1

GRANDEZZE EL. e MAGN. MISURABILI

in C.C. ; V, I, R, P, E, H

(non solo El.)

in a.c : usando un opportuno circuito equiv

sono misurabili V, I, Z, P, E, H

R, L, C

in r.f. e μW : possono esistere V, I, Z in qualche caso

esistono sempre E, H, P

e al posto di Z sempre coeff. di rifless. e trasm

T e F sempre misurabili, ma non specificamente elettriche anche se - - - -

2

GIUNZIONI

in cc e in a.c : circuiti elettrici

in r.f e μw : ~~circuiti~~ $\longrightarrow$ GIUNZIONI

per ogni modo di propagazione

$$\vec{E}_t = v(z,t)\ \vec{e}(x,y)$$
$$\vec{H}_t = i(z,t)\ \vec{h}(x,y)$$

da MAXWELL
EQUAZ. ONDE
$e^{j\omega t}$

v e i (complesse) : tensione e corrente generali

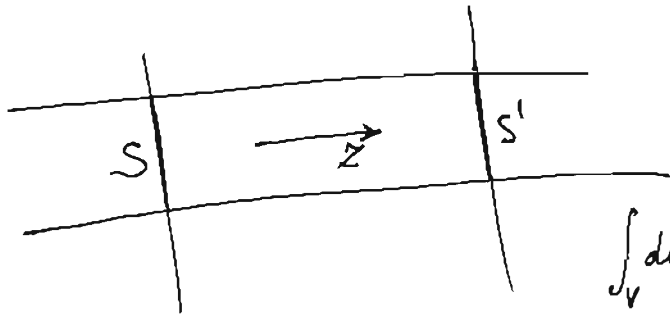
$\vec{e}$ e $\vec{h}$ (vettori reali) : mappa del c.e.m. trasversale

Arbitrarietà nella separazione tra v e $\vec{e}$ e tra i e $\vec{h}$

3

v e i — proprietà simili a tens. e co
— dimensioni di tensione e corrent

attraverso normalizzazioni di
potenze e di impedenze



Poynting

$$\int_V \mathrm{div}\,(\vec{E}_t \times \vec{H}_t^*)\, dV =$$
$$\int_{S+S'} \vec{E}_t \times \vec{H}_t^* \cdot d\vec{s} = -\frac{d}{dt}$$

$$P = \frac{1}{2}\,\mathrm{Re}\left[\int_S (\vec{E}_t \times \vec{H}_t^* \cdot \vec{u}_z)\, ds\right] = \frac{1}{2}\,\mathrm{Re}\,(v\, i^*)\underbrace{\int_S (\vec{e} \times \vec{h} \cdot \vec{u}_z)\, ds}_{W_0}$$

cost. adimens.

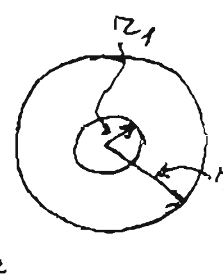
$$\begin{cases} \vec{E}_t = \vec{E}_t^+ + \vec{E}_t^- & \vec{H}_t = \vec{H}_t^+ + \vec{H}_t^- \\ v = v^+ + v^- & i = i^+ + i^- \end{cases}$$

$$Z_W = \frac{E_t^+}{H_t^+} = \frac{v^+}{i^+}\,\frac{e(x,y)}{h(x,y)} = Z_0\, z_w$$

Z_0 in Ω
z_w adim.

impedenza d'onda.

$$Z_{W(TEM)} = \sqrt{\frac{\mu}{\varepsilon}}$$



Scelto $W_0 = 1$

$$Z_{0(TEM)} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\,\ln\frac{r_2}{r_1}$$

IN ARMONIA CON L'EQUAZIONE DELLE ONDE

$\vec{E}^+, \vec{E}^-, \vec{H}^+, \vec{H}^-, \quad v^+, v^-, i^+, i^-$

SI INTRODUCONO DELLE <u>ONDE VIAGGIANTI</u>

$$\frac{v^+}{\sqrt{Z_0}} = \underset{(+z)}{a} \quad e \quad \underset{(-z)}{b} = \frac{v^-}{\sqrt{Z_0}}$$

$$\frac{v}{\sqrt{Z_0}} = a + b \qquad [V/\sqrt{\Omega}] \qquad \frac{v^+ + v^-}{\sqrt{Z_0}}$$

$$\sqrt{Z_0}\, i = a - b \qquad \sqrt{Z_0}(i^+ - i^-)$$

$$a = \frac{1}{2}\left(\frac{v}{\sqrt{Z_0}} + \sqrt{Z_0}\, i\right)$$

$$b = \frac{1}{2}\left(\frac{v}{\sqrt{Z_0}} - \sqrt{Z_0}\, i\right)$$

a e b <u>non</u> ha
significato fisico
(in questa <u>defin.</u> sim

ma .. $P = \underset{\substack{\uparrow \\ 1}}{W_0}\ \frac{1}{2} \operatorname{Re}(v\, i^*) = \frac{1}{2}\left(|a|^2 - |b|^2\right)$

quindi $P(+z) = \frac{1}{2}|a|^2$ e $P(-z) = \frac{1}{2}|b|^2$

$$\frac{b}{a} = \frac{v - Z_0 i}{v + Z_0 i} = \frac{Z - Z_0}{Z + Z_0} = \text{coeff. di riflessione } (Z =$$

$$DEF: \quad Z_0 = \frac{V^+}{I^+} = \frac{V^-}{I^-} \quad \Gamma = \frac{V^-}{V^+}$$

$$Z = \frac{V}{I} \qquad V = V^+ + V^-$$

$$I = I^+ - I^-$$ UNICO ATTO DI FEDE

$$Z = \frac{V^+ + V^-}{I^+ - I^-} = Z_0 \frac{V^+ + V^-}{V^+ - V^-} = Z_0 \frac{1 + \frac{V^-}{V^+}}{1 - \frac{V^-}{V^+}} = Z_0 \frac{1+}{1-}$$

$$(1 - \Gamma) Z = (1 + \Gamma) Z_0$$

$$Z - \Gamma Z = Z_0 + \Gamma Z_0 \Rightarrow \boxed{\Gamma = \frac{Z - Z_0}{Z + Z_0}}$$

⑤ CIRCUITI LINEARI A n PORTE

$$v_1 = Z_{11} i_1 + Z_{12} i_2 + \dots Z_{1n} i_n$$

$$\dots$$

$$v_i = Z_{i1} i_1 + Z_{i2} i_2 + \dots Z_{in} i_n$$

$$\dots$$

$$v_n = Z_{n1} i_1 + Z_{n2} i_2 + \dots Z_{nn} i_n$$

$$v = Z\, i \qquad\qquad i = Y v$$

matrici colonne

matrice quadrata

Se nel sistema a n coppie di morsetti sono presenti solo R, L, M, C vale la

<u>reciprocità</u>

$$Z_{ik} = Z_{ki} \quad ; \quad Y_{ik} = Y_{ki}$$

sostituendo in v_k e i_k le corrispondenti a_k e b_k si ottiene la relazione lineare che introduce

$$b_1 = S_{11} a_1 + S_{12} a_2 + \dots S_{1n} a_n$$

la matrice di diffus (scattering) S

$$b_2 = S_{21} a_1 + S_{22} a_2 + \dots S_{2n} a_n$$

$$\dots$$

$$b_n = S_{n1} a_1 + S_{n2} a_2 + \dots S_{nn} a_n$$

(6) a_k sono onde entranti o incidenti sulla giunzione

b_k sono onde uscenti

Significato fisico:

$i \neq k$ $\quad b_i = S_{ik} a_k \quad$ se $a_1 = a_2 = \dots a_{k-1} = a_{k+1} = \dots a_n$

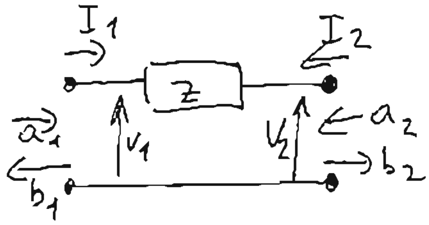
$S_{ik} = b_i / a_k$ coeff. di trasmissione dalla porta k alla porta i quando tutte le a_i per $i \neq k$ sono nulle (chiusura su terminazioni adattate

$i = k$ $\quad b_k = S_{kk} a_k \quad$ se $a_1 = a_2 = a_{k-1} = a_{k+1} = a_n = 0$

$S_{kk} = \dfrac{b_k}{a_k}$ coeffic. di riflessione Γ

alla porta k quando a tutte e le altre porte $i \neq k$ le $a_i = 0$ (chiusura su terminazioni adattate)

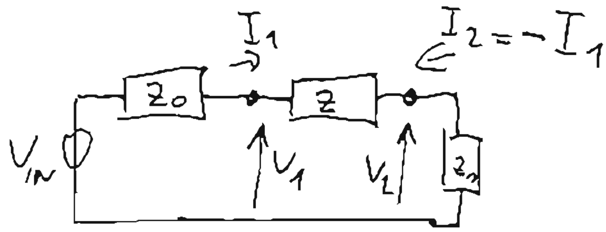
CALCOLO DI [S] PER Z-SERIE



$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \Rightarrow$$



$$V_1 = \frac{Z+Z_0}{Z+2Z_0} V_{IN} \qquad V_2 = \frac{Z_0}{Z+2Z_0} V_{IN}$$

$$I_1 = \frac{V_{IN}}{Z+2Z_0} = -I_2$$

$$a_1 = \frac{1}{2}\left(\frac{V_1}{\sqrt{Z_0}} + \sqrt{Z_0} I_1\right)$$

$$b_1 = \frac{1}{2}\left(\frac{V_1}{\sqrt{Z_0}} - \sqrt{Z_0} I_1\right)$$

$$\Rightarrow a_1 = \frac{1}{2}\left[\frac{Z+Z_0}{Z+2Z_0} \cdot \frac{V_{IN}}{\sqrt{Z_0}} + \sqrt{Z_0} \cdot \frac{V_{IN}}{Z+2Z_0}\right] = \frac{V_{IN}}{2\sqrt{Z_0}} \; \frac{Z+2Z_0}{Z+2Z_0} = \frac{V_{IN}}{2\sqrt{Z_0}}$$

$$b_1 = \frac{1}{2}\left[\frac{Z+Z_0}{Z+2Z_0} \frac{V_{IN}}{\sqrt{Z_0}} - \sqrt{Z_0} \frac{V_{IN}}{Z+2Z_0}\right] = \frac{V_{IN}}{2\sqrt{Z_0}} \; \frac{Z}{Z+2Z_0}$$

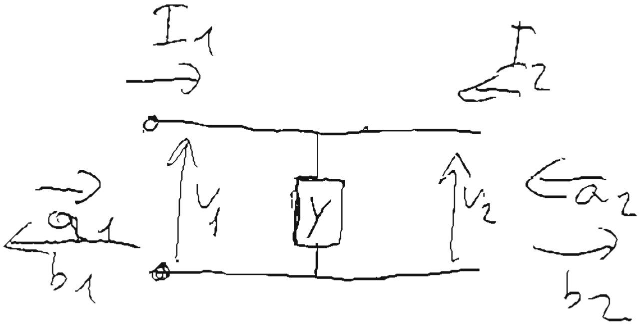
$$\Rightarrow S_{11} = \frac{b_1}{a_1} = \frac{Z}{Z+2Z_0} = \Gamma_1 = \frac{Z_{IN}-Z_0}{Z_{IN}+Z_0} \text{ con } Z_{IN} = Z+Z_0$$

$$b_2 = \frac{1}{2}\left(\frac{V_2}{\sqrt{Z_0}} - \sqrt{Z_0} I_2\right) = \frac{1}{2}\left[\frac{V_{IN}}{\sqrt{Z_0}} \frac{Z_0}{Z+2Z_0} - \sqrt{Z_0} \cdot \left(-\frac{V_{IN}}{Z+2Z_0}\right)\right] =$$

$$= \frac{V_{IN}}{2\sqrt{Z_0}} \cdot \frac{2Z_0}{Z+2Z_0} \qquad \Rightarrow S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \frac{2Z_0}{Z+2Z_0}$$

$$a_2 = \frac{1}{2}\left(\frac{V_2}{\sqrt{Z_0}} + \sqrt{Z_0} \cdot I_2\right) = 0 \leftarrow \text{Come voluto}$$

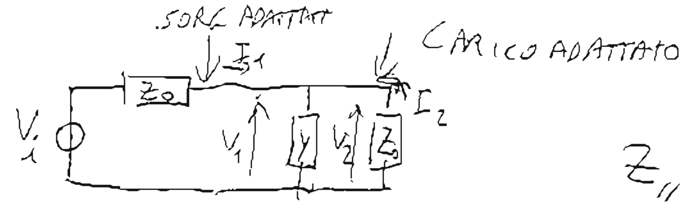
Calcolo di [S] per Y Parallela



$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} \cdot a_2$$

$\Rightarrow$



$$Z_{/\!/} = \frac{Z_0 / Y}{Z_0 + \frac{1}{Y}} = \frac{Z_0}{1 + Y \cdot Z_0}$$

$$V_1 = V_2 = \frac{Z_{/\!/}}{Z_0 + Z_{/\!/}} \cdot V_i$$

$$I_1 = \frac{V_1}{Z_{/\!/}} = \frac{\cancel{Z_{/\!/}}}{Z_0 + Z_{/\!/}} \cdot V_i \cdot \frac{1}{\cancel{Z_{/\!/}}} = \frac{V_i}{Z_0 + Z_{/\!/}}$$

$$I_2 = -\frac{V_2}{Z_0} = -\frac{Z_{/\!/}}{Z_0} \cdot \frac{1}{Z_0 + Z_{/\!/}} \cdot V_i$$

$$a_1 = \frac{1}{2\sqrt{Z_0}}\left(V_1 + Z_0 I_1\right) = \frac{1}{2\sqrt{Z_0}}\left[\frac{Z_{/\!/}}{Z_0 + Z_{/\!/}} V_i + \frac{Z_0 \cdot V_i}{Z_0 + Z_{/\!/}}\right] = \frac{V_i}{2\sqrt{Z_0}}$$

$$b_1 = \frac{1}{2\sqrt{Z_0}}\left(V_1 - Z_0 I_1\right) = \frac{1}{2\sqrt{Z_0}}\left[\frac{Z_{/\!/} - Z_0}{Z_{/\!/} + Z_0}\right] \cdot V_i$$

$$b_2 = \frac{1}{2\sqrt{Z_0}}\left(V_2 - Z_0 I_2\right) = \frac{1}{2\sqrt{Z_0}}\left[\frac{Z_{/\!/}}{Z_0 + Z_{/\!/}} V_i + \frac{\cancel{Z_0} \cdot Z_{/\!/}}{\cancel{Z_0}(Z_0 + Z_{/\!/})} V_i\right] = \frac{V_i}{2\sqrt{Z_0}} \cdot \frac{2Z}{Z_0 +}$$

$$a_2 = \frac{1}{2\sqrt{Z_0}}\left(V_2 + Z_0 I_2\right) = 0 \quad \text{(come Voluto)}$$

$$\Rightarrow S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} = \frac{Z_{11}-Z_0}{Z_{11}+Z_0} = \frac{\frac{Z_0}{1+YZ_0} - Z_0}{\frac{Z_0}{1+YZ_0}+Z_0} = \frac{-Y\cdot Z_0^2}{2Z_0+YZ_0^2} = \frac{-YZ_0}{2+YZ_0}$$

$$\overset{Y_0=\frac{1}{Z_0}}{=} \frac{-Y}{2Y_0+Y}$$

$$S_{21} = \left.\frac{b_2}{a_1}\right|_{a_2=0} = \frac{2Z_{11}}{Z_0+Z_{11}} = \frac{\frac{2Z_0}{1+YZ_0}}{Z_0+\frac{Z_0}{1+YZ_0}} = \frac{2Z_0}{2Z_0+YZ_0^2} = \frac{2}{2+YZ_0} = \frac{2Y_0}{2Y_0+Y}$$

$S_{22} = S_{11}$ e $S_{12} = S_{21}$

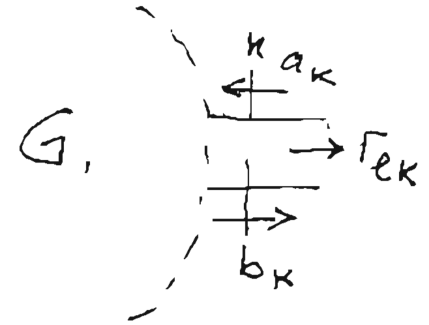
ANCHE QUI $\quad S_{11} = \Gamma_1 = \frac{Z_{in}-Z_0}{Z_{in}+Z_0} = \boxed{\frac{Z_{11}-Z_0}{Z_{11}+Z_0}} =$ PRIMA

$Z_{in} = Z_{11}$

7

Casi speciali: se la porta k è chiusa su Γ_{eK}

$$\Gamma_{eK} = \frac{a_K}{b_K}$$



GIUNZIONE SENZA PERDITE (UTILE COME GUIDA)

S è unitaria cioè $S\tilde{S}^* = I$

$$\left|\det S_{ik}\right| = 1 \qquad \sum_k S_{ik} S_{jk}^* = \delta_{ij} \qquad \sum_i S_{ik} S_{ij}^* = \delta_{kj}$$

PROPRIETÀ DI SIMMETRIA (UTILE COME GUIDA)

(ESERCITAZIONI)

RECIPROCITÀ (questa è usata!)

$$\frac{S_{ik}}{Z_{0i}} = \frac{S_{ki}}{Z_{0k}}$$

e per T. C. uguali $Z_{0i} = Z_{0K} \rightarrow S_{ik} = S_{ki}$

8

PROPAGAZIONE DI $a(z)$, $b(z)$

$$\sin(\omega t \pm \beta z) = \sin[\Phi(t)] \qquad [\beta] =$$

fissato $\Phi = \Phi_0 =$ costante. $+$ prop. $(-z)$ / $-$ prop. $(+z)$

PERIODICITÀ TEMPORALE A $z = \text{cost}$

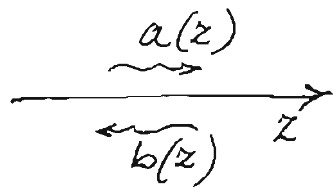
PERIODICITÀ SPAZIALE A $t = \text{cost}$

$$\omega T = 2\pi \longrightarrow \pm \beta \lambda = 2\pi$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{\lambda f} = \frac{\omega}{\lambda / T} = \frac{\omega}{v}$$

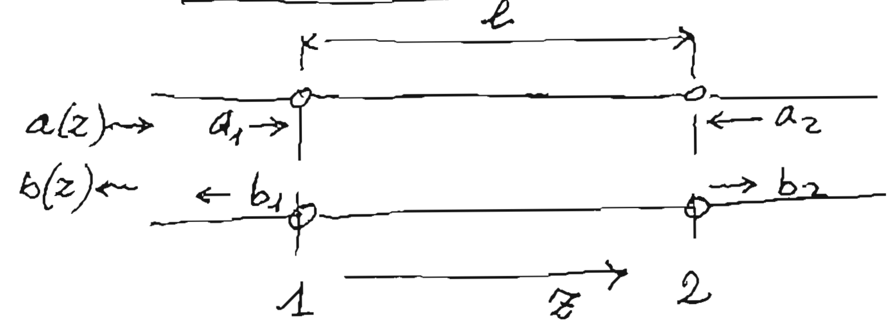
Usando la notazione complessa

$$a(z) = a(z_0)\, e^{j[\cancel{\omega t} - \beta(z - z_0)]}$$



$$b(z) = b(z_0)\, e^{j\beta(z - z_0)}$$

(9) GIUNZIONE COSTITUITA DA LINEA CON PERDITE



$\ell = z_2 - z_1$

PERDITE $\quad -j\beta \longrightarrow -\gamma = -(\alpha + j\beta)$

$\qquad\qquad +j\beta \longrightarrow \gamma = \alpha + j\beta$

$$a(z_2) = a(z_1)\, e^{-\gamma(z_2 - z_1)} \quad \longrightarrow \quad b_2 = a_1\, e^{-\gamma\ell}$$

$$b(z_2) = b(z_1)\, e^{+\gamma(z_2 - z_1)} \quad \longrightarrow \quad b_1 = a_2\, e^{-\gamma\ell}$$

$$S_{21} = e^{-\gamma\ell} = e^{-(\alpha + j\beta)\ell} \qquad S_{11} = S_{22} = 0$$

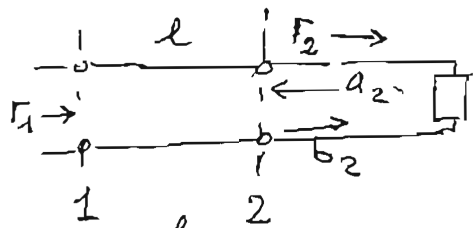
$$S_{12} = e^{-\gamma\ell} = e^{-(\alpha + j\beta)\ell}$$

$$S = \begin{vmatrix} 0 & e^{-\gamma\ell} \\ e^{-\gamma\ell} & 0 \end{vmatrix}$$

per cavi dB/m

TRASF. DI COEFF. DI RIFLESS.

Sia $\quad \Gamma_2 = \Gamma(z_2) = \dfrac{a_2}{b_2}$



in 1 $\quad \Gamma_1 = \dfrac{b_1}{a_1} = \dfrac{a_2\, e^{-\gamma\ell}}{b_2\, e^{\gamma\ell}} = \Gamma_2\, e^{-2\gamma\ell}$

$$|\Gamma_1| = |\Gamma_2|\, e^{-2\alpha\ell} \qquad \varphi_1 = \varphi_2 - 2\beta\ell + 2n\pi$$

Es. TEMA D'ESAME 06-2005

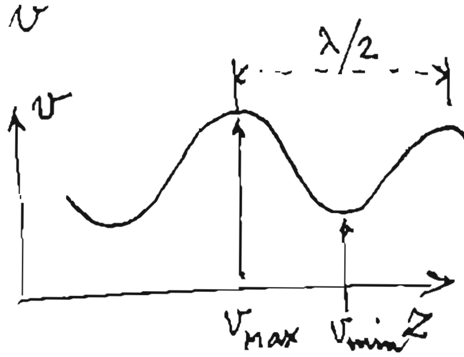
(10)

RAPPORTO D'ONDA STAZIONARIA

VSWR (σ)

(VOLTAGE STANDING WAVE RATIO)

FA RIFERIMENTO A v^+ e v^-



LINEA SENZA PERDITE

$$VSWR \equiv \sigma \equiv \frac{v_{Max}}{v_{min}} = \frac{|v^+|+|v^-|}{|v^+|-|v^-|}$$

$$= \frac{|a|+|b|}{|a|-|b|} = \frac{1+|\Gamma|}{1-|\Gamma|}$$

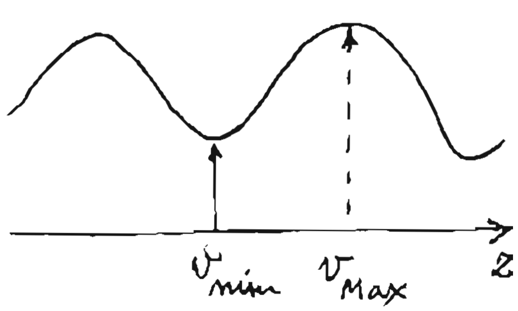
LINEA CON PERDITE

$$a(z) = a\,e^{-\gamma z}$$

$$b(z) = b\,e^{\gamma z}$$

$$\sigma_A = \frac{v_{Max}}{v_{min}} = \frac{|a(z_M)|+|b(z_M)|}{|a(z_M)|\,e^{\alpha\lambda/4} - |b(z_M)|^{-\alpha\lambda/4}}$$

$$= \frac{1+|\Gamma|}{1-|\Gamma|\,e^{-\alpha\lambda/2}}\,e^{-\alpha\lambda/4}$$



QUI RECIPROC

SENZA PERDITE

MISURE A RADIOFREQUENZA **25 febbraio 2005**

ESERCIZIO SULLE GIUNZIONI

1a) Si dimostri come un coefficiente di riflessione si trasferisce in generale dall'uscita all'ingresso di un due porte lineare attraverso una trasformazione bilineare.
1b) Si costruisca la matrice di diffusione di una linea di trasmissione perfettamente adattata, di lunghezza pari a 21.5 λ, che perde 0.1 dB.
1c) Si calcoli il coefficiente di riflessione all'ingresso di questa linea se la seconda porta è lasciata a circuito aperto (si supponga nullo l'irraggiamento).

SOLUZIONE

1a) Il legame tra il coefficiente di riflessione di chiusura $\Gamma_l = |\Gamma_l| e^{j\phi}$ e quello d'ingresso Γ_{IN} vale (si vedano le dispense del corso):

$$\frac{b_1}{a_1} = \Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_l}{1 - S_{22}\Gamma_l} = \frac{S_{11} + (S_{12}S_{21} - S_{11}S_{22})\Gamma_l}{1 - S_{22}\Gamma_l} = \frac{a\Gamma_l + b}{c\Gamma_l + d}$$

1b) La linea è perfettamente adattata, per cui abbiamo 0 sulla diagonale principale. Sicuramente vale la reciprocità, per cui la matrice è simmetrica.
La perdita di 0.1 dB in lineare corrisponde alla moltiplicazione dell'ampiezza per $G = 10^{-(0.1/20)} = 0.988$ ($20\log_{10}(G) = -0.1$).
La fase dei termini incrociati è data da $-\frac{2\pi}{\lambda}L = -\frac{2\pi}{\lambda}21.5\lambda = -43\pi$, che quindi corrisponde ad uno sfasamento di 180°.
La matrice di diffusione vale quindi $S = \begin{bmatrix} 0 & -0.988 \\ -0.988 & 0 \end{bmatrix}$

1c) Applichiamo la formula del punto a), considerando che il coefficiente di riflessione di un circuito aperto vale $\Gamma_l = 1$ (si ricordi che $\Gamma = \frac{Z_{carico} - Z_0}{Z_{carico} + Z_0}$)

$$\Gamma_{IN} = S_{11} + \frac{S_{12}S_{21}\Gamma_l}{1 - S_{22}\Gamma_l} = 0 + \frac{0.988^2\,\Gamma_l}{1-0} = 0.976 \times \Gamma_l = 0.976$$

3.11.4 Use of *s*-parameters for active elements

Example 3.18

A 50 Ω microwave integrated circuit (MIC) amplifier has the following *s*-parameters:

$$s_{11} = 0.12 \angle -10^\circ \qquad s_{12} = 0.002 \angle -78^\circ$$
$$s_{21} = 9.8 \angle 160^\circ \qquad s_{22} = 0.01 \angle -15^\circ$$

Calculate: (a) input VSWR, (b) return loss, (c) forward insertion power gain and (d) reverse insertion power loss.

Given: $s_{11} = 0.12 \angle -10^\circ$ $\quad s_{12} = 0.002 \angle -78^\circ$
$s_{21} = 9.8 \angle 160^\circ$ $\quad s_{22} = 0.01 \angle -15^\circ$

Required: (a) Input VSWR, (b) return loss, (c) forward insertion power gain, (d) reverse insertion power loss.

Solution

(a) From Equation 2.38

$$\text{VSWR} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+|s_{11}|}{1-|s_{11}|} = \frac{1+0.12}{1-0.12}$$
$$= 1.27$$

(b) Return loss (dB) $= -20 \log_{10} 0.12 = 18.42$ dB
(c) Forward insertion gain $= |s_{21}|^2 = (9.8)^2 = 96.04$ or
Forward insertion gain $= 10 \log_{10} (9.8)^2$ dB $= 19.83$ dB
(d) Reverse insertion gain $= |s_{12}|^2 = (0.002)^2 = 4 \times 10^{-6}$ or
Reverse insertion gain $= 10 \log_{10} (0.002)^2$ dB $= -53.98$ dB

The amplifier is virtually unilateral with (53.98 − 19.83) or 34.15 dB of output to input isolation.

3.12 Summary of scattering parameters

Section 3.10 has been devoted to the understanding of two-port scattering networks. Section 3.11 has been devoted to the use of two-port scattering networks. You should now be able to manipulate two-port networks skilfully and have the ability to change two-port parameters given in one parameter set to another parameter set.

An excellent understanding of scattering parameters is vitally important in microwave engineering because most data given by manufacturers are in terms of these parameters. In fact, you will find it difficult to proceed without a knowledge of *s*-parameters. This is the reason why we have provided you with several examples of *s*-parameter applications. The examples will be repeated using a software program called PUFF which has been supplied to you with this book. The purpose of these software exercises is to reinforce the concepts you have learnt and also to convince you that what we have been doing is correct.

Do *not* be unduly perturbed if you initially found *s*-parameters difficult to understand.

GIUNZIONE SENZA PERDITE

Potenza che entra = potenza che esce

$$\sum_{i=1}^{N} |b_i|^2 - \sum_{i=1}^{N} |a_i|^2 = 0 \qquad |a_i|^2 = a_i \cdot a_i^*$$

trasposto

$$[b] = [S][a]$$

$$\sum_{i=1}^{N} |b_i|^2 = \{b_1, b_2, \dots b_N\} \begin{bmatrix} b_1^* \\ b_2^* \\ \vdots \\ b_N^* \end{bmatrix} = \overline{[b]}\,[b]^* =$$

$$= \overline{[S][a]}\;[S^*][a^*]$$

$$\sum_{i=1}^{N} |a_i|^2 = \overline{[a]} \cdot [a^*]$$

$$\Rightarrow \sum_{i=1}^{N} |b_i|^2 - \sum_{i=1}^{N} |a_i|^2 = \overline{[S]\cdot[a]} \cdot [S^*] \cdot [a^*] - \overline{[a]} \cdot [a^*]$$

$$= \overline{[a]} \cdot \overline{[S]} \cdot [S^*][a^*] - \overline{[a]} \cdot [I] \cdot [a^*] =$$

$$= \overline{[a]} \cdot \underbrace{\left(\overline{[S]} \cdot [S^*] - [I]\right)}_{\text{MATRICE UNITARIA}} \cdot [a^*] = 0 \quad \forall a$$

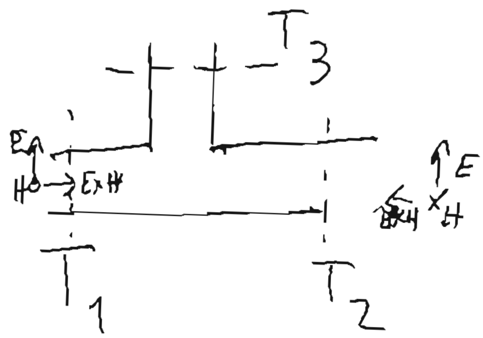
$$\Rightarrow \boxed{\overline{[S]} \cdot [S^*] = [I]} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ & 1 & & 0 \\ 0 & & 1 & \\ & & & \ddots \end{bmatrix}$$

$\Rightarrow \det[S] = 1$

↑ MENO STRINGENTE

SIMMETRIE

3 Porte reciproche e con simmetria tra 1 e 2



Reciproca $\Rightarrow S_{21} = S_{12}$, $S_{31} = S_{13}$

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} \\ S_{21} & S_{22} & S_{23} \\ S_{31} & S_{32} & S_{33} \end{bmatrix} \qquad S_{32} = S_{23}$$

Simmetria $\Rightarrow S_{11} = S_{22}$, $S_{13} = \pm S_{23}$ (se piano H / se piano E)

9 NUMERI, 5 equazioni $\Rightarrow$ solo 4 Parametri

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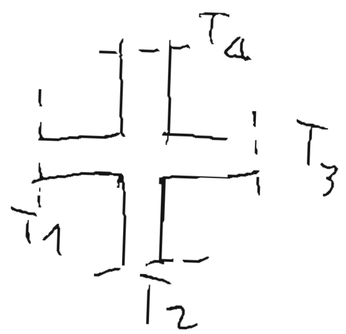
4 Porte reciproche e simmetrica



Reciproca $\Rightarrow$ 6 uguaglianze

Simmetria $S_{11} = S_{22} = S_{33} = S_{44}$

$S_{12} = S_{23} = S_{34} = S_{41}$

$S_{13} = S_{24}$ $\Rightarrow$ 7 uguaglianze (PIANO H)

16 Numeri $-(7+6)$ equazioni $= 3$ Parametri (S_{11}, S_{12}, S_{13})

NON FIDARSI DELLE SIMMETRIE,

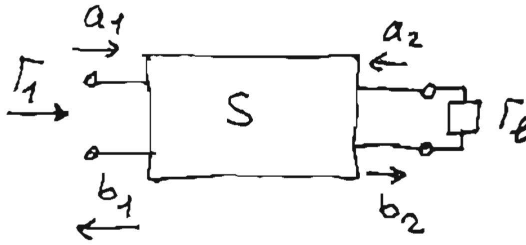
NÉ DEL ~~...~~ "SENZA PERDITE"

TRASFORMAZIONI IMPORTANTI

$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

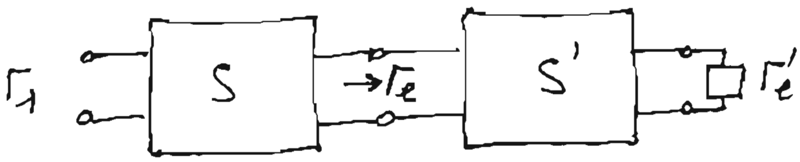
$$\Gamma_\ell = \frac{a_2}{b_2}$$



$$\frac{b_1}{a_1} \equiv \Gamma_1 = S_{11} + \frac{S_{12} S_{21} \Gamma_\ell}{1 - S_{22}\Gamma_\ell} = \frac{S_{11} + (S_{12} S_{21} - S_{11} S_{22})\Gamma_\ell}{1 - S_{22}\Gamma_\ell}$$

è una <u>relazione bilineare</u>

genera nuove <u>relazioni bilineari</u>



$$\Gamma_1 = \frac{P + Q\Gamma'_\ell}{R + S\Gamma'_\ell}$$

TRASFORMAZIONE BILINEARE
(CIRCONF. IN CIRCONF.)

DATE DUE VARIABILI COMPLESSE w e z

$$w = \frac{Az + B}{Cz + D} = \frac{\frac{A}{C}(Cz + D) + B - \frac{AD}{C}}{Cz + D} = \frac{A}{C} + \frac{B - \frac{AD}{C}}{Cz + D}$$

$$\frac{w - A/C}{B - \frac{AD}{C}} \equiv W = \frac{1}{Cz + D} \equiv \frac{1}{Z}$$

(12)

Se z descrive circonf.

$Z = Cz + D$ descrive circonf.

$$\underset{\substack{\uparrow \\ \text{centro}}}{(Z - S)}(Z^* - S^*) = \underset{\substack{\uparrow \\ \text{raggio}}}{R^2}$$

$$ZZ^* - SZ^* - S^*Z + \underbrace{SS^* - R^2}_{>0 \text{ o } <0} = 0 \longrightarrow \frac{S^*S}{(S^*S - R^2)^2} - R'^2$$

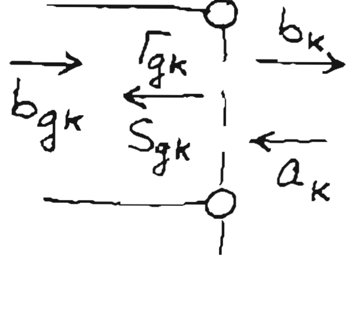
$$W = \frac{1}{Z} \longrightarrow WW^* - \frac{SW}{SS^* - R^2} - \frac{S^*W^*}{SS^* - R^2} + \left(\frac{1}{SS^* - R^2}\right) = 0$$

centro $S' = \dfrac{S^*}{SS^* - R^2}$ raggio $R' \to R'^2 = \dfrac{R^2}{(SS^* - R^2)^2}$

ma se W descrive una circonferenza, lo stesso sarà per w.

RAPPRESENTAZ. EQUIV. DI UNA SORGENT
(Thevenin, Norton)

$$b_k = \underset{\substack{\uparrow \\ \Gamma_{gk}}}{S_{gk}}\, a_k + b_{gk}$$

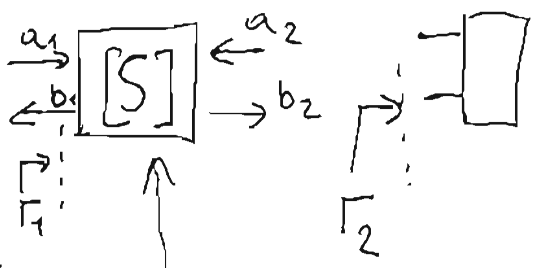


1ª ESERCITAZIONE M.R.F.

Trasformazioni bilineari

Giunzione 2 PORTE

Può essere un carico o qualsiasi altra cosa



$$\frac{b_1}{a_1} \equiv \Gamma_1 \qquad \Gamma_2 = \frac{a_2}{b_2}$$

$$\begin{cases} b_1 = S_{11} a_1 + S_{12} a_2 \\ b_2 = S_{21} a_1 + S_{22} a_2 \end{cases} \Rightarrow \Gamma_1 = \frac{b_1}{a_1} = S_{11} + \frac{S_{12} \cdot S_{21} \cdot \Gamma_2}{1 - S_{22} \cdot \Gamma_2} =$$

Trasformazione bilineare $\rightarrow \Gamma_1 = \dfrac{S_{11} + (S_{12} S_{21} - S_{11} S_{22}) \Gamma_2}{1 - S_{22} \Gamma_2}$

del tipo $X_1 = \dfrac{A + B X_2}{C + D X_2}$ A, B, C, D numeri complessi (come X_1 e X_2)

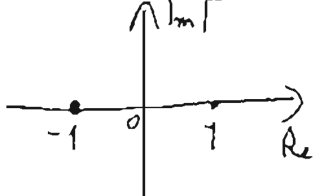
> Se X_2 varia seguendo una circonferenza $\Rightarrow$
> $\Rightarrow$ Anche X_1 varia seguendo una circonferenza.

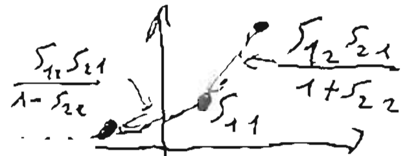
Per i conti a mano conviene riscriverla così:

$$\Gamma_1 = S_{11} + \frac{S_{12} S_{21}}{\frac{1}{\Gamma_2} - S_{22}}$$

Esempio: $\Gamma_2 = -1 \rightarrow 0 \rightarrow 1$

$$\Gamma_1 = S_{11} - \frac{S_{12} S_{21}}{1 + S_{22}} \rightarrow S_{11} \rightarrow S_{11} + \frac{S_{12} S_{21}}{1 - S_{22}}$$



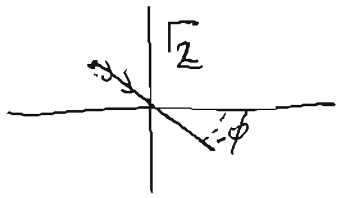


OPERAZIONE

$\boxed{\Gamma \text{ VARIA SOLO IL MODULO}}$

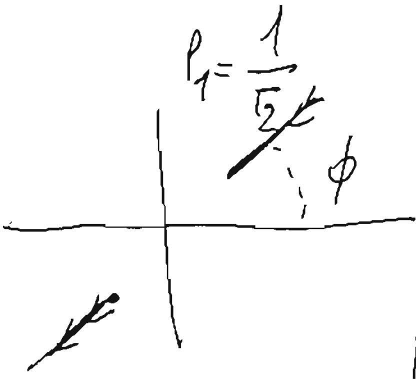
GRAFICO NEL PIANO COMPLESSO

$$P_1 = \frac{1}{\Gamma_2}$$

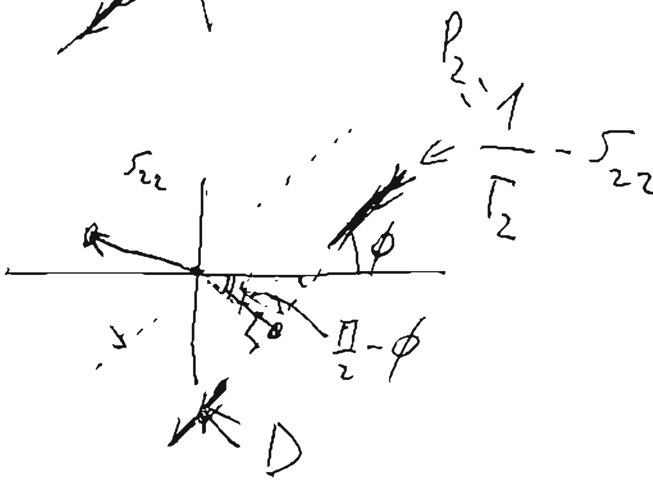


$$\Gamma_2 = |\Gamma_2| e^{-i\phi}$$

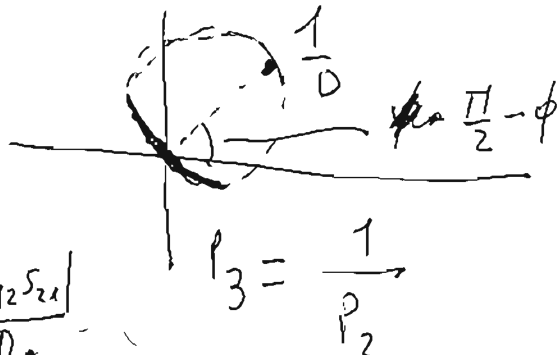
varia



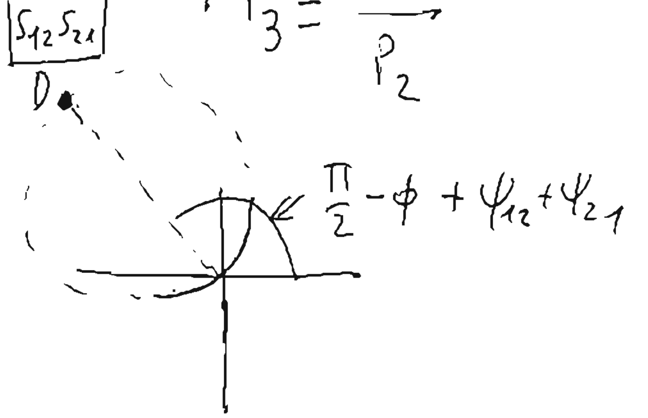
$$P_2 = P_1 - S_{22} = \frac{1}{\Gamma_2} - S_{22}$$



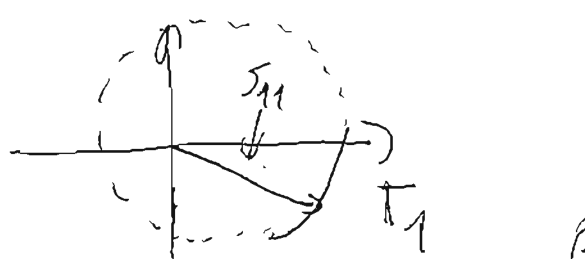
$$P_3 = \frac{1}{P_2} = \frac{1}{\frac{1}{\Gamma_2} - S_{22}}$$



$$P_4 = S_{12} \cdot S_{21} \cdot P_3 = \frac{S_{12} \cdot S_{21}}{\frac{1}{\Gamma_2} - S_{22}}$$



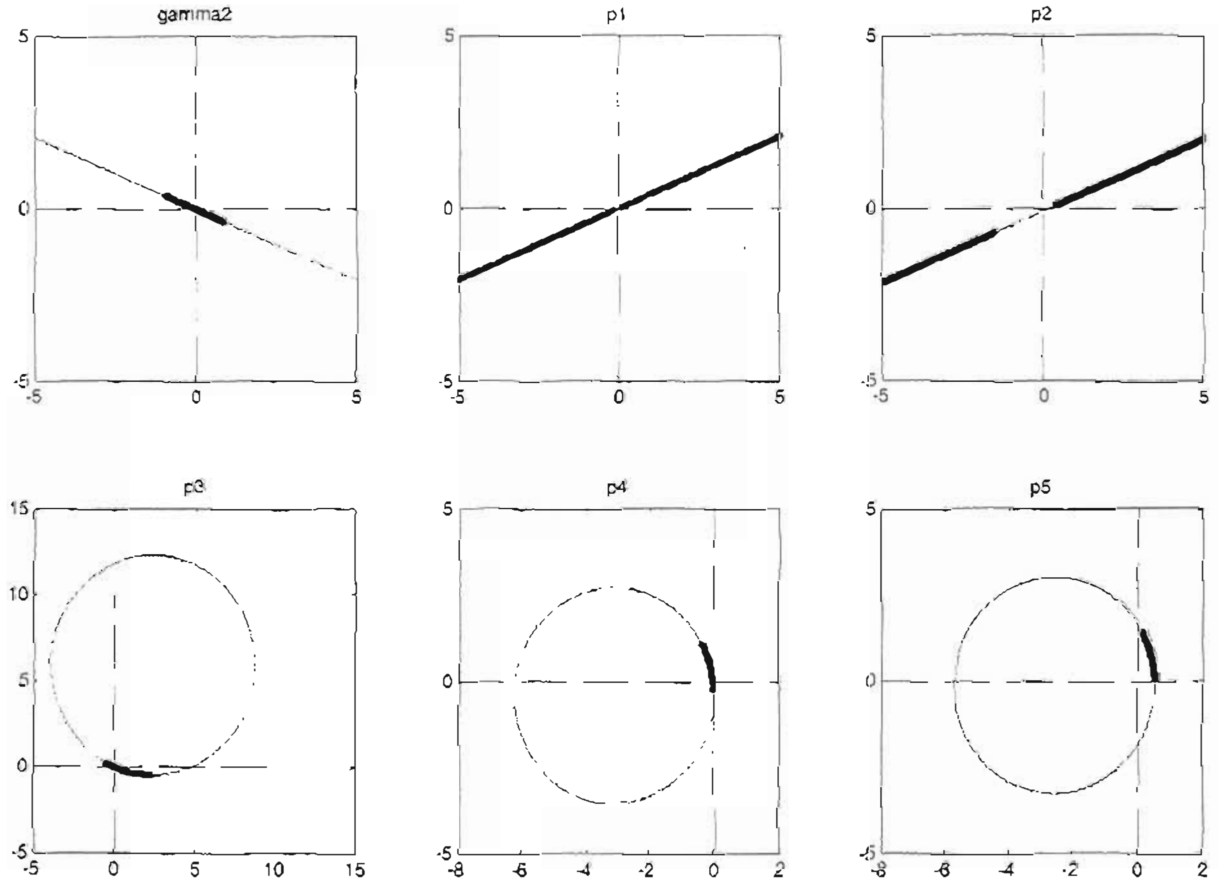
$$\Gamma_1 = S_{11} + P_4 = S_{11} + \frac{S_{12} \cdot S_{21}}{\frac{1}{\Gamma_2} - S_{22}}$$



$$S = \begin{bmatrix} 0.6e^{i\pi/6} & 0.7e^{i\pi/3} \\ 0.7e^{i\pi/3} & 0.6e^{i\pi/6} \end{bmatrix}$$

Det(S)= 0.4250 - 0.1126i

Γ2=[-0.999:0.001:1]*exp(-iπ/8)



Teorema dell'adattamento (4 Porte)

Se in una giunzione a 4 porte senza perdite e reciproca, non c'è accoppiamento tra 2 porte, entrambe adattate (quando le altre 3 vedono un carico adattato), ⟹ le altre 2 porte sono disaccoppiate e adattate

RECIPROCA

$$S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & 0 \\ S_{14} & S_{24} & 0 & 0 \end{bmatrix}$$

senza perdite

$\Downarrow$

$[S]$ unitaria

$$\Rightarrow \quad |S_{11}|^2 + |S_{12}|^2 + |S_{13}|^2 + |S_{14}|^2 = 1 \qquad ①$$

$$|S_{12}|^2 + |S_{22}|^2 + |S_{23}|^2 + |S_{24}|^2 = 1 \qquad ②$$

$$|S_{13}|^2 + |S_{23}|^2 = 1 \qquad ③$$

$$|S_{14}|^2 + |S_{24}|^2 = 1 \qquad 4$$

Sommando (① e ②) e (③ e ④)

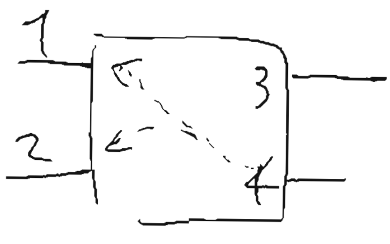
$$\Rightarrow \quad |S_{11}|^2 + 2|S_{12}|^2 + |S_{22}|^2 + |S_{13}|^2 + |S_{23}|^2 + |S_{14}|^2 + |S_{24}|^2 = 2$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{14}|^2 + |S_{24}|^2 = 2$$

$\Rightarrow S_{11} = S_{12} = S_{22} = 0$ DIMOSTRATO

È UN ACCOPPIATORE DIREZIONALE

$$S = \begin{bmatrix} 0 & 0 & \alpha & \beta \\ 0 & 0 & \beta & \alpha \\ \alpha & \beta & 0 & 0 \\ \beta & \alpha & 0 & 0 \end{bmatrix}$$



GIUNZIONI COMPLETAMENTE ADATTATE:

hanno la proprietà che ogni porta presenta un carico adattato al suo ingresso, se tutte le altre porte sono terminate su carichi adattati.

TEOREMA DELL'ADATTAMENTO (3 porte)

Non è possibile fare una giunzione a 3 porte senza perdite, reciproca, completamente adattata.

DIMO: Se fosse completamente adattata $\Rightarrow$ $S_{11}=S_{22}=S_{33}=0$ (NO RIFLE.

$$S=\begin{bmatrix} 0 & S_{12} & S_{13} \\ S_{21} & 0 & S_{23} \\ S_{31} & S_{32} & 0 \end{bmatrix}$$

introduciamo la condizione di assenza di perdite: $\overline{[S]\cdot[S^*]}=[I]$

$$\begin{bmatrix} 0 & S_{21} & S_{31} \\ S_{12} & 0 & S_{32} \\ S_{13} & S_{23} & 0 \end{bmatrix}\cdot\begin{bmatrix} 0 & S_{12}^* & S_{13}^* \\ S_{21}^* & 0 & S_{23}^* \\ S_{31}^* & S_{32}^* & 0 \end{bmatrix}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \quad |S_{21}|^2+|S_{31}|^2=1$$

$$S_{31}\cdot S_{32}^*=0$$

$$S_{21}\cdot S_{23}^*=0$$

$$|S_{12}|^2+|S_{32}|^2=1$$

$$S_{12}\,S_{13}^*=0$$

$$|S_{13}|^2+|S_{23}|^2=1$$

$\Rightarrow$ Uniche soluzioni possibili:

$S_{12}=S_{23}=S_{31}=0$,

$|S_{21}|=|S_{32}|=|S_{13}|=1$

oppure

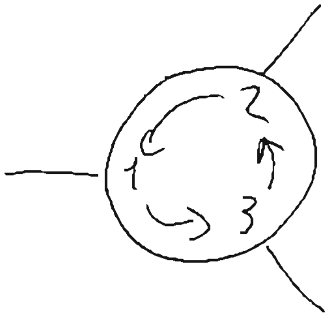
CIRCOLA

$S_{21}=S_{32}=S_{13}=0$

$|S_{12}|=|S_{23}|=|S_{31}|=1$

$\Rightarrow$ NON RECIPROCHE

$$S = \begin{bmatrix} 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \\ e^{i\gamma} & 0 & 0 \end{bmatrix}$$

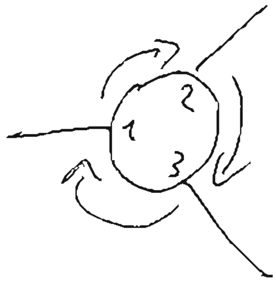


$$b_1 = e^{i\alpha} \cdot a_2$$
$$b_2 = e^{i\beta} \cdot a_3$$
$$b_3 = e^{i\gamma} \cdot a_1$$

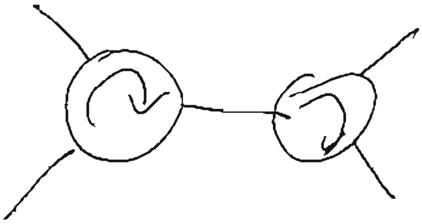
oppure

$$S = \begin{bmatrix} 0 & 0 & e^{i\alpha} \\ e^{i\beta} & 0 & 0 \\ 0 & e^{i\gamma} & 0 \end{bmatrix}$$

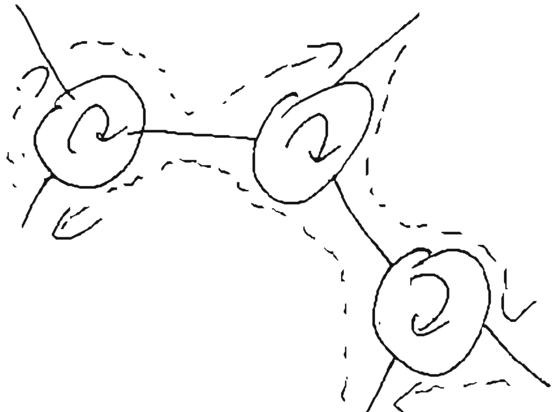
$$b_1 = e^{i\alpha} \cdot a_3$$
$$b_2 = e^{i\beta} \cdot a_1$$
$$b_3 = e^{i\gamma} \cdot a_2$$



Per fare un circolatore a + porte basta attaccarne tanti da 3 :



= 4 Porte



5 Porte