Domanda 1 (5 punti)

Sia dato il circuito di Fig. 1, funzionante in regime alternato sinusoidale alla frequenza di 50Hz, in cui: $R_L = 50\Omega$, $R_2 = 5\Omega$, $X_L = 30\Omega$, $X_2 = 1\Omega$. Sia inoltre nota la corrente $I_L = 5$A rms assorbita dal carico (serie del resistore R_L e dell'induttore X_L). (rms: valore efficace)

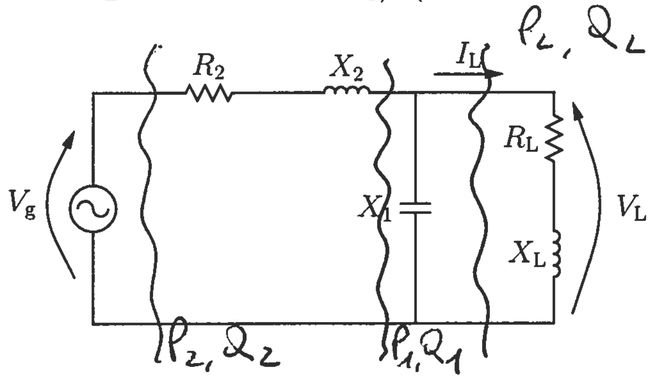


Figura 1: Circuito in regime alternato sinusoidale della Domanda 1.

(a) Si calcoli il valore della capacità C del condensatore tale da dimezzare la potenza reattiva assorbita dal carico.

(b) Si calcoli la tensione V_g del generatore.

$$P_L = R_L I_L^2 = 1{,}25\ \text{kW}$$

$$Q_L = X_L I_L^2 = 750\ \text{VAr}$$

$$V_L I_L = \sqrt{P_L^2 + Q_L^2}$$

$$V_L = \frac{\sqrt{P_L^2 + Q_L^2}}{I_L} = 231{,}55\ \text{V}$$

$$P_1 = P_L$$

$$Q_1 = Q_L + Q_{X_1} = X_L I_L^2 + \frac{V_L^2}{X_1}$$

$$Q_1 = \frac{1}{2} X_L I_L^2$$

$$-\frac{1}{2} X_L I_L^2 = \frac{V_L^2}{X_1} = -\omega C V_L^2$$

$$C = \frac{X_L I_L^2}{2\omega V_L^2} = 14{,}04\ \mu\text{F}$$

$$V_L I = \sqrt{P_1^2 + Q_1^2}$$

$$I = \frac{\sqrt{P_1^2 + Q_1^2}}{V_L} = 4{,}48\ \text{A}$$

$$P_2 = P_L + R_2 I^2 = 1{,}35\ \text{kW}$$

$$Q_2 = Q_1 + X_2 I^2 = 395{,}04\ \text{VAr}$$

$$V_g I = \sqrt{P_2^2 + Q_2^2}$$

$$V_g = \frac{\sqrt{P_2^2 + Q_2^2}}{I} = 314{,}28\ \text{V}$$

FASORI

$$\dot{I}_L = I_L$$

$$\dot{V}_L = Z_L I_L = (R_L + jX_L) I_L$$

$$\dot{I}_1 = j\omega C \dot{V}_L$$

$$A_1 = \dot{V}_2 \dot{I}_2^* = \dot{V}_L (\dot{I}_1 + I_L)^*$$
$$= \dot{V}_L (j\omega C \dot{V}_L + I_L)^*$$
$$= \dot{V}_L (-j\omega C \dot{V}_L^* + I_L)$$

$$\mathrm{Im}\{A_1\} = \frac{Q_L}{2}$$

$$\frac{Q_L}{2} = -\omega C \dot{V}_L^2 + Q_L$$

$$\frac{Q_L}{2} = \omega C \dot{V}_L^2 \Rightarrow C = \frac{Q_L}{2\omega \dot{V}_L^2}$$

$$= \frac{X_L \cancel{I_L^2}}{2\omega (R_L^2 + X_L^2) \cancel{I_L^2}} = 14{,}04\,\mu F$$

$$Z_2 = R_2 + jX_2$$

$$\dot{V}_g = Z_2 (j\omega C \dot{V}_L + I_L) + \dot{V}_L$$

$$|\dot{V}_g| = I_L \sqrt{(R_L + R_2(1-\omega C X_L) - \omega C R_L X_2)^2 + (X_L + (1-\omega C X_L) X_2 + \omega C R_L R_2)^2}$$

$$= 314{,}28\,V$$

Domanda 2 (6 punti)

Sia dato il circuito in regime transitorio di Fig. 2 in cui: $V = 12\text{V}$, $R_1 = 1\Omega$, $R_2 = 5\Omega$, $R_3 = 1\Omega$, $R_4 = 0.01\Omega$, $L = 1\text{mH}$. Si consideri il circuito a regime per $t < 0$.

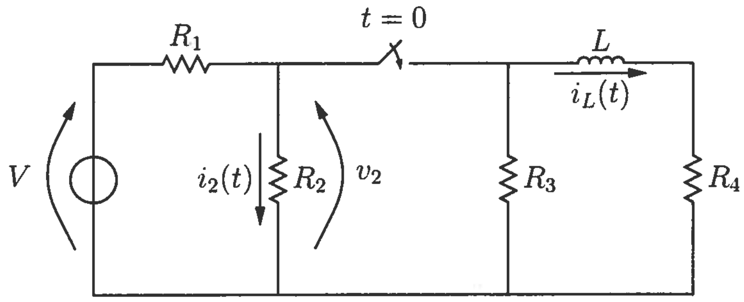


Figura 2: Circuito in regime transitorio della Domanda 2.

Si determini l'espressione analitica della corrente $i_2(t)$ ai capi del resistore R_2 e se ne fornisca la rappresentazione grafica.

$t = 0^-$

$$v_2 = \frac{R_2 V}{R_1 + R_2} \; ; \; i_2 = \frac{V}{R_1 + R_2} = 2A$$

$i_L(0^-) = 0\,A$

$t = 0^+$

$i_L(0^+) = i_L(0^-) = 0A$

$$v_2 = \frac{R_2 R_3 V}{R_2 R_3 + R_1 (R_2 + R_3)}$$

$$i_2(0^+) = \frac{v_2}{R_2} = 1{,}09\,A$$

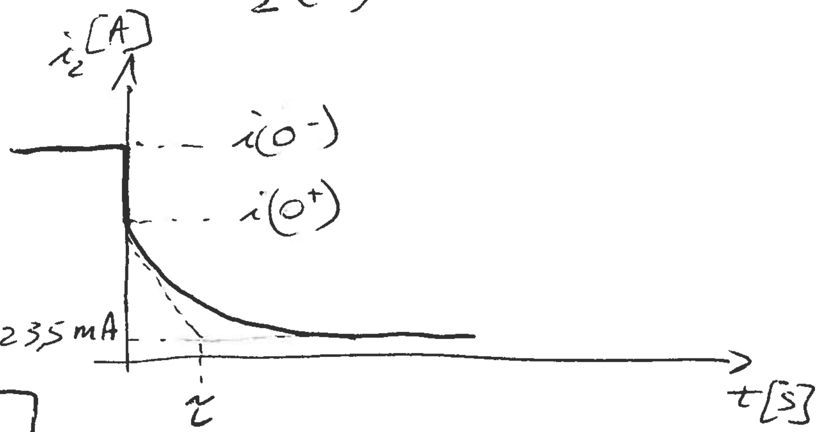
$$R_{eq} = R_1 // R_2 // R_3 + R_4 = \frac{R_1 R_2 R_3}{R_2 R_3 + R_1 R_3 + R_1 R_2} + R_4 = 9{,}46\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = 2{,}2\,ms$$

$t \to \infty$

$$i_2(\infty) = \frac{R_3 R_4 V}{R_2 R_3 R_4 + R_1 (R_3 R_4 + R_2 R_4 + R_2 R_3)} = 23{,}5\,mA$$

$$i_2(t) = [i_2(0^+) - i_2(\infty)] \exp\left(-\frac{t}{\tau}\right) + i_2(\infty)$$

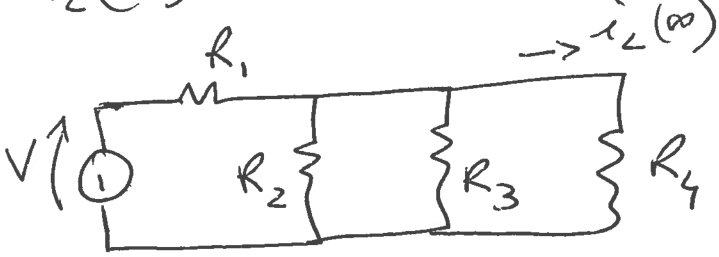


SOVRAPPOSIZIONE DEGLI EFFETTI

$i_L(t)$

$i_L(0^-) = i_L(0^+) = 0\,A$

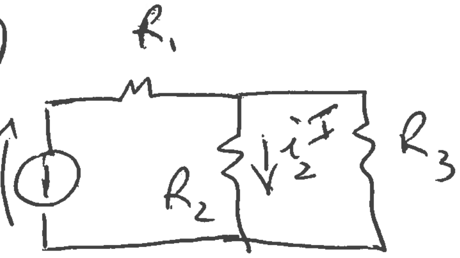
$i_L(\infty)$



$$i_L(\infty) = V \frac{R_2 // R_3 // R_4}{R_2 // R_3 // R_4 + R_1} \frac{1}{R_4} = 11{,}74\,A$$

$$i_L(t) = -i_L(\infty)\exp\left(-\frac{t}{\tau}\right) + i_L(\infty) = i_L(\infty)\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$$

I

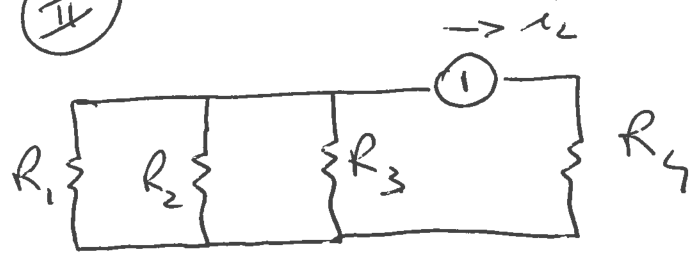


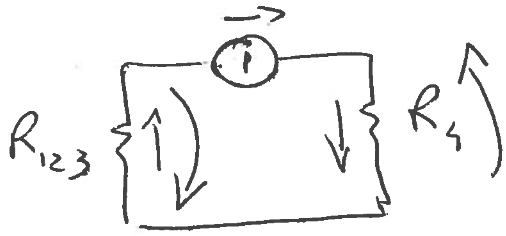
$$i_2^{I} = V \frac{R_2 // R_3}{R_2 // R_3 + R_1} \frac{1}{R_2}$$

$$= \frac{V R_3}{R_2 R_3 + R_1 (R_2 + R_3)}$$

$$= 1{,}09\,A$$

II





$$R_{123} = R_1 // R_2 // R_3$$

$$i_2^{II} = -\frac{R_{123}}{R_2} i_L$$

$$= -1{,}0673\left[1 - \exp\left(-\frac{t}{\tau}\right)\right]$$

$$i_2 = i_2^{I} + i_2^{II} = 1{,}0673\exp\left(-\frac{t}{\tau}\right) + 0{,}0235\,A$$

$i_2(0^-)$ come calcolato prima $= \dfrac{V}{R_1 + R_2} = 2\,A$

Domanda 3 (4 punti)

Sia dato il circuito di Fig. 3 in cui: $I = 5\text{A}$, $R_1 = 2\Omega$, $R_2 = 4\Omega$, $R_3 = 6\Omega$.

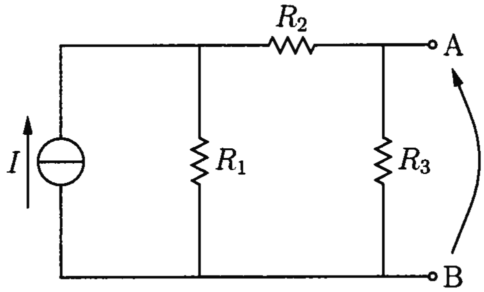
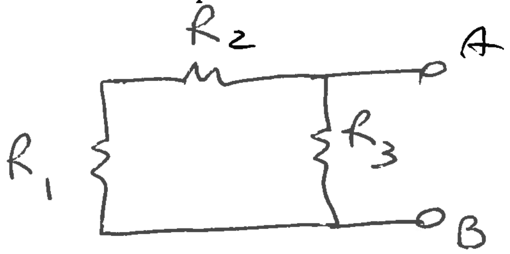


Figura 3: Circuito della Domanda 3.

Si determini il circuito equivalente di Thévenin ai morsetti A, B.

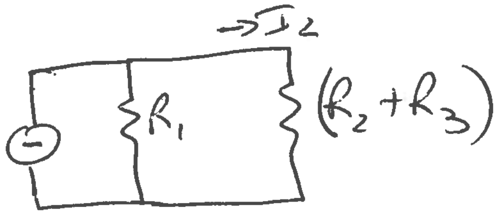
$\cancel{R_{eq}}\ R_{TH}$



$$R_{TH} = (R_1 + R_2) \parallel R_3$$

$$= \frac{R_3 (R_1 + R_2)}{R_1 + R_2 + R_3} = 3\Omega$$

V_{TH}

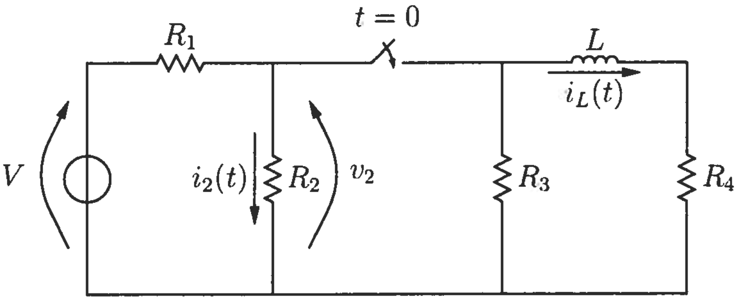


$$I_2 = \frac{R_1}{R_1 + R_2 + R_3} I$$

$$V_{TH} = R_3 I_2 = \frac{R_1 R_3}{R_1 + R_2 + R_3} I = 5V$$

Domanda 7 (5 punti)

Facendo riferimento al circuito della Domanda 2, riportato di seguito per comodità, si calcoli la potenza istantanea dissipata dal resistore R_2 in $t =$ 1ms.



$t = 0^-$

$$v_2(0^-) = i_2(0^-) R_2 = 10\,V$$

$t = 0^+$

$$v_2(0^+) = i_2(0^+) R_2 = 5{,}45\,V$$

$t > 0^+$

$$v_2(t) = R_2 \, i_2(t)$$

$$= (5{,}45 - 0{,}117) \exp\left(-\frac{t}{\tau}\right) + 0{,}117$$

$t \to \infty$

$$v_2(\infty) = R_2 \, i_2(\infty) = 0{,}117\,V$$

$$p(t)\Big|_{t=1ms} = v_2(1ms)\, i_2(1ms)$$

$$= \left[5{,}337 \exp\left(-\frac{0{,}001}{\tau}\right) + 0{,}117\right] \left[\underset{1{,}0674}{\cancel{1{,}0909}} \exp\left(-\frac{0{,}001}{\tau}\right) + 0{,}0235\right] W$$

$$= 2{,}41\ W$$