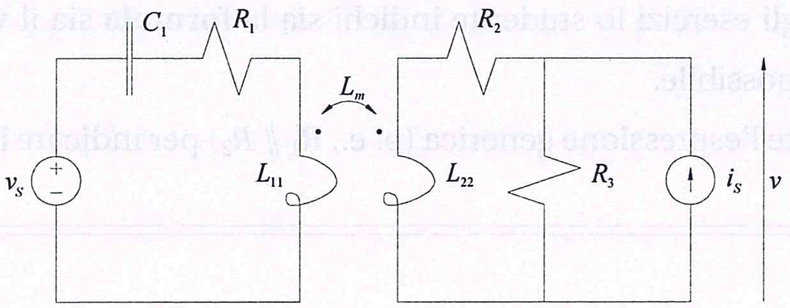
Domanda 1 (10 punti)

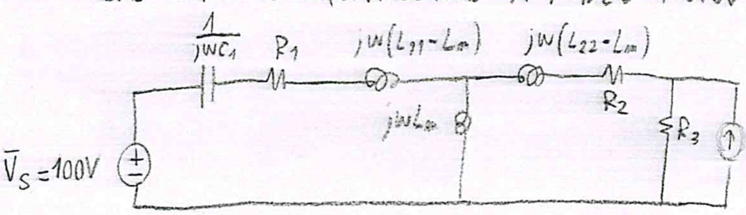
Si consideri il circuito in regime alternato sinusoidale alla frequenza $f = 1000\,\text{Hz}$ della figura seguente; siano: $v_S = \sqrt{2}\cdot 100\cos(2\pi f t)\,\text{V}$, $i_S = \sqrt{2}\cdot 0.3\sin(2\pi f t)\,\text{A}$, $R_1 = 50\,\Omega$, $R_2 = 100\,\Omega$, $R_3 = 500\,\Omega$, $L_{11} = 30\,\text{mH}$, $L_{22} = 50\,\text{mH}$, $L_\text{m} = 25\,\text{mH}$, $C_1 = 1\,\mu\text{F}$.



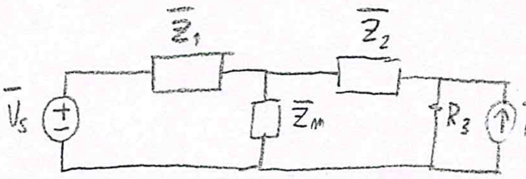
(a) Si calcoli il fasore della tensione v ai capi del generatore di corrente, utilizzando il teorema di Norton;

(b) Si determini l'espressione analitica della potenza istantanea assorbita dal resistore R_2 e si tracci il suo andamento nel tempo.

DAL CIRCUITO EQUIVALENTE A T DEL MUTUO INDUTTORE



$\bar{I}_S = -j0.3\text{A}$ CON $\omega = 2\pi f$

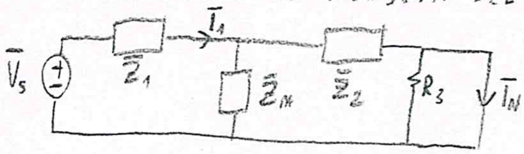


$$\bar{Z}_1 = \frac{1}{j\omega C_1} + R_1 + j\omega(L_{11} - L_m) = (50 - j127{,}7)\,\Omega$$

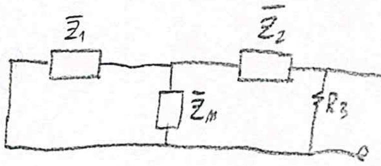
$$\bar{Z}_2 = R_2 + j\omega(L_{22} - L_m) = (100 + j157{,}1)\,\Omega$$

$$\bar{Z}_m = j\omega L_m = j157.1\,\Omega$$

EQUIVALENTE NORTON AI MORSETTI DEL GENERATORE DI CORRENTE

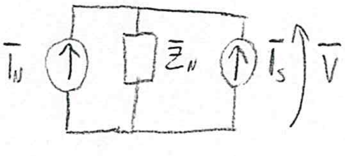


$$\bar{I}_1 = \frac{\bar{V}_S}{\bar{Z}_1 + \dfrac{\bar{Z}_m \bar{Z}_2}{\bar{Z}_m + \bar{Z}_2}} \qquad \bar{I}_N = \bar{I}_1 \frac{\bar{Z}_m}{\bar{Z}_2 + \bar{Z}_m} = (0.3823 + j0.4195)\,\text{A}$$



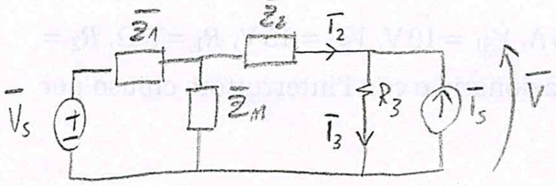
$$\bar{Z}_{eq} = \bar{Z}_2 + \frac{\bar{Z}_1 \bar{Z}_m}{\bar{Z}_1 + \bar{Z}_m} \qquad \bar{Z}_N = \frac{\bar{Z}_{eq} R_3}{\bar{Z}_{eq} + R_3} = (244.2 + j26.13)\,\Omega$$

DUNQUE:



$$\bar{V} = \bar{Z}_N(\bar{I}_N + \bar{I}_S) = (90.22 + j39.16)\,\text{V}$$

SI CALCOLI ORA LA CORRENTE $\bar{I}_2$ IN R_2



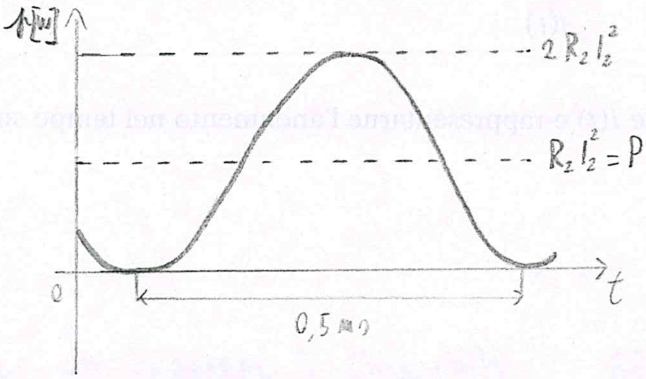
$$\bar{I}_3 = \frac{\bar{V}}{R_3} \qquad \bar{I}_2 = \bar{I}_3 - \bar{I}_S = (0.1804 + j0.3783)\,A =$$

$$= 0.4191\,A \angle 1.126\,[\text{Rad}]$$

$$i_2(t) = \sqrt{2} \cdot I_2 \cos(\omega t + \angle \bar{I}_2)$$

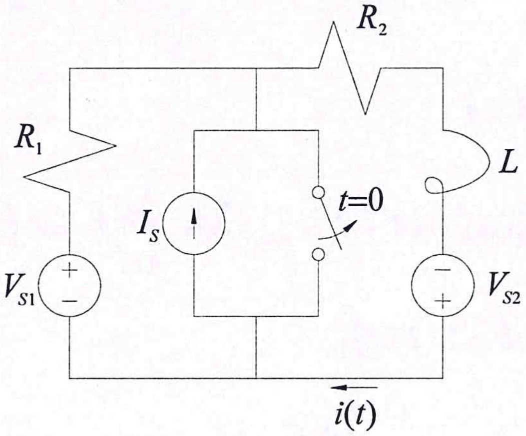
$$p(t) = R_2 i_2^2(t) = 2 I_2^2 R_2 \cos^2(\omega t + \angle \bar{I}_2) = R_2 I_2^2 \left[1 + \cos(2\omega t + 2\angle \bar{I}_2)\right] =$$

$$= 17.57\,W \left[1 + \cos(2\omega t + 2.252)\right]$$



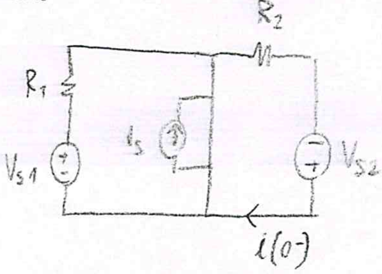
Domanda 2 (6 punti)

Sia dato il circuito mostrato nella figura seguente, in cui: $I_S = 5\,\text{A}$, $V_{S1} = 10\,\text{V}$, $V_{S2} = 15\,\text{V}$, $R_1 = 5\,\Omega$, $R_2 = 3\,\Omega$, $L = 5\,\text{mH}$. Si consideri il circuito inizialmente in regime stazionario e con l'interruttore chiuso per $t < 0$, mentre in $t = 0$ si verifica l'apertura dell'interruttore.



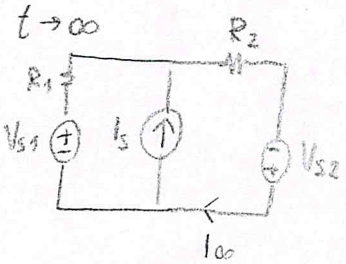
Determinare l'espressione analitica della corrente $i(t)$ e rappresentarne l'andamento nel tempo su un grafico a partire da $t < 0$.

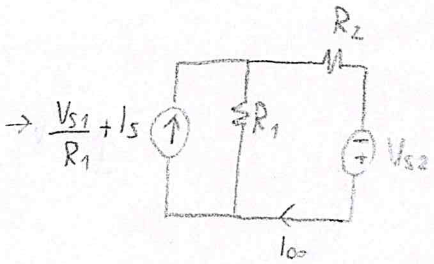
$t=0^-$



$i(0^-) = \frac{V_{S2}}{R_2} = 5\,A$ $\quad i(0^+) = i(0^-)$ È VARIABILE DI STATO

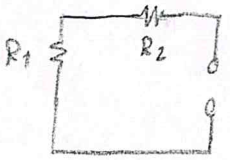
$t \to \infty$





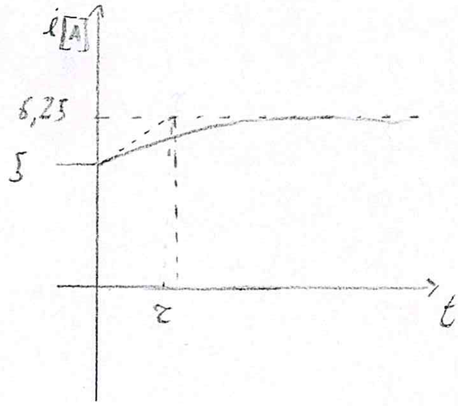
$$I_\infty = \left(\frac{V_{S1}}{R_1} + I_S\right)\frac{R_1}{R_1+R_2} + \frac{V_{S2}}{R_1+R_2} = 6{,}25\,A$$

CALCOLO DI τ



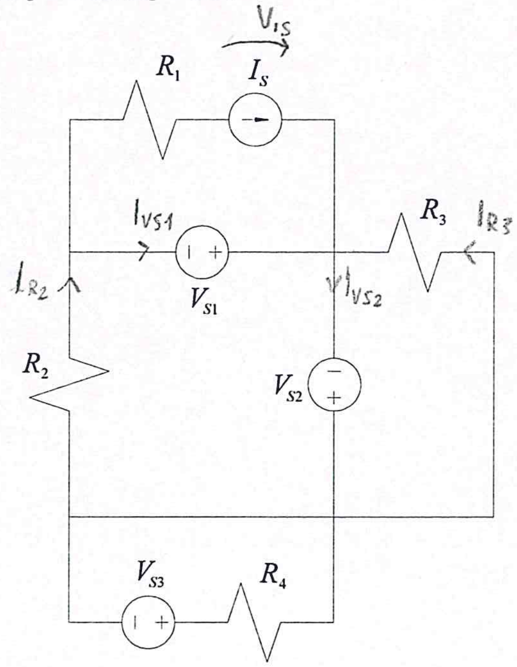
$R_{eq} = R_1 + R_2$ $\qquad \tau = \frac{L}{R_{eq}} = 0{,}625\,ms$

$$i(t) = \left(i(0) - I_\infty\right)e^{-\frac{t}{\tau}} + I_\infty = \left(-1{,}25\,e^{-\frac{t}{\tau}} + 6{,}25\right)A$$



Domanda 3 (6 punti)

Sia dato il circuito in regime stazionario della figura seguente in cui: $I_S = 5\,\text{A}$, $V_{S1} = 10\,\text{V}$, $V_{S2} = 15\,\text{V}$, $V_{S3} = 18\,\text{V}$, $R_1 = 5\,\Omega$, $R_2 = 3\,\Omega$, $R_3 = 8\,\Omega$, $R_4 = 6\,\Omega$.



Si calcoli la potenza erogata da ogni generatore.

$P_{VS_3} = \dfrac{V_{S3}^2}{R_4} = 54\,W$ $\quad I_{R2} = \dfrac{V_{S1}+V_{S2}}{R_2} = 8,333\,A$ $\quad I_{VS1} = I_{R2} - I_S = 3,333\,A$

$P_{VS1} = V_{S1} I_{VS1} = 33,33\,W$ $\quad V_{IS} = R_1 I_S + V_{S1} = 35\,V$

$P_{IS} = V_{IS} I_S = 175\,W$ $\quad I_{R2} = I_{VS2} - I_{R3}$ $\quad I_{VS2} = I_{R2} + \dfrac{V_{S2}}{R_3} = 10,21\,A$

$P_{VS2} = V_{S2} I_{VS2} = 153,1\,W$