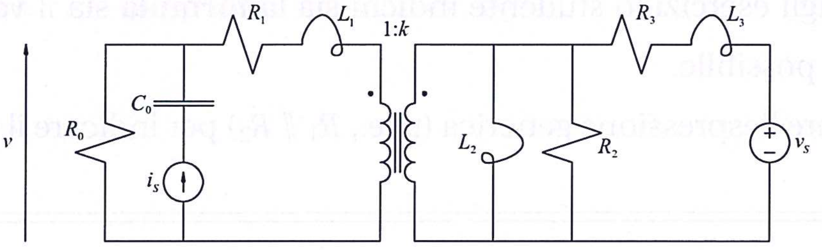
Domanda 1 (10 punti)

Si consideri il circuito in regime alternato sinusoidale alla frequenza $f = 1000$ Hz mostrato nella figura seguente, in cui: $v_S = \sqrt{2}\cdot 50\sin(2\pi ft)$ V, $i_S = \sqrt{2}\cdot 3\cos(2\pi ft)$ A, $R_0 = 6\,\Omega$, $R_1 = 3\,\Omega$, $R_2 = 40\,\Omega$, $R_3 = 20\,\Omega$, $L_1 = 300\,\mu$H, $L_2 = 5$ mH, $L_3 = 2$ mH, $C_0 = 40\,\mu$F, $k = 5$.

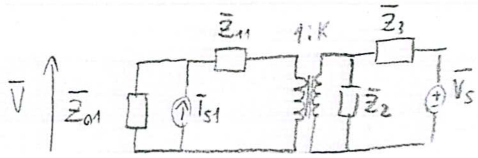


(a) Si calcolino potenza attiva e reattiva erogate dal generatore di tensione v_S;

(b) Si tracci su un grafico l'andamento nel tempo della tensione v.

$\omega = 2000\pi$ rad/s $\quad \bar I_{S1} = 3$ A $\quad \bar V_S = -j50$ V $\quad \bar Z_{11} = R_1 + j\omega L_1 = (3+j1{,}885)\,\Omega \quad \bar Z_2 = \dfrac{j\omega L_2 R_2}{R_2 + j\omega L_2} = (15{,}26+j19{,}43)\,\Omega$

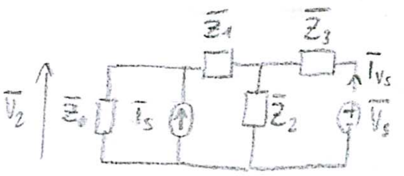
$\bar Z_3 = R_3 + j\omega L_3 = (20+j12{,}57)\,\Omega \quad \bar Z_{01} = R_0 = 6\,\Omega$ $\quad C_0$ PUÒ ESSERE SOSTITUITO CON UN CORTO CIRCUITO



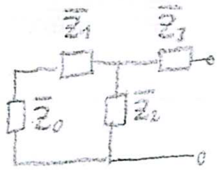
RIPORTO TUTTO AL SECONDARIO

$\bar Z_0 = k^2 \bar Z_{01} = 150\,\Omega \quad \bar I_S = \dfrac{\bar I_{S1}}{k} = 0{,}6$ A $\quad \bar Z_1 = k^2 \bar Z_{11} = (75+j47{,}12)\,\Omega$

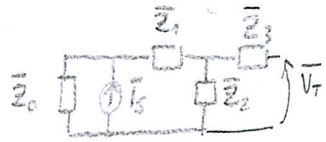
$\bar V_2 = k\bar V_1$

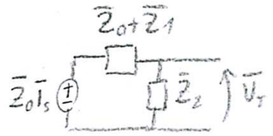


EQ. THEVENIN AI MORSETTI DI $\bar V_S$

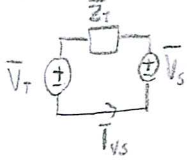


$$\bar Z_T = \frac{(\bar Z_0+\bar Z_1)\bar Z_2}{\bar Z_0+\bar Z_1+\bar Z_2} + \bar Z_3 = (35{,}18+j29{,}55)\,\Omega$$



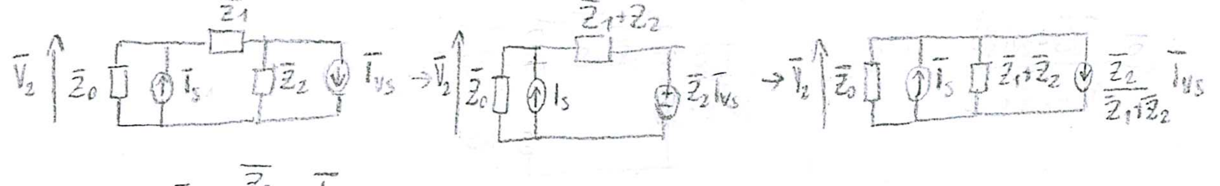


$$\bar V_T = \bar Z_0 \bar I_S \frac{\bar Z_2}{\bar Z_0+\bar Z_1+\bar Z_2} = (7{,}182+j5{,}289)\text{ V}$$



$$\bar I_{VS} = \frac{\bar V_S - \bar V_T}{\bar Z_T} = (-0{,}8936 - j0{,}8209)\text{ A} \qquad \bar S = \bar V_S \bar I_{VS}^* = 41{,}27\text{ W} + j32{,}16\text{ VAr}$$

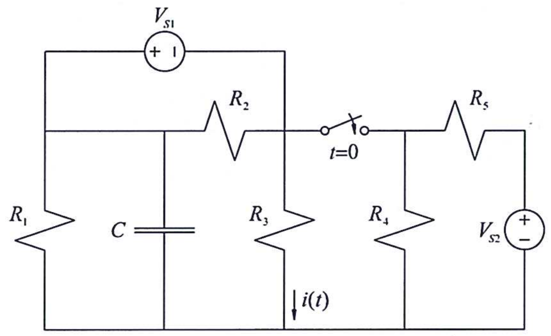
UTILIZZANDO IL TEOREMA DI SOSTITUZIONE:



$$\bar V_2 = \frac{\bar I_S - \dfrac{\bar Z_2}{\bar Z_1+\bar Z_2}\bar I_{VS}}{\dfrac{1}{\bar Z_0}+\dfrac{1}{\bar Z_1+\bar Z_2}} = (41{,}27+j32{,}16)\text{ V} \qquad \bar V = \frac{\bar V_2}{k} = (8{,}255+j6{,}432)\text{ V}$$

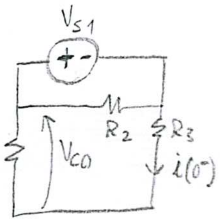
Domanda 2 (6 punti)

Sia dato il circuito mostrato nella figura seguente, in cui: $V_{S1} = 20\,\mathrm{V}$, $V_{S2} = 50\,\mathrm{V}$, $R_1 = 30\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 8\,\Omega$, $R_4 = 15\,\Omega$, $R_5 = 7\,\Omega$, $C = 1\,\mathrm{mF}$. Si consideri il circuito inizialmente in regime stazionario e con l'interruttore aperto per $t < 0$, mentre in $t = 0$ si verifica la commutazione.



Si determini l'espressione analitica della corrente $i(t)$ a partire da $t < 0$ e si rappresenti graficamente il suo andamento.

$t \to 0^-$

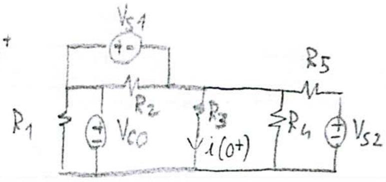


$$V_{c0} = V_{S1} \frac{R_1}{R_1 + R_3} = 15{,}79\,V$$

SI IMPONE LA CONTINUITÀ DELLA V.D.S.

$$i(0^-) = \frac{-V_{S1}}{R_1 + R_3} = -0{,}5263\,A$$

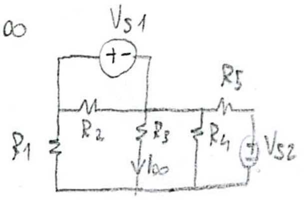
$t \to 0^+$

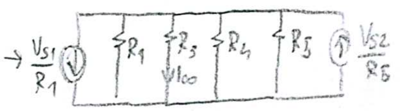


$$i(0^+) = \frac{V_{c0} - V_{S1}}{R_3} = -0{,}5263\,A$$

ANCHE $i(t)$ È CONTINUA

$t \to \infty$



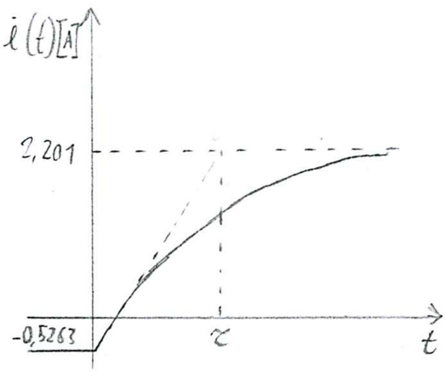


$$I_\infty = \left(\frac{V_{S1}}{R_1} + \frac{V_{S2}}{R_5}\right) \frac{\frac{1}{R_3}}{\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} = 2{,}201\,A$$

CALCOLO τ ; RETE PASSIVA

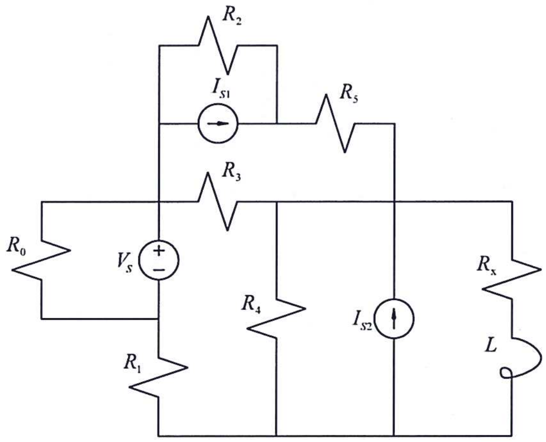
$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}} = 2{,}718\,\Omega \qquad \tau = R_{eq} C = 2{,}718\,ms$$

$$i(t) = \left(i(0^+) - I_\infty\right) e^{-t/\tau} + I_\infty = \left(-2{,}727\, e^{-t/\tau} + 2{,}201\right) A$$



Domanda 3 (6 punti)

Sia dato il circuito in regime stazionario della figura seguente, in cui: $V_S = 100\,\mathrm{V}$, $I_{S1} = 10\,\mathrm{A}$, $I_{S2} = 8\,\mathrm{A}$, $R_0 = 30\,\Omega$, $R_1 = 20\,\Omega$, $R_2 = 5\,\Omega$, $R_3 = 7\,\Omega$, $R_4 = 10\,\Omega$, $R_5 = 3\,\Omega$, $L = 0.2\,\mathrm{H}$.

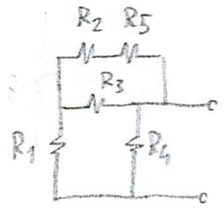


(a) Si calcoli il valore del resistore R_x che massimizza il trasferimento di potenza dal resto della rete;

(b) In tali condizioni, si calcoli l'energia immagazzinata nell'induttore.

OCCORRE RICAVARE LA RESISTENZA EQUIVALENTE DELLA RETE VISTA DA R_X

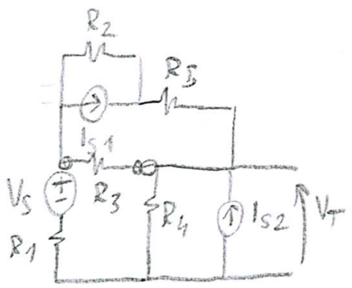
CALCOLO DI R_T



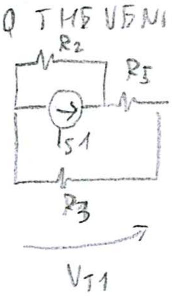
$$R_e = \frac{(R_2+R_5)R_3}{R_2+R_3+R_5} = 3{,}733\,\Omega \qquad R_T = \frac{\frac{R_1 R_e}{R_1+R_e} R_4}{\frac{R_1 R_e}{R_1+R_e} + R_4} = 7{,}036$$

CONDIZIONE DI MASSIMO TRASFERIMENTO DI POTENZA: $R_X = R_T$

PER TROVARE LA CORRENTE, COMPLETO IL CALCOLO DELL'EQUIVALENTE THEVENIN DELLA RETE VISTA DA R_X

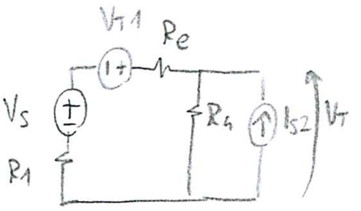


EQ THEVENIN DELLA RETE AI MORSETTI AB : $R_{T1} = R_e$



$$V_{T1} = I_{S1} \frac{R_2 R_3}{R_2+R_3+R_5} = 23{,}33\,\mathrm{V}$$

SOSTITUENDO:



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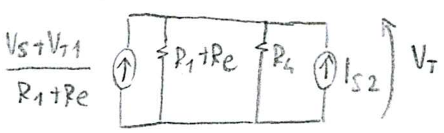
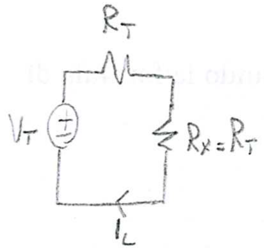


$$V_T = \frac{\frac{V_S+V_{T1}}{R_1+R_e} + I_{S2}}{\frac{1}{R_1+R_e} + \frac{1}{R_4}} = 92{,}85\,\mathrm{V}$$



$I_L = \frac{V_T}{2R_T} = 6,598\,A$ $\quad W = \frac{1}{2} L I_L^2 = 4,354\,J$