Effective Number of Bits

Sistemi di Acquisizione Dati
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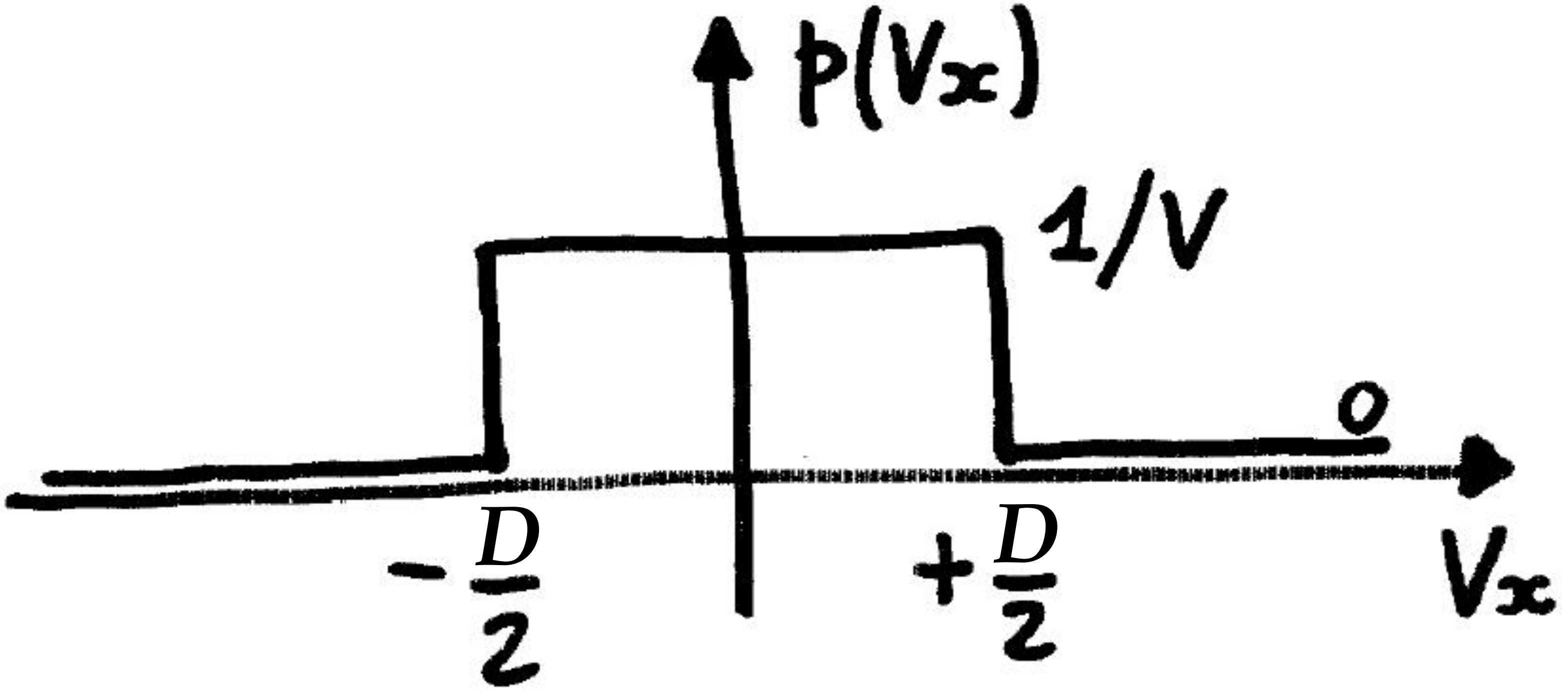
Effective number of bits (1/7)

signal $s(t) = V_x \in [-D/2, +D/2]$



The signal dynamic is
$D = \pm D/2$

$$\Rightarrow \qquad \sigma_s^2 = \frac{D^2}{12}$$

Effective number of bits (2/7)

Using a converter / voltmeter that quantizes the signal $s(t)$ of n bits, it will be

STEP QUANTIZATION $$Q = \Delta V = \frac{D}{2^n}$$

[D **is the dynamic** of the voltmeter] and therefore it has a variance ("uncertainty of quantization")

$$\sigma_q^2 = \frac{Q^2}{12} = \frac{1}{12}\left(\frac{D}{2^n}\right)^2 = \frac{\sigma_s^2}{2^{2n}}$$

$$\Rightarrow \frac{\sigma_s^2}{\sigma_q^2} = constant = 2^{2n} \quad = \frac{S}{N}$$

For an ideal quantizer

Effective number of bits (3/7)

In an ideal converter

$$n = \frac{1}{2}\log_2\left(\frac{\sigma_s^2}{\sigma_q^2}\right)$$

$$S = \sigma_s^2$$

$$N = N_q = \sigma_q^2$$

$$\rightarrow \boxed{n = \frac{1}{2}\log_2\left(\frac{S}{N_q}\right) \qquad \textbf{“ideal” case}}$$

In a real converter

$$N = N_q + \overset{\text{added noise}}{\underleftrightarrow{N_{A/D} + N_{ext}}} > N_q$$

$$\sigma_c^2 = \sigma_q^2 + \underset{\text{real converter}}{\sigma_{n,A/D}^2} + \underset{\text{external noise}}{\sigma_{n,ext}^2} > \sigma_q^2$$

Effective number of bits (4 / 7)

It is defined **the effective number of bits** as

$$n_e \equiv \frac{1}{2}\log_2\left(\frac{\sigma_s^2}{\sigma_c^2}\right) < n \quad \rightarrow \quad \boxed{n_e = \frac{1}{2}\log_2\left(\frac{S}{N}\right) \quad \textbf{real case}}$$

converter noise

$$n_e = \frac{1}{2}\log_2\left(\frac{\sigma_s^2}{\sigma_q^2}\frac{\sigma_q^2}{\sigma_c^2}\right) = \frac{1}{2}\log_2\left(\frac{\sigma_s^2}{\sigma_q^2}\right) + \frac{1}{2}\log_2\left(\frac{\sigma_q^2}{\sigma_c^2}\right) =$$

added noise

$$= n - \frac{1}{2}\log_2\left(\frac{\sigma_c^2}{\sigma_q^2}\right) = n - \frac{1}{2}\log_2\left(1 + \frac{\sigma_{n,A/D}^2 + \sigma_{n,ext}^2}{\sigma_q^2}\right)$$

Effective number of bits (5/7)

added noise

$$\frac{\overset{\sigma_n^2\text{ added noise}}{\overleftrightarrow{\sigma^2_{\mathrm{n,A/D}}+\sigma^2_{\mathrm{n,ext}}}}}{\sigma^2_{\mathrm{q}}}=2^{2n}\frac{N}{S}\qquad\qquad \sigma^2_{\mathrm{q}}=\frac{\sigma^2_{\mathrm{s}}}{2^{2n}}$$

There are then two limiting conditions

“high” $\frac{S}{N} >> 2^{2n}$ $\qquad n_{\mathrm{e}} \cong n$

“low” $\frac{S}{N} << 2^{2n}$

$$n_{\mathrm{e}} \cong n-\frac{1}{2}\log_2\left(2^{2n}\right)-\frac{1}{2}\log_2\left(\frac{\sigma^2_{\mathrm{n}}}{\sigma^2_{s}}\right)=$$

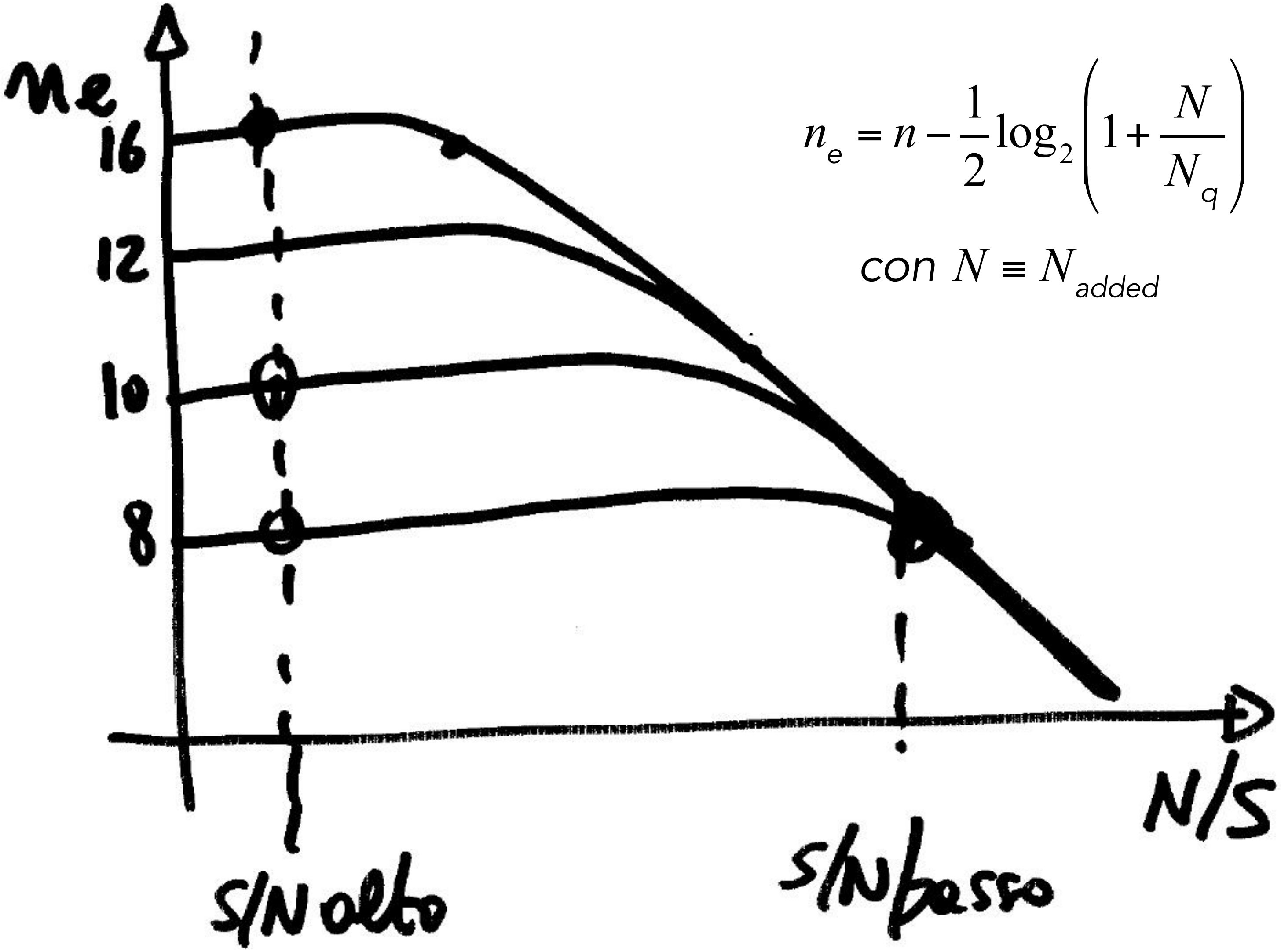
$$=\frac{1}{2}\log_2\left(\frac{\sigma^2_{\mathrm{s}}}{\sigma^2_{\mathrm{n}}}\right)=\frac{1}{2}\log_2\left(\frac{S}{N}\right)$$

It loses 1 bit for each -6 dB decrease of S/N (factor 4)

Effective number of bits (6 / 7)

$$n_e = n - \frac{1}{2}\log_2\left(1 + \frac{N}{N_q}\right)$$

con $N \equiv N_{added}$



Effective number of bits (7/7)

Losing a factor 4 = 6 dB of S/N it is lost 1 bit.
For each factor 2 = 3 dB, always lost in S/N, you lose instead ½ bit.

If instead of losing in S/N, S/N is gained (S/N increases), then it is gained the same increment in effective bit (**+0.5 bit every ×2 in S/N**)

Exercise: in the area of "low" S/N, changing from $(S/N)_1$ to $(S/N)_2$ and 30 dB are lost. How do the equivalent bits change?

$$\text{if} \quad \frac{(S/N)_2}{(S/N)_1} = -30\ \text{dB} = \frac{1}{1000}$$

$$n_{e,1} = \frac{1}{2}\log_2\left(\frac{S}{N}\right)_1 \qquad n_{e,2} = \frac{1}{2}\log_2\left(\frac{S}{N}\right)_2$$

$$n_{e,2} = n_{e,1} + \frac{1}{2}\log_2\frac{(S/N)_2}{(S/N)_1} = n_{e,1} - \frac{10}{2} = n_{e,1} - 5\ \text{bit}$$