NOME: COGNOME: MATRICOLA:

POLITECNICO DI MILANO

Insegnamento di ELETTROTECNICA - Ing. Fisica - Prof. L. Di Rienzo
Prova d'esame del 20 novembre 2009

Esercizio 1 (8 punti)**:** Nel circuito in regime transitorio di Fig. 1 determinare l'andamento analitico e grafico della tensione v(t) e della corrente $i(t)$. Dati: V_{S1}= 40 V; V_{S2}= 10 V; I_{S1}= 2 A; R_1 = 4 Ω; R_2 = 2 Ω; R_3 = 6 Ω; L = 5 H.

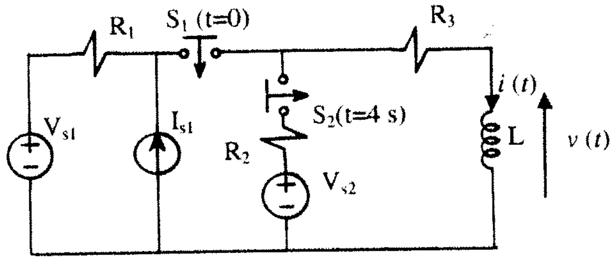
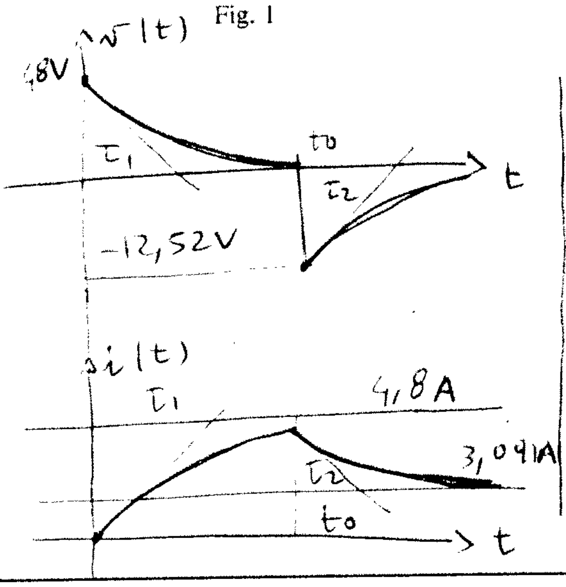


Fig. 1



$0 < t < t_0 = 4\,s$

$i(0^-) = 0, \quad v(0^-) = 0$

$i(\infty) = \left(\frac{V_s}{R_1} + I_{s1}\right)\frac{R_1}{R_1+R_3} = 4,8\,A$

$\tau_1 = \frac{L}{R_1+R_3} = \frac{1}{2}\,s \qquad i(t) = -i(\infty)e^{-t/\tau_1} + i(\infty)$

$v(t) = L\frac{di}{dt} = R\,i(\infty)e^{-t/\tau_1} \qquad i(t_0^-) = 4,798\,A$

$v(t_0^-) = 4,661 \cdot 10^{-3}\,V$

$t_0 < t < +\infty$

$i(\infty) = \left(\frac{V_{s1}}{R_1} + I_{s1} + \frac{V_{s2}}{R_2}\right) \cdot \frac{R_1/\!/R_2}{R_1/\!/R_2 + R_3} = 3,091\,A$

$i(t) = [i(t_0) - i(\infty)]e^{-\frac{t-t_0}{\tau_2}} + i(\infty)$

$v(t) = L\frac{di}{dt} = -R_{eq_2}[i(t_0) - i(\infty)]e^{-\frac{t-t_0}{\tau_2}}$

$\tau_2 = \frac{L}{R_{eq_2}} = 0,6825 \qquad R_{eq_2} = R_1/\!/R_2 + R_3$

$v(t_0^+) = -12,52\,V$

Esercizio 2 (8 punti): Nel circuito di Fig. 2, funzionante in regime sinusoidale, calcolare la potenza attiva e reattiva erogate dal generatore di corrente Is_1.

Dati: $\bar{V}_{s1} = 5\,V$; $\bar{I}s_1 = 10e^{j\frac{\pi}{3}}\,A$; R_1 =3 Ω; R_2 =6 Ω; R_3 = 10 Ω; X_{L1} = 4 Ω; X_C = -5 Ω.

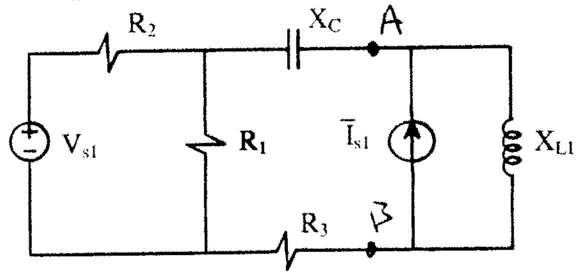
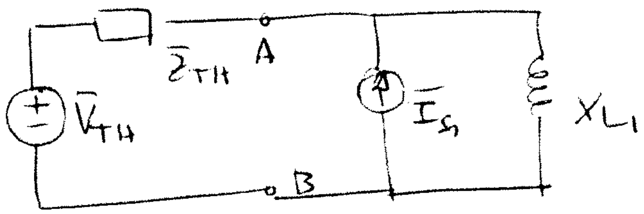


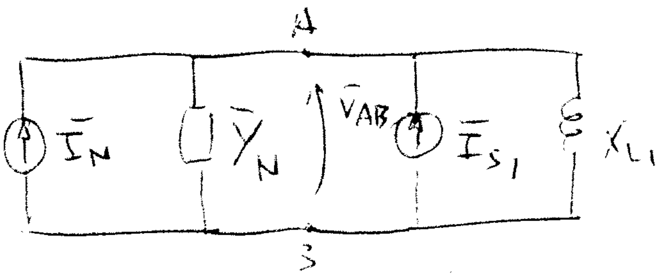
Fig. 2



$\bar{Z}_{TH} = R_1/\!/R_2 + R_3 + jX_c = 12 - j5\,\Omega$

$\bar{V}_{TH} = V_{s1}\frac{R_1}{R_1+R_2} = 1,\bar{6}\,V$

$\bar{I}_N = \frac{\bar{V}_{TH}}{\bar{Z}_{TH}}, \qquad \bar{Y}_N = \frac{1}{\bar{Z}_{TH}}$



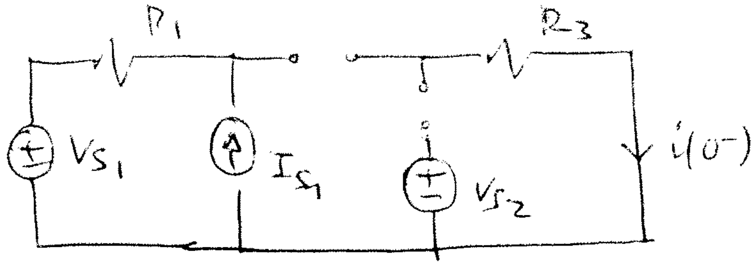
$\bar{V}_{AB} = \frac{\bar{I}_{s1} + \bar{I}_N}{\bar{Y}_N + \frac{1}{jX_{L1}}} = -29,02 + j32,57$

$\bar{S}_{I_{s1}} = \bar{V}_{AB} \cdot \bar{I}_{s1}^* = 136,9 + j414,2$

transitorio:

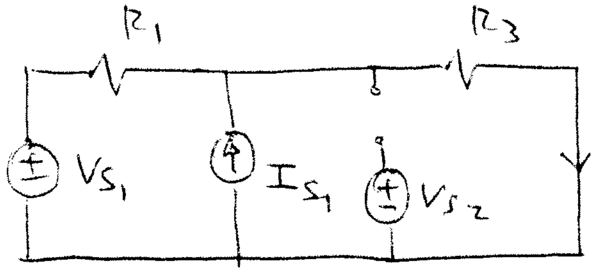
$0 < t < t_0 = 4\,s$

$t = 0^-$]



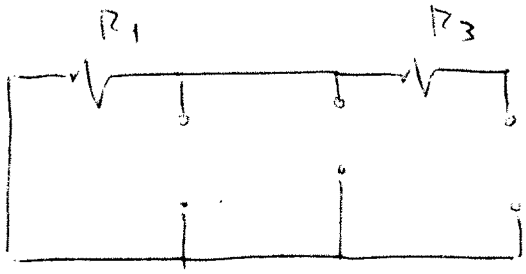
$i(0^-) = 0$

$t \to +\infty$]



$$i(\infty) = \left(\frac{V_s}{R_1} + I_{S_1}\right)\frac{R_1}{R_1+R_3} = 4{,}8\,A$$

τ]



$\leftarrow R_{eq_1} = R_1 + R_3 = 10\,\Omega$

$$\tau_1 = \frac{L}{R_{eq_1}} = \frac{1}{2}\,s$$

$$i(t) = -i(\infty)e^{-t/\tau_1} + i(\infty)$$

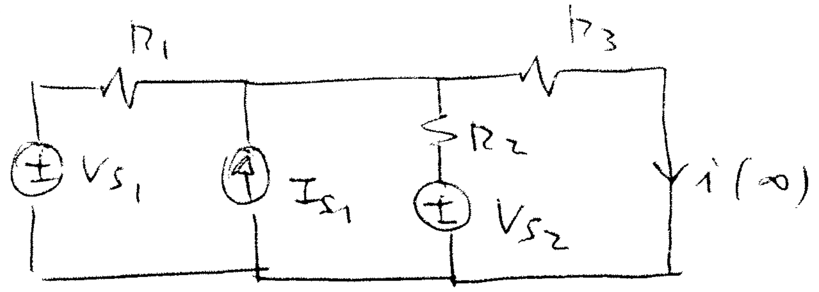
$$v(t) = L\frac{di(t)}{dt} = R\,i(\infty)e^{-t/\tau_1} = 48\,e^{-t/\tau_1}$$

$$i(t_0^-) = -i(\infty)e^{-t_0/\tau_1} + i(\infty) = 4{,}798\,A$$

$$v(t_0^-) = R\,i(\infty)\,e^{-t_0/\tau_1} = 9{,}661\cdot 10^{-3}\,V$$

do transitorio

$t \to +\infty$



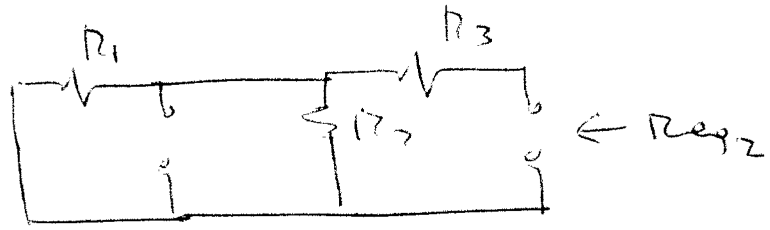
$$i(\infty) = \left(\frac{V_{S1}}{R_1} + I_{S1} + \frac{V_{S2}}{R_2} \right) \frac{R_{12}}{R_{12}+R_3} = 3,091 \text{ A}$$

$$i(t) = [i(t_0) - i(\infty)] e^{-\frac{t-t_0}{\tau}} + i(\infty) =$$

$$v(t) = L \frac{di}{dt} = -R_{eq_2} [i(t_0) - i(\infty)] e^{-\frac{t-t_0}{\tau}}$$

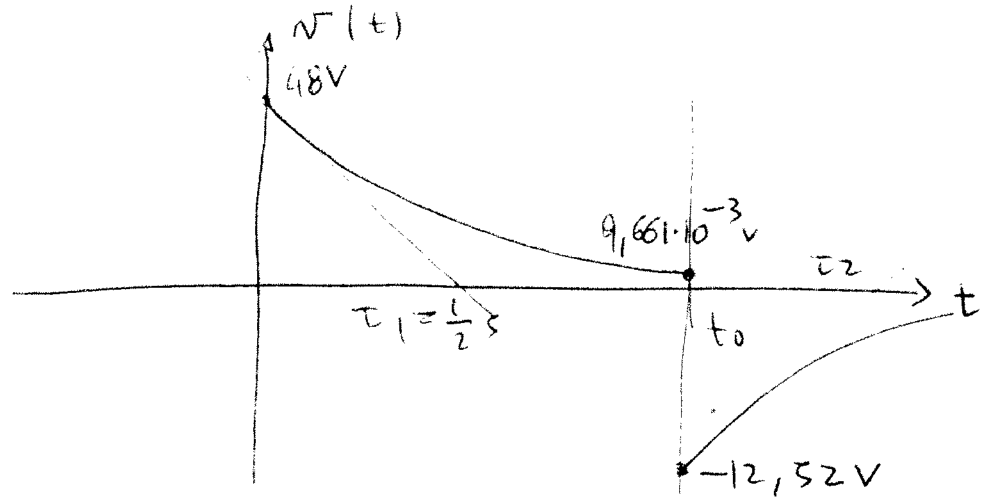
$$v(t_0^+) = -R_{eq_2} [i(t_0) - i(\infty)] = -12,52 \text{ V}$$

$$\tau_2 = \frac{L}{R_{eq_2}}$$

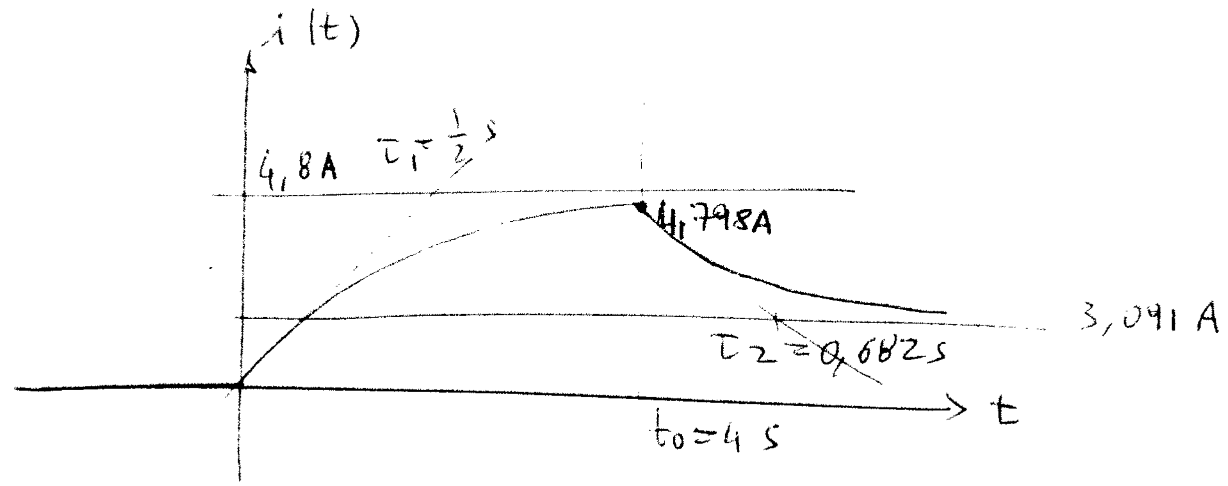


$$R_{eq_2} = R_1 // R_2 + R_3 = 7,\bar{3} \ \Omega$$

$$\tau_2 = \frac{L}{R_{eq_2}} = 0,682 \text{ s}$$



v(t)
48V
9,661·10^-3 V
τ2
t
τ1 = 1/2 s
t0
-12,52 V



i(t)
4,8 A
τ1 = 1/2 s
4,798A
3,091 A
τ2 = 0,682 s
t
t0 = 4 s