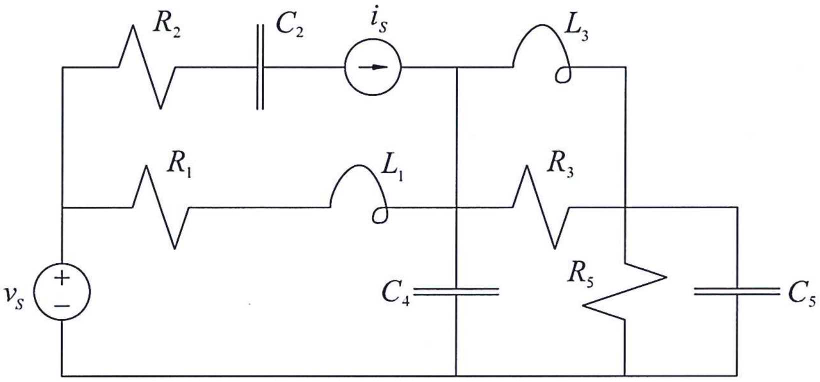
Domanda 1 (9 punti)

Si consideri il circuito in regime alternato sinusoidale alla frequenza $f = 100\,\text{Hz}$ della figura seguente, in cui: $v_S = \sqrt{2}\cdot 100\cos(2\pi f t)\,\text{V}$, $i_S = \sqrt{2}\cdot 5\cos(2\pi f t - \pi/3)\,\text{A}$, $R_1 = 5\,\Omega$, $R_2 = 30\,\Omega$, $R_3 = 20\,\Omega$, $R_5 = 20\,\Omega$, $L_1 = 30\,\text{mH}$, $L_3 = 20\,\text{mH}$, $C_2 = 100\,\mu\text{F}$, $C_4 = 40\,\mu\text{F}$, $C_5 = 50\,\mu\text{F}$.



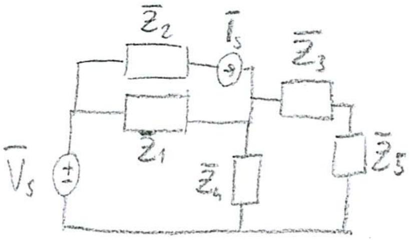
(a) Si ricavi l'equivalente di Thevenin della rete vista dal generatore di tensione v_S;

(b) Si ricavi l'equivalente di Norton della rete vista dal generatore di corrente i_S. $\omega = 200\pi$ rad/s

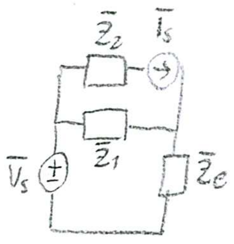
$\bar{V}_S = 100\,V$ $\quad \bar{I}_S = (2,5 - j4,330)\,A$ $\quad \bar{Z}_1 = R_1 + j\omega L_1 = (5 + j18,85)\,\Omega$

$\bar{Z}_2 = R_2 - \dfrac{j}{\omega C_2} = (30 - j15,92)\,\Omega$ $\quad \bar{Z}_3 = \dfrac{j\omega L_3 R_3}{R_3 + j\omega L_3} = (5,661 + j9,010)\,\Omega$

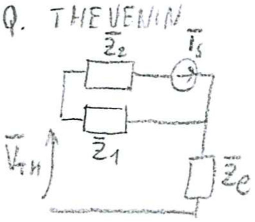
$\bar{Z}_4 = \dfrac{-j}{\omega C_4} = -j39,79\,\Omega$ $\quad \bar{Z}_5 = \dfrac{\dfrac{-jR_5}{\omega C_5}}{R_5 - \dfrac{j}{\omega C_5}} = (14,34 - j9,010)\,\Omega$



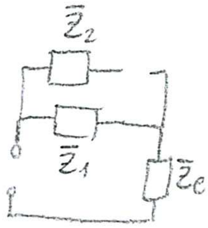
$$\bar{Z}_e = \frac{(\bar{Z}_3 + \bar{Z}_5)\bar{Z}_4}{\bar{Z}_3 + \bar{Z}_4 + \bar{Z}_5} = (15,97 - j8,025)\,\Omega$$



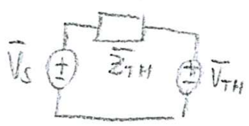
EQ. THEVENIN



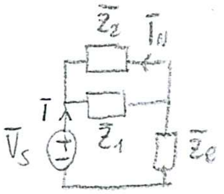
$$\bar{V}_{TH} = -\bar{Z}_1\bar{I}_S = (-94,12 - j25,47)\,V$$



$$\bar{Z}_{TH} = \bar{Z}_1 + \bar{Z}_e = (20,97 + j10,82)\,\Omega$$

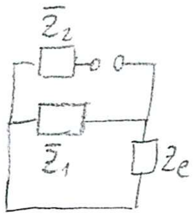


EQ. NORTON

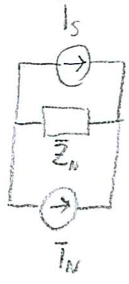


$$\bar{I} = \frac{\bar{V}_S}{\dfrac{\bar{Z}_1\bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} + \bar{Z}_e} = (3,265 - j0,5124)\,A$$

$$\bar{I}_N = -\bar{I}\,\frac{\bar{Z}_1}{\bar{Z}_1 + \bar{Z}_2} = (-0,8776 - j1,612)\,A$$

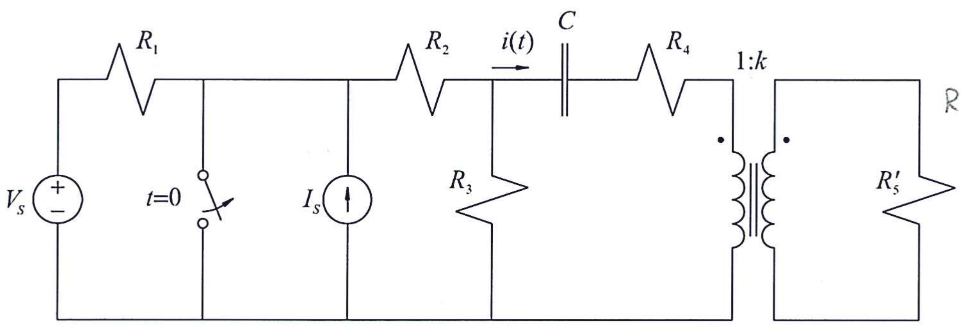


$$\bar{Z}_N = \bar{Z}_2 + \frac{\bar{Z}_1\bar{Z}_e}{\bar{Z}_1 + \bar{Z}_e} = (43,77 - j10,59)\,\Omega$$



Domanda 2 (7 punti)

Sia dato il circuito mostrato nella figura seguente, in cui: $V_S = 80\,\mathrm{V}$, $I_S = 2\,\mathrm{A}$, $R_1 = 5\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 40\,\Omega$, $R_4 = 20\,\Omega$, $R_5' = 100\,\Omega$, $C = 1\,\mathrm{mF}$, $k = 5$. Si consideri il circuito inizialmente in regime stazionario e con l'interruttore chiuso per $t < 0$, mentre in $t = 0$ si verifica la commutazione.

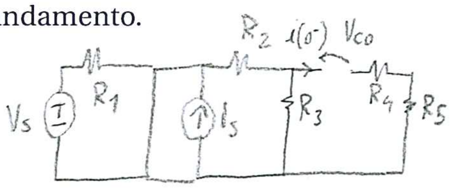


RIPORTO AL PRIMARIO

$$R_5 = \frac{R_5'}{k^2} = 4\,\Omega$$

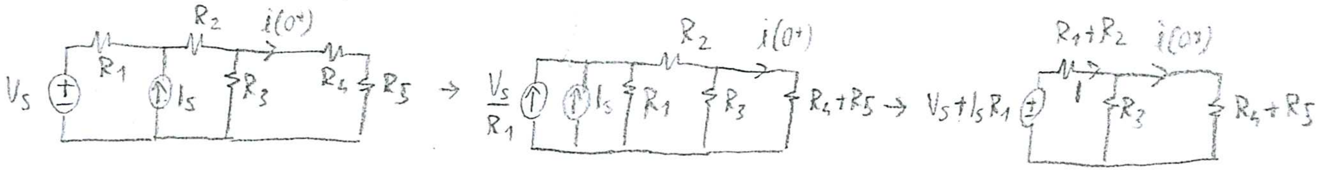
Si determini l'espressione analitica della corrente $i(t)$ a partire da $t < 0$ e si rappresenti graficamente il suo andamento.

$t \to 0^-$



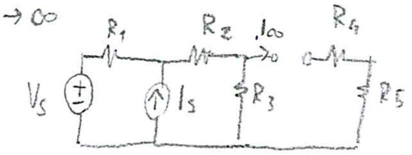
$i(0^-) = 0$ R_3, R_4, R_5 NON SONO ATTRAVERSATE DA CORRENTE $\rightarrow V_{C0} = 0$

$t \to 0^+$, IMPONGO LA CONTINUITÀ DELLA TENSIONE SU C ; $V_{C0} = 0$, LO SOSTITUISCO CON UN CORTO CIRCUITO



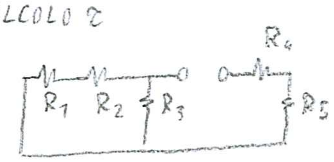
$$I = \frac{V_S + I_S R_1}{R_1 + R_2 + \frac{R_3(R_4+R_5)}{R_3+R_4+R_5}} = 3\,\mathrm{A} \qquad i(0^+) = I\,\frac{R_3}{R_3+R_4+R_5} = 1{,}875\,\mathrm{A}$$

$t \to \infty$



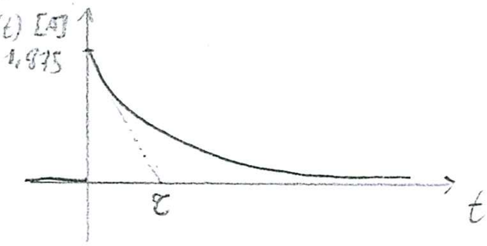
$I_\infty = 0$

CALCOLO τ



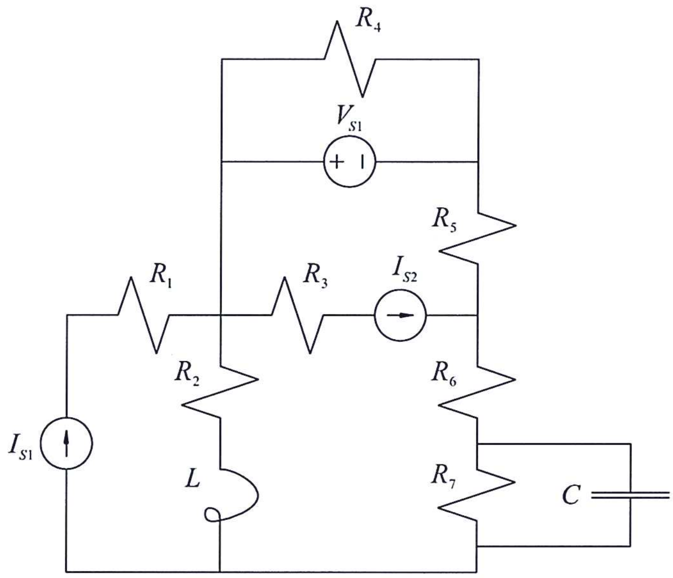
$$R_{eq} = \frac{(R_1+R_2)R_3}{R_1+R_2+R_3} + R_4 + R_5 = 34{,}90\,\Omega \qquad \tau = R_{eq}C = 34{,}9\,\mathrm{ms}$$

$$i(t) = \begin{cases} 0 & t < 0 \\ 1{,}875\, e^{-\frac{t}{34{,}9\,\mathrm{ms}}} & t > 0 \end{cases}$$



Domanda 3 (6 punti)

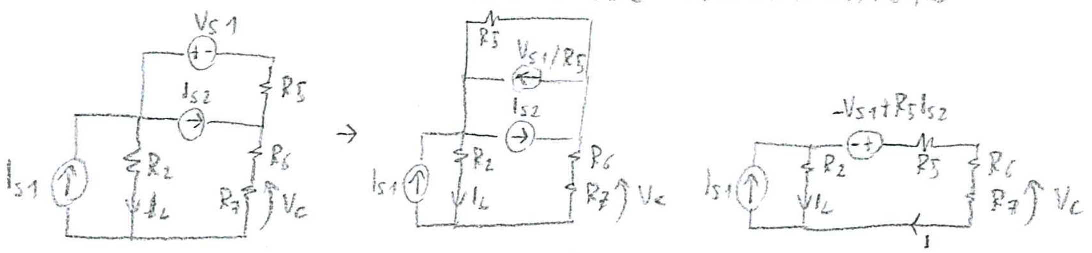
Sia dato il circuito in regime stazionario della figura seguente, in cui: $V_{S1} = 10\,\text{V}$, $I_{S1} = 2\,\text{A}$, $I_{S2} = 4\,\text{A}$, $R_1 = 2\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 1\,\Omega$, $R_4 = 5\,\Omega$, $R_5 = 3\,\Omega$, $R_6 = 2\,\Omega$, $R_7 = 4\,\Omega$, $L = 10\,\text{mH}$, $C = 400\,\mu\text{F}$.

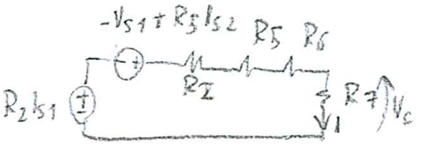


(a) Si calcoli l'energia immagazzinata nell'induttore;

(b) Si calcoli l'energia immagazzinata nel condensatore.

R_1 e R_3 IN SERIE A I_{S1} E I_{S2}; R_4 IN PARALLELO A V_{S1}; NON HANNO EFFETTO AI MORSETTI DELL'INDUTTORE E DEL CONDENSATORE





$I = \dfrac{-V_{S1} + R_2 I_{S1} + R_5 I_{S2}}{R_2 + R_5 + R_6 + R_7} = 1{,}158\,A$ $\quad V_C = R_7 I = 4{,}632\,V$

$W_{COND} = \dfrac{1}{2} C V_C^2 = 4{,}290\,mJ$

$I_L = I_S - I = 0{,}8421\,A$ $\quad W_{IND} = \dfrac{1}{2} L I_L^2 = 3{,}546\,A$