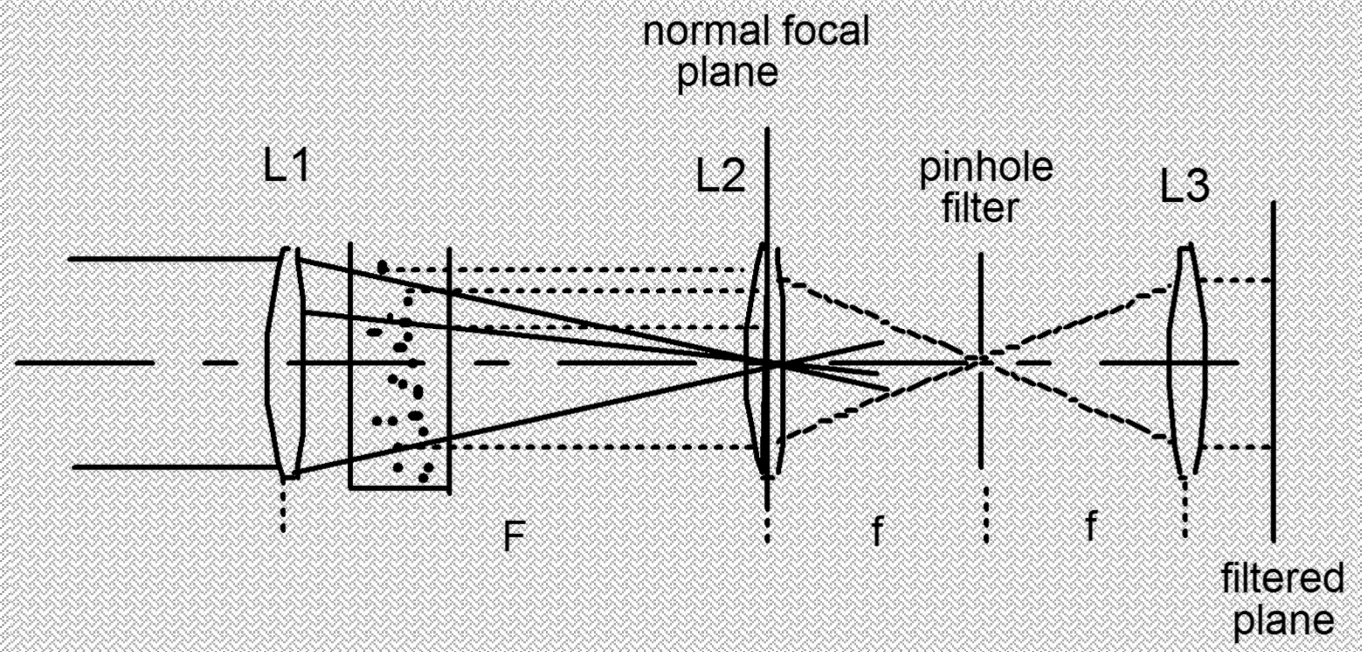
$$p_{k+1} = (S_{nk}/\Sigma_{n=1..N} S_{nk})(I_{k.meas}/I_{k.calc})\; p_k$$

In this way, spurious peaks found in Chahine's method are suppressed, and resolution and dynamic range are improved

Particle Size Measurement 5 *

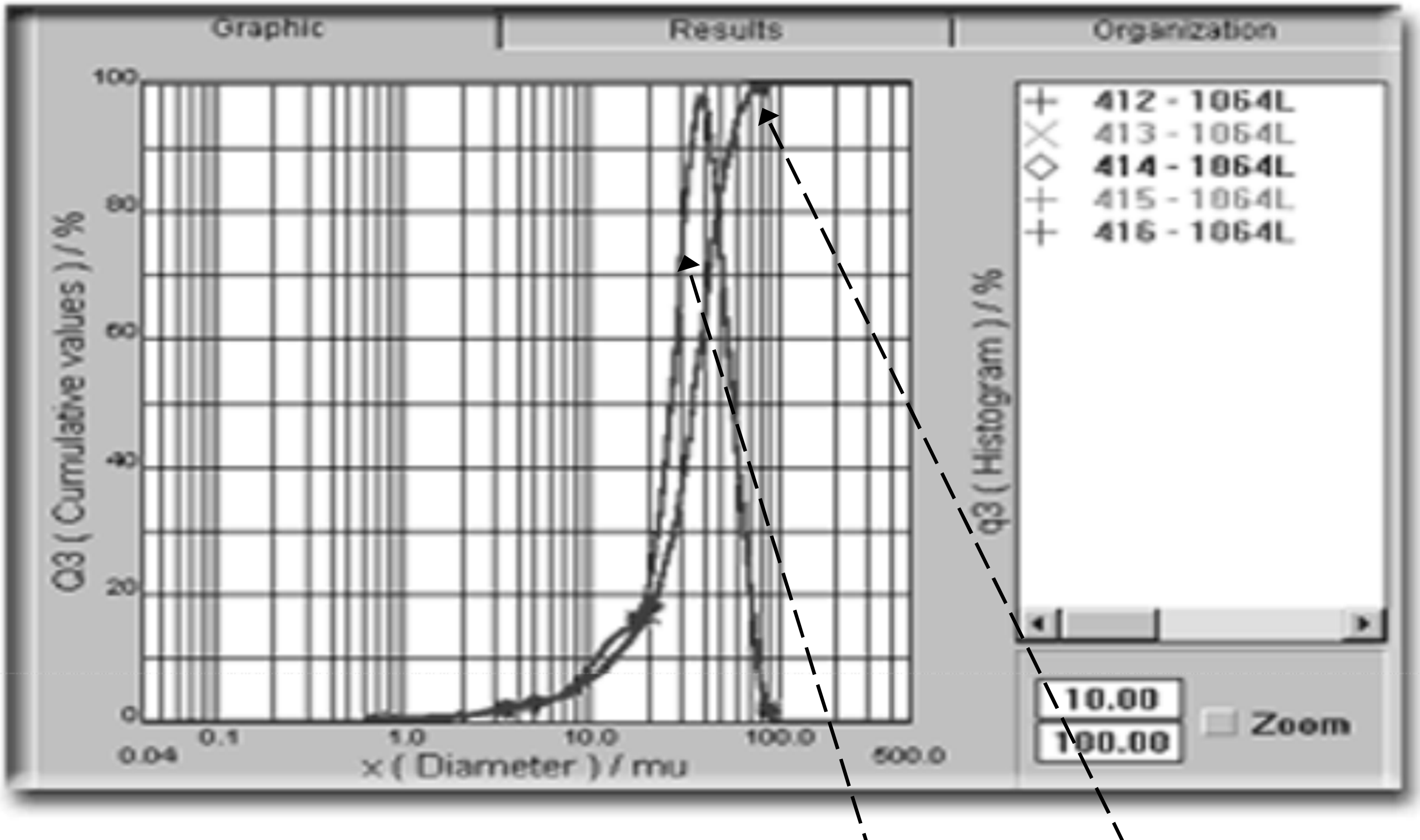
Common errors in the PSM: *finite size* of detector, *beam waist* effects, *lens vignetting*, and *undiffracted beam* (θ=0), important for small θ (large D).

Better than a stop to block it out, we can use the filtering known as reverse Fourier-transform illumination, with a convergent beam to illuminate the cell. Diffracted rays (dotted lines) are focused on axis, and pass through the pinhole, whereas undiffracted rays arrive out-of-axis and are blocked.



Particle Size Measurement 6

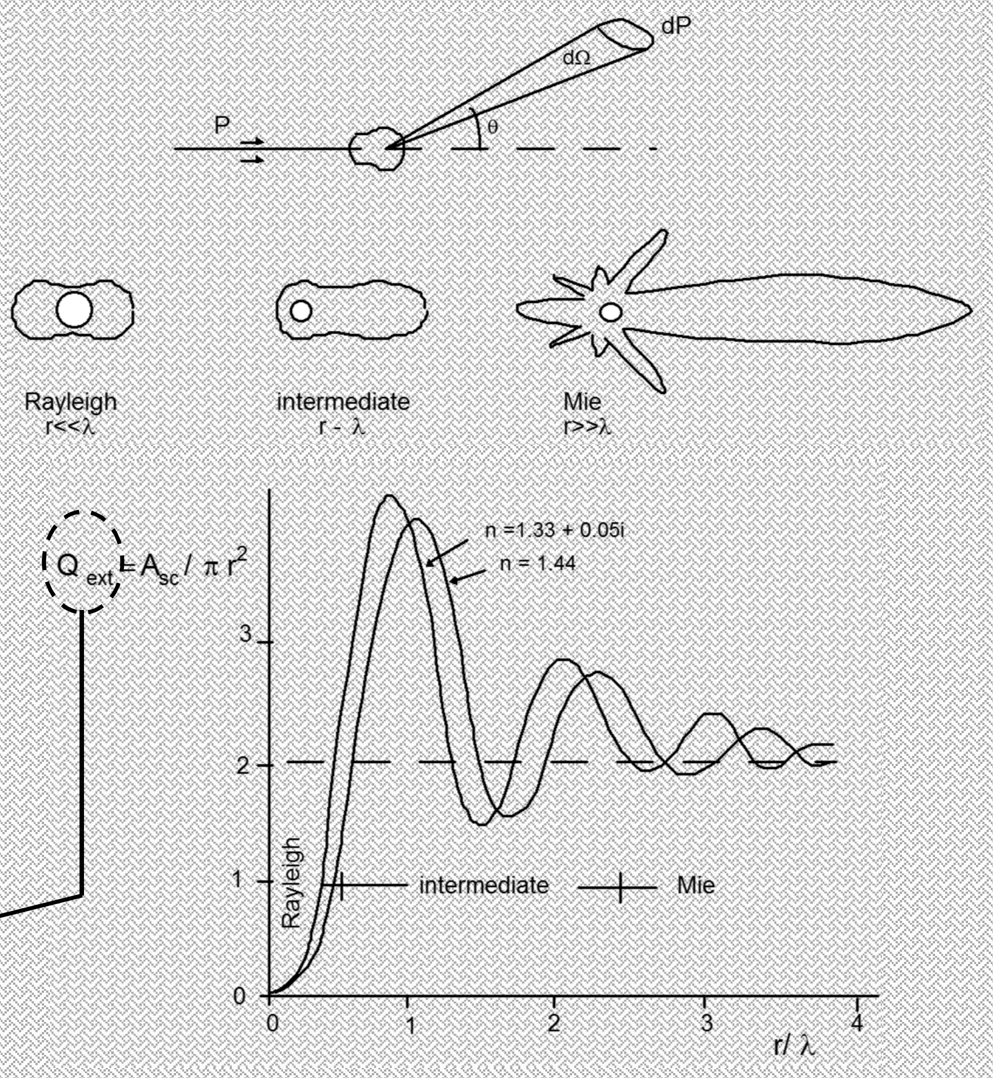
A modern, general purpose particle-size analyzer based on diffraction performs diameter measurements from 1 to 2500 µm [by CILAS, France]



An example of an-easy-to-get **particle size pdf** $p(D)$ and **cdf** $P(D)$ distribution measured by a commercial instrument (courtesy of CILAS)

Particle Size Measurement 8 *

- In the **Rayleigh** regime $r \ll \lambda$, the scattering is nearly **isotropic** in angle and the extinction factor $\boldsymbol{Q_{ext}}$ varies as $\boldsymbol{(r/\lambda)^4}$.
- When r increases up to about $r \approx \lambda$, (**intermediate** regime) the scattering function $f(\theta)$ is mainly **forward** and the extinction factor increases up to $Q_{ext} \approx 2\text{-}4$.
- For $r \gg \lambda$ we enter in the **Mie** regime, extinction Q_{ext} is nearly constant (in λ) at ≈2 and $f(\theta)$ is strongly peaked **forward**



$\boldsymbol{Q_{ext}}$ tells how much the light ***extinction cross section*** (due to *scattering*) is larger than the **physical area** (πr^2) of the diffusing **particle**

Particle Size Measurement 9 *

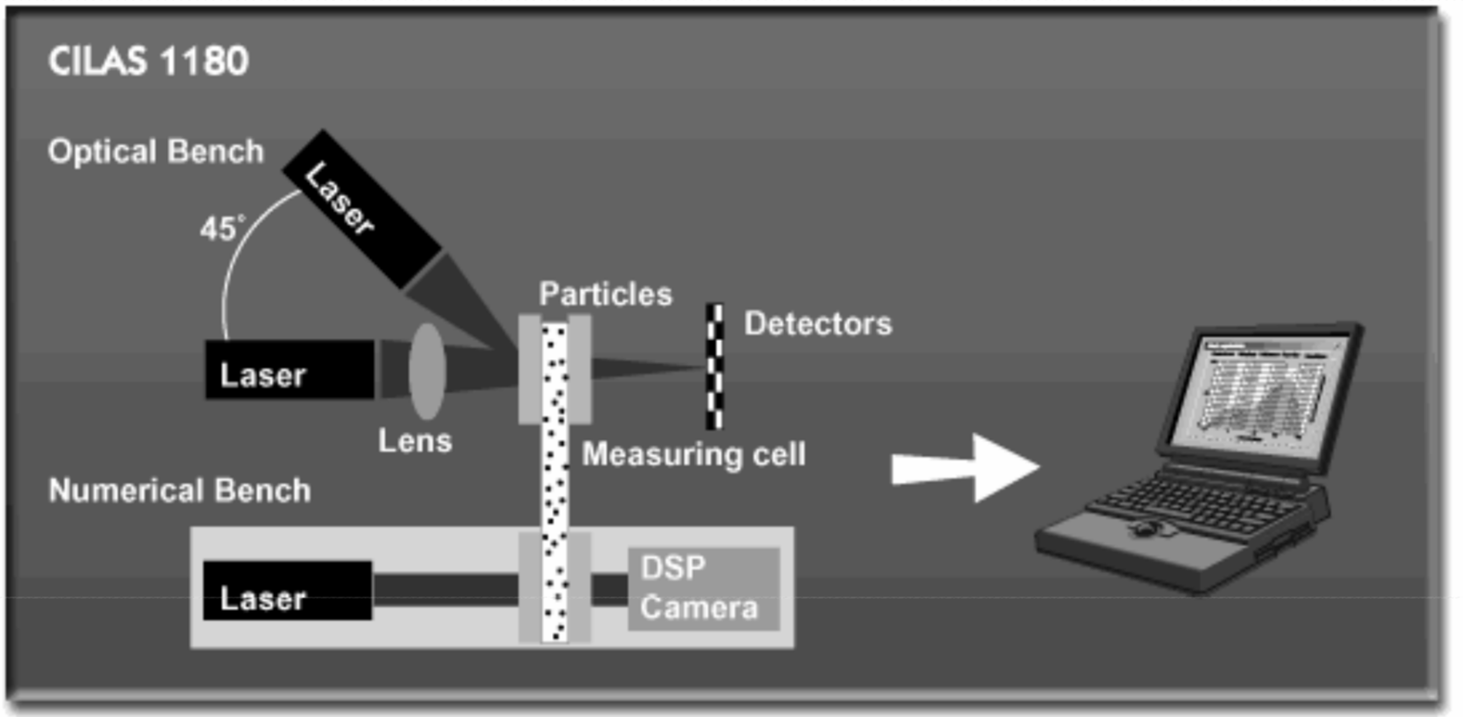
• Another method is **SEAS** (***Spectral Extinction*** *Aerosol Sizing*). It is based on measuring light scattered from the cell **at a fixed angle**, while scanning λ instead of θ. By varying the ratio D/λ, the extinction factor $Q_{ext}(D,\lambda,n)$ varies and the scattered power too, according to:

$$I(\lambda,45°) = f(45°)\ (\Delta\Omega/4\pi)\ I_0 \int_{0-\infty} Q_{ext}(D,\lambda,n)\ p(D)\ dD$$

where $f(\theta)$=scattering function, Q_{ext}=extinction factor. The equation is the counterpart of that for extinction-related measurement, and all the methods of inversion of the Fredholm's integral can now be applied on D_k and λ_n. With **SEAS we may to go down to 0.02–0.1µm** as the minimum measurable size, overlapping with the LAELS low-range (≈ 2-5 µm).

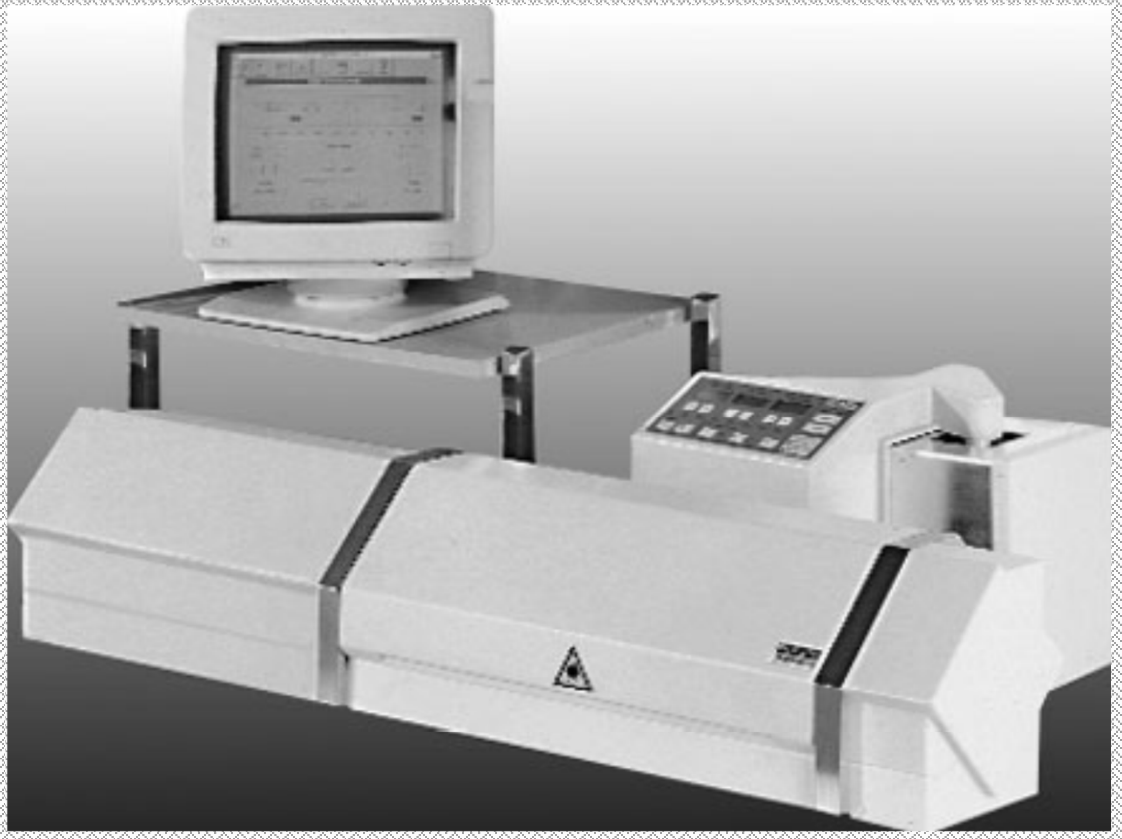
• A last method is the ***Dynamical Scattering*** *Size Analyzer* (**DSSA**), useful **for very small (1..100-nm) particles**. Based on the frequency shift due to Doppler effect $(\underline{k}_o-\underline{k}_i)\cdot\underline{v}$, it is measured by the time-domain autocorrelation function $C(\tau)=(1/T)\int_{0-T} i(t)i(t+\tau)dt$ which depends from the diffusion constant δ of particles according to: $C(\tau)=C_0 \exp -\delta(k_o-k_i)^2\tau$.

Particle Size Measurement 10



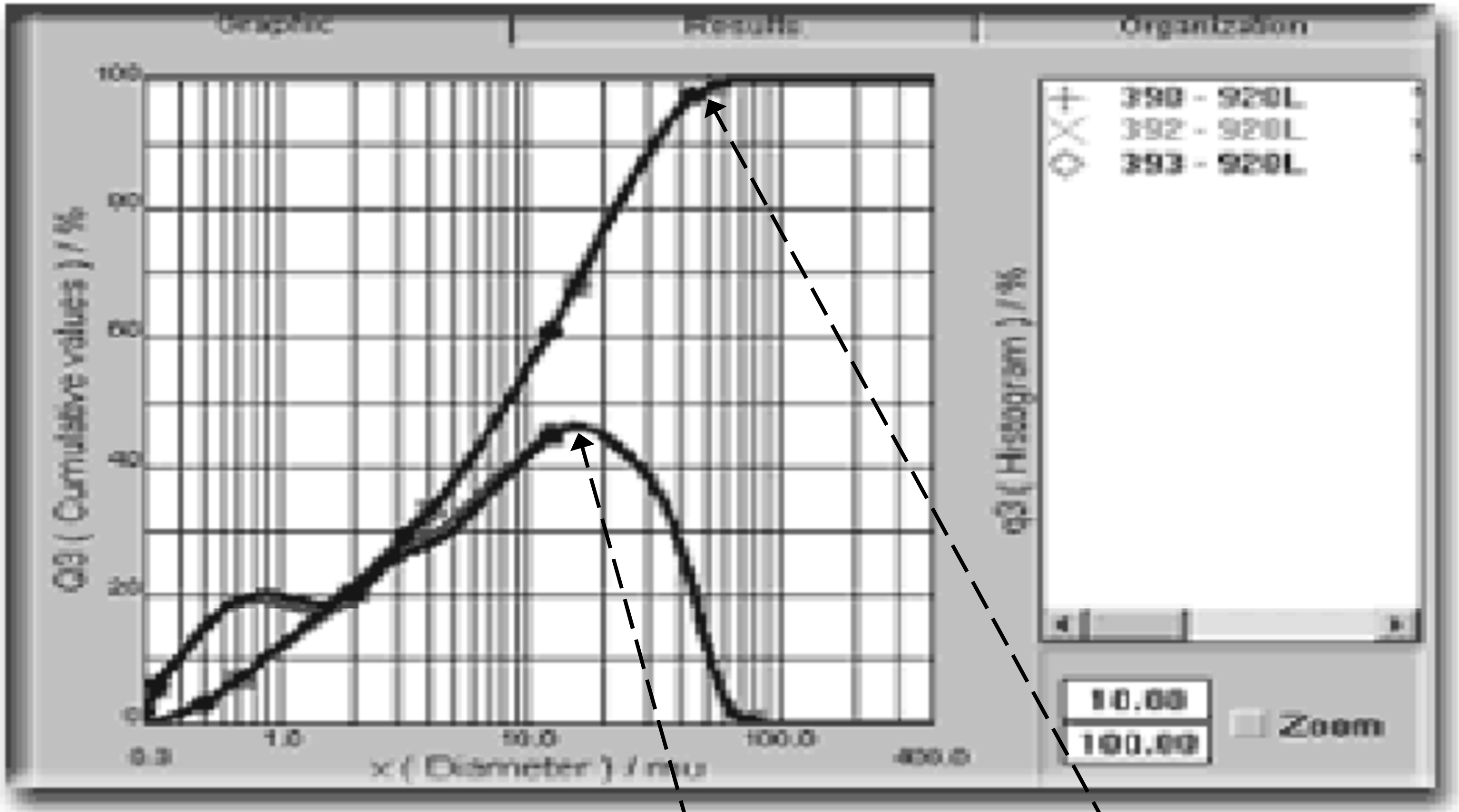
A SEAS particle size analyzer uses scattering data at 135° to sort submicronic particulate (typ. of metropolitan area pollutants) from 0.02 to 0.1 µm [by CILAS, France]

Particle Size Measurement 11

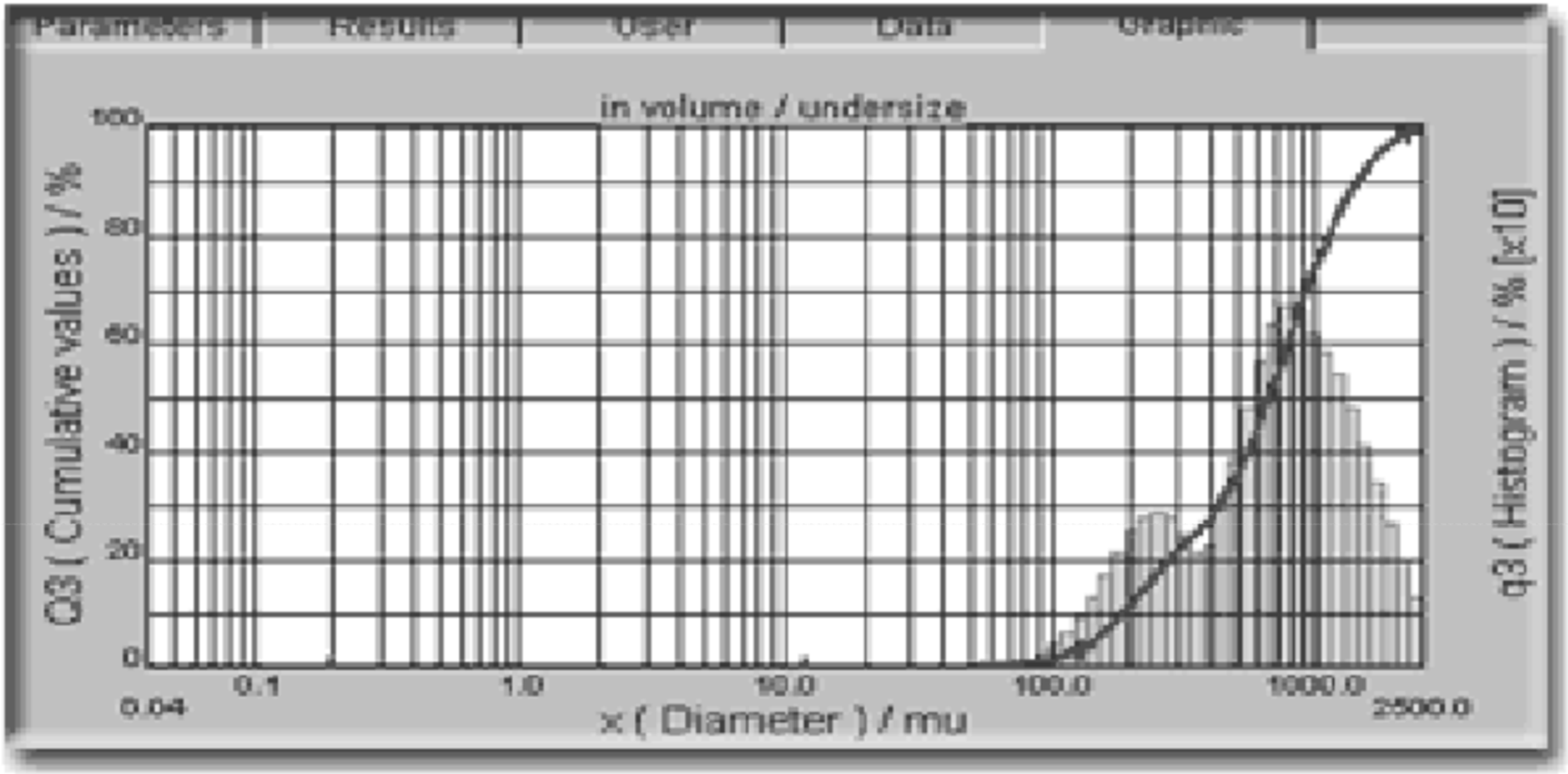


A modern particle-size analyzer based on diffraction and extinction (LAELS + SEAS), performs diameter measurements from 0.05 to 2500 μm [by CILAS, France]

Particle Size Measurement 12



A second example of particle size pdf $p(D)$ and cumulative $P(D)$ of a bi-modal distribution, more difficult because with both small and large particles, as measured by a commercial instrument (courtesy of CILAS)



A third example of particle size pdf $p(D)$ and cumulative $P(D)$ of a distribution with two populations of very large particles (powders) as measured by a commercial granulometer (courtesy of CILAS)