Radiofrequency Measurements

Insertion loss and mismatch error

Insertion Loss

When considering RF attenuators, it is important to underline the difference between "attenuation" and "insertion loss".

The **insertion loss** L_I of a device is the power loss due to the insertion of the device before the load. It could be induced by a real attenuation (a loss), but also by mismatch and resulting power reflection.

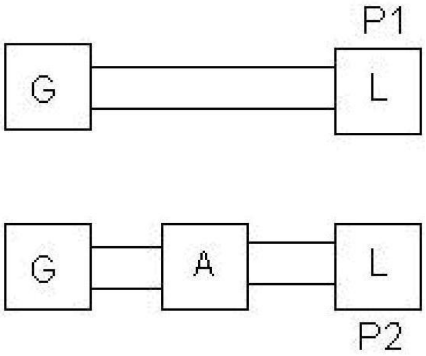
The value L_I is given by the ration, in logarithmic units, between the received power in two conditions:

Direct connection between the load L and the source G,

Insertion of the device (A in figure 1) between load and source.

the insertion loss L_I is defined as:

$$L_I = 10\log\left(\frac{P_1}{P_2}\right)$$



Attenuation coefficient

When both load and source are matched, the insertion loss is equal to the **attenuation coefficient** α:

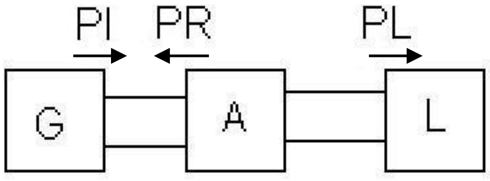
$$\alpha = L_I \big|_{\Gamma_G = \Gamma_L = 0}$$

Reflection contribution: $$\alpha_R = 10\log\left(\frac{P_I}{P_I - P_R}\right)$$

Loss contribution: $$\alpha_D = 10\log\left(\frac{P_I - P_R}{P_L}\right)$$

$$\alpha = \alpha_R + \alpha_D$$

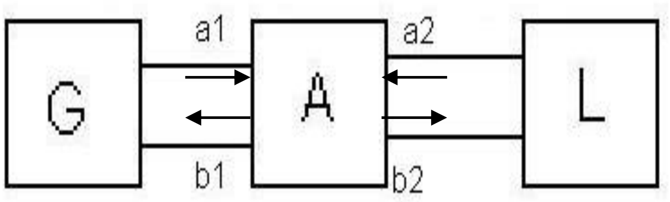
Where P_I, P_R, P_L are respectively the incident power, the power reflected by the attenuator and the power at the load.



Attenuation coefficient

With respect to the attenuator scattering matrix, the attenuation parameters are given by:

$$\alpha = 10\log\left(\frac{P_I}{P_L}\right) = 10\log\frac{1}{|S_{21}|^2}$$



$$P_I = \frac{|a_1|^2}{2}$$

$$P_R = \frac{|b_1|^2}{2}$$

$$P_L = \frac{|b_2|^2}{2}$$

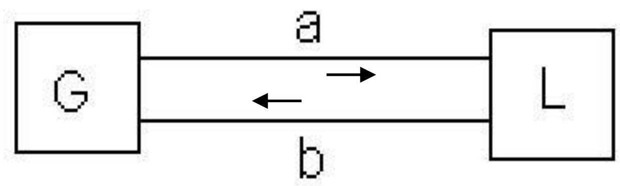
$$\alpha_D = 10\log\frac{1-|S_{11}|^2}{|S_{21}|^2}$$

$$\alpha_R = 10\log\frac{1}{1-|S_{11}|^2}$$

Mismatch Error

We want to evaluate the error on the insertion loss, induced by the mismatch of the attenuator and, mostly, by the mismatch of the system

$$S_{11}, S_{22} \neq 0 \qquad\qquad \Gamma_G, \Gamma_L \neq 0$$



$$\begin{cases} a = e_g + \Gamma_G b \\ b = \Gamma_L a \\ P_L = \dfrac{|a|^2}{2} - \dfrac{|b|^2}{2} \end{cases}$$

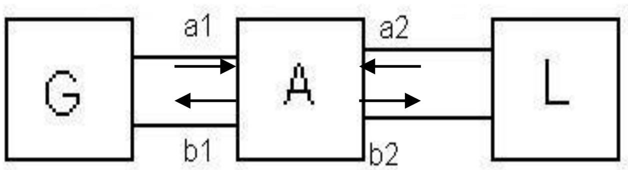
$$a = e_g \frac{1}{1 - \Gamma_G \Gamma_L}$$

$$P_L = \frac{|a|^2}{2}\left(1 - |\Gamma_L|^2\right)$$

$$\Rightarrow \quad \boxed{P_{L,1} = \frac{|e_g|^2}{2} \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_G \Gamma_L|^2}}$$

Mismatch Error

Now we insert the attenuator and calculate the resulting power received by the load. We have 4 equations: 1 at the source, 1 at the load, and 2 from the scattering matrix.



$$\begin{cases} a_1 = e_g + \Gamma_G b_1 \\ a_2 = \Gamma_L b_2 \\ b_1 = S_{11} a_1 + S_{12} a_2 \\ b_2 = S_{21} a_1 + S_{22} a_2 \\ P_L = \dfrac{|b_2|^2}{2}\left(1 - |\Gamma_L|^2\right) \end{cases} \Rightarrow \begin{cases} b_1 = S_{11}\left(e_G + \Gamma_G b_1\right) + S_{12}\Gamma_L b_2 \\ b_2 = S_{21}\left(e_G + \Gamma_G b_1\right) + S_{22}\Gamma_L b_2 \end{cases} \Rightarrow \begin{cases} b_1\left(1 - \Gamma_G S_{11}\right) = S_{11} e_G + S_{12}\Gamma_L b_2 \\ b_1\left(-\Gamma_G S_{21}\right) = S_{21} e_G + \left(S_{22}\Gamma_L - 1\right) b_2 \end{cases}$$

$$\frac{S_{11} e_G + S_{12}\Gamma_L b_2}{1 - \Gamma_G S_{11}}\left(-\Gamma_G S_{21}\right) = S_{21} e_G + \left(S_{22}\Gamma_L - 1\right) b_2$$

$$\Downarrow$$

$$b_2 = \frac{S_{11} e_G S_{21}\Gamma_G + S_{21} e_G\left(1 - \Gamma_G S_{11}\right)}{-S_{21}\Gamma_L\Gamma_G S_{12} + \left(1 - \Gamma_L S_{22}\right)\left(1 - \Gamma_G S_{11}\right)} = \frac{e_G S_{21}}{\left(1 - \Gamma_L S_{22}\right)\left(1 - \Gamma_G S_{11}\right) - S_{12} S_{21}\Gamma_G\Gamma_L}$$

Mismatch Error

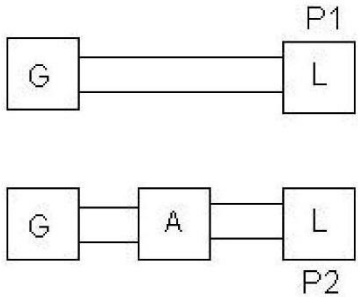
In conclusion, the load power is

$$P_{L,2}=\frac{|e_G|^2}{2}\frac{|S_{21}|^2\left(1-|\Gamma_L|^2\right)}{\left|(1-\Gamma_L S_{22})(1-\Gamma_G S_{11})-S_{12}S_{21}\Gamma_L\Gamma_G\right|^2}$$

The insertion loss is given by the ratio of the power values in the two situations:

$$L_I=10\log\frac{P_{L1}}{P_{L2}}=10\log\frac{\left|(1-\Gamma_L S_{22})(1-\Gamma_G S_{11})-S_{12}S_{21}\Gamma_L\Gamma_G\right|^2}{|S_{21}|^2\,|1-\Gamma_G\Gamma_L|^2}=$$

$$=10\log\frac{1}{|S_{21}|^2}+10\log\frac{\left|(1-\Gamma_L S_{22})(1-\Gamma_G S_{11})-S_{12}S_{21}\Gamma_L\Gamma_G\right|^2}{|1-\Gamma_G\Gamma_L|^2}$$



The first term represents the attenuation coefficient, corresponding to the insertion loss when the system is matched ($\Gamma_G=\Gamma_L=0$).

Mismatch Error

We can define the **mismatch error** *M* as:

$$M = L_I - \alpha = 10\log\frac{\left|(1-\Gamma_L S_{22})(1-\Gamma_G S_{11}) - S_{12}S_{21}\Gamma_L\Gamma_G\right|^2}{\left|1-\Gamma_G\Gamma_L\right|^2}$$

Typically, the *M* coefficient is plotted as a function of the standing wave ratio of the system, under some simplifying assumptions:

$$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|}$$

- The plot represents the maximum error (we consider the phases of the complex parameters such as to maximize the error);
- The attenuator is assumed symmetric ⟺ $S_{11} \cong S_{22}$
- The system is assumed symmetric ⟺ $\Gamma_G \cong \Gamma_L$
- The plot is parametric with respect to the attenuator VSWR, with the assumption of loss > 20 dB.

Mismatch Error

$$M = L_I - \alpha = 10\log\frac{\left|(1-\Gamma_L S_{22})(1-\Gamma_G S_{11}) - S_{12}S_{21}\Gamma_L\Gamma_G\right|^2}{\left|1-\Gamma_G\Gamma_L\right|^2}$$