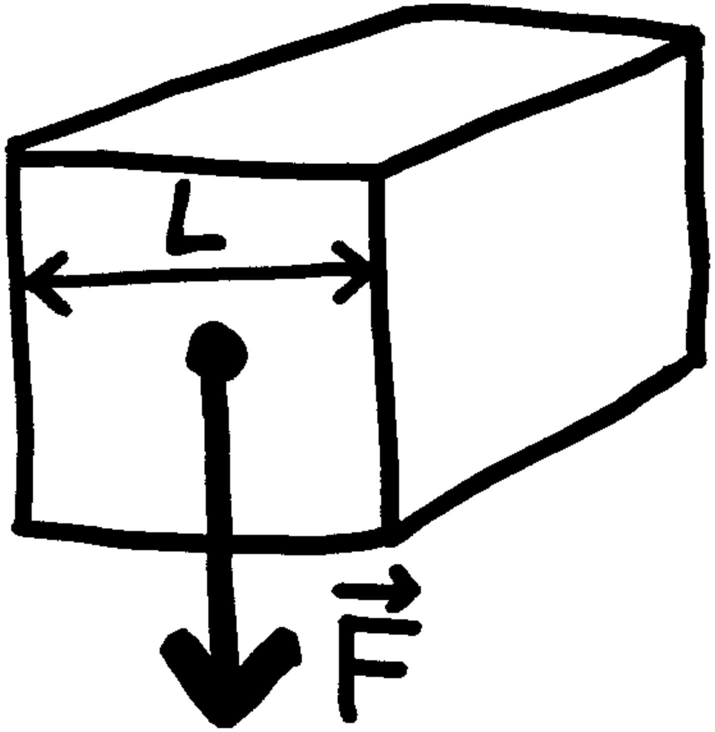
Es. 3 Lez. 11

ESERCIZIO media pesata e incertezza



Cubo di Al di lato ~1 m
Calcolare il suo PESO

$$g = 9.806\,65(33)\ m/s^2$$

L misurato mediante conteggio di frange a $\lambda = 500$ nm ottenendo un numero di conteggi N sempre compreso tra $N_1 = 2 \times 10^6$ e $N_2 = 2 \times 10^6 + 40$

Per la densità ρ del materiale disponiamo di 3 possibili valori:

"C" secondo il costruttore:

$\rho_C = 2.71$ Kg/dm^3 con incertezza di 2×10^{-5}

"T" da diversi testi di meccanica e materiali si ricavano $n=9$ valori ρ_K con $\rho_T = \overline{\rho_K} = 2.73$ Kg/dm^3 e dev. $s(\rho_K) = 9$ g/dm^3

"L" da una misura di laboratorio si ha

$\rho_L = 2.69$ Kg/dm^3 con $u(\rho_L) = 0.02$ Kg/dm^3

$$\vec{F} = m\vec{a}$$

$$F = mg = (\rho V)g = \rho L^3 g$$

relazione funzionale

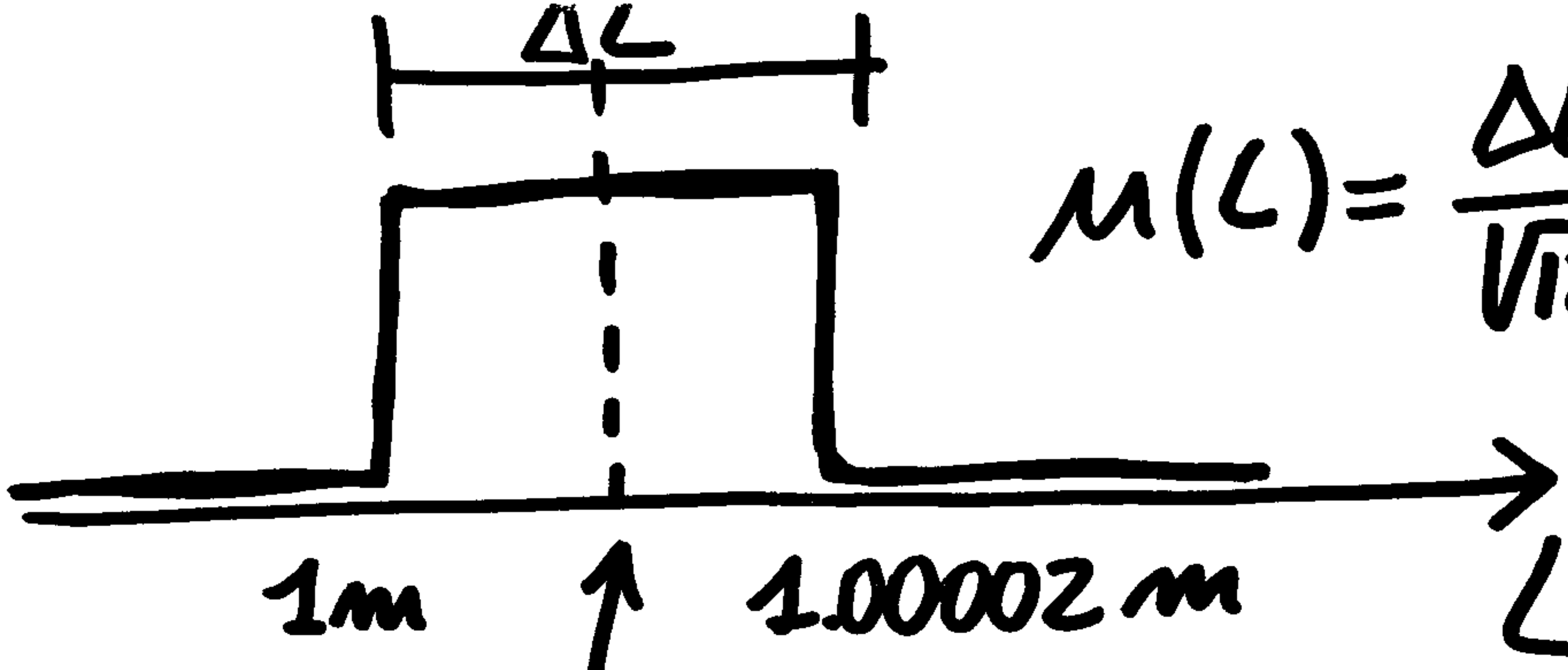
$$y = f(x_1, x_2, \ldots x_n)$$

$$\bar{F} = \bar{\rho}\,\bar{L}^3\,\bar{g}$$

$$\mu(F) = \sqrt{\sum_{i=1}^{3} \left[\frac{\partial f}{\partial x_i}\right]^2 \mu^2(x_i)}$$

o anche

$$\mu_r(F) = \sqrt{\sum_{i=1}^{3} n_i^2\,\mu_r^2(x_i)}$$



$$u(L) = \frac{\Delta L}{\sqrt{12}}$$

$$\Delta L = 20\,\mu m$$

$\bar{L} =$

$1m + 10\mu m$

L_1 $\quad$ L_2

$N_1 \lambda$ $\quad$ $N_2 \lambda$

$2 \times 10^6 \times 0.5 \times 10^{-6} m$

$$u(\bar{L}) = u(L) = \frac{20\,\mu m}{\sqrt{12}} \cong 5.8\,\mu m$$

$$u_r(L) = \frac{u(L)}{L} \cong 6 \times 10^{-6}$$

$$u(g) = 33 \times 10^{-5}\ m/s^2$$

$$u_r(g) = \frac{u(g)}{g} = \frac{33}{980\,665} \cong 3.4 \times 10^{-5}$$

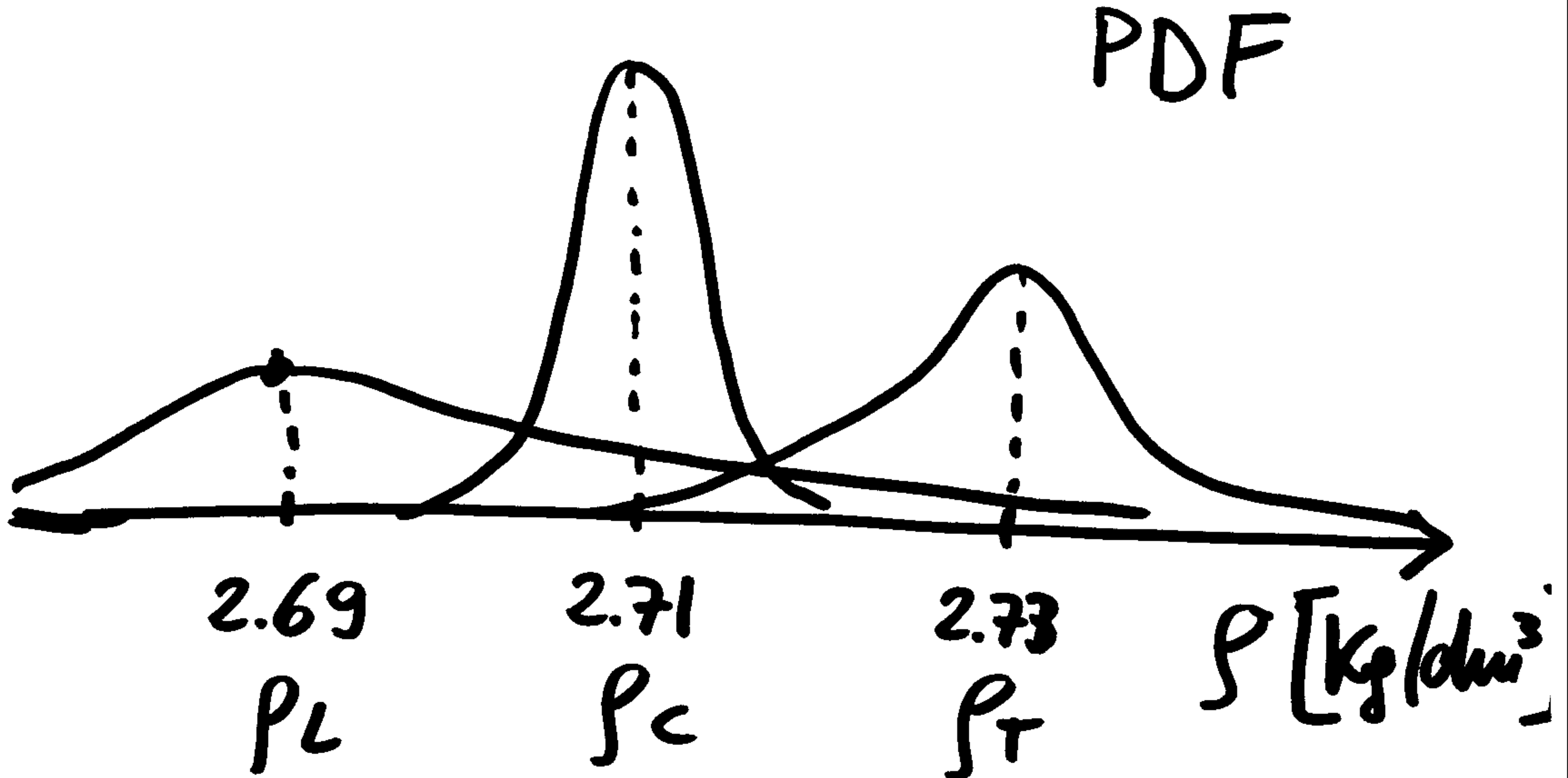
densità ρ

$\rho_C = 2.71\ Kg/dm^3 \quad \mu(\rho_C) \cong 5.4 \times 10^{-5}\ Kg/dm^3$

$\rho_T = 2.73\ Kg/dm^3 \quad \mu(\rho_T) = \frac{s(\rho_K)}{\sqrt{n}} =$

$= \frac{9 \times 10^{-3}\ Kg/dm^3}{3} = 3 \times 10^{-3}\ Kg/dm^3$

$\rho_L = 2.69\ Kg/dm^3 \quad \mu(\rho_L) = 2 \times 10^{-2}\ Kg/dm^3$



COMPATIBILITÀ fra ρ_α e ρ_β

$$|\rho_\alpha - \rho_\beta| \leq K\sqrt{u^2(\rho_\alpha) + u^2(\rho_\beta)}$$

con $K = 1$ risultano comp.
le misure ρ_L e ρ_C mentre
è incompatibile ρ_T

Per la stima di ρ e $u(\rho)$
ricorro al criterio della
MEDIA PESATA tra mis
comp.

$$\rho = \frac{\dfrac{\rho_L}{\mu^2(\rho_L)} + \dfrac{\rho_C}{\mu^2(\rho_C)}}{\dfrac{1}{\mu^2(\rho_L)} + \dfrac{1}{\mu^2(\rho_C)}} \cong \rho_C = 2.71 \text{ Kg/dm}^3$$

$$\mu^2(\rho) = \frac{1}{\dfrac{1}{\mu^2(\rho_L)} + \dfrac{1}{\mu^2(\rho_C)}} = \mu^2(\rho_C)$$

$$\mu(\rho) \cong \mu(\rho_C) \equiv 5.4 \times 10^{-5} \text{ Kg/dm}$$

$$\mu_\rho(\rho) = \mu(\rho)/\rho \simeq 2 \times 10^{-5}$$

$$F = 2.71 \times 10^3 \, \frac{Kg}{\cancel{m^3}} \times (1.00001)^3 \, \cancel{m^3} \times 9.80665 \, m/s^2$$

$$F = 26\,576.8 \, \underbrace{Kg \frac{m}{s^2}}_{N} \approx 26.6 \, KN$$

$$\mu^2(F) = \left[\frac{\partial F}{\partial \rho}\right]^2 \mu^2(\rho) + \left[\frac{\partial F}{\partial L}\right]^2 \mu^2(L) +$$

$$+ \left[\frac{\partial F}{\partial g}\right]^2 \mu^2(g) =$$

$$= \left[L^3 g\right]^2 \mu^2(\rho) + \left[3\rho L^2 g\right]^2 \mu^2(L) +$$

$$\left[\rho L^3\right]^2 \mu^2(g)$$

$$\frac{\mu^2(F)}{F^2} = \frac{\mu^2(\rho)}{\rho^2} + \frac{9\mu^2(l)}{l^2} + \frac{\mu^2(g)}{g^2}$$

$$\mu_r(F) = \sqrt{\mu_r^2(\rho) + 9\mu_r^2(l) + \mu_r^2(g)} \cong$$

$$\cong \sqrt{(2\times10^{-5})^2 + 9(6\times10^{-6})^2 + (3.4\times10^{-5})^2}$$

$$= \sqrt{4+3.24+11.56} \times 10^{-5} =$$

$$= \sqrt{18.8} \times 10^{-5} \cong 4.4 \times 10^{-5}$$

$$u(F) = u_r(F) \times F \cong 1.2\,N$$